Does competitive pressure foster innovation? In addressing this important question, prior studies ignored a distinction between discrete innovation aiming at entirely new technology and continuous improvement consisting of numerous incremental improvements and modifications made upon the existing technology. This paper shows that distinguishing between these two types of innovation will lead to a much richer understanding of the interplay between firm’s incentives to innovate and competitive pressure. In particular, our model predicts that, in contrast to previous theoretical findings, an increase in competitive pressure measured by product substitutability may decrease firms’ incentives to conduct continuous improvement, and that an increase in the size of discrete innovation may decrease firms’ incentives to conduct continuous improvement.

A unique feature of this paper is its exploration of the model’s real-world relevance and usefulness through field research. Motivated by recent declines in levels of continuous improvement in Japanese manufacturing, we conducted extensive field research at two Japanese manufacturing firms. After presenting our findings, we demonstrate that our model guides us to focus on several key changes taking place at these two firms; discover their interconnectedness; and finally ascertain powerful underlying forces behind each firm’s decision to weaken its investment in traditional continuous improvement activities.

Keywords: Competitive pressure, continuous improvement, discrete innovation, field research, location model, product substitutability, small group activities, technical progress.

JEL classification numbers: L10, L60, M50, O30
1 Introduction

Technical progress consists of innovative activities aiming at entirely new products and processes, and numerous minor improvements made upon the existing technology (see, for example, Kuznets, 1962; Rosenberg, 1982). In this paper, we label the former type of technical progress as *discrete innovation*, while the latter as *continuous improvement*. For example, in the petroleum refining industry, five major new processes (thermal cracking, thermal reforming, polymerization, catalytic cracking, and catalytic reforming) had been introduced between the 1910s and the 1950s. Enos (1958) pointed out that production efficiency of each process was improved after its initial introduction in a variety of ways such as better equipment design and the elimination of “bottlenecks”, and found that such improvement resulted in substantial cost reductions. Similarly, in his study of DuPont’s rayon plants, Hollander (1965) distinguished between major and minor technical change, where a minor technical change involved an “evolutionary” alternation in the existing techniques whereas a major change involved a significant departure from existing methods.

This paper investigates the interplay between discrete innovation and continuous improvement in the presence of competitive pressure, and studies how firms’ incentives to conduct continuous improvement are affected by changes in the degree of competition and the nature of discrete innovation.

We consider a Hotelling style duopoly model in which firms’ locations are fixed. Each firm makes decisions concerning its investments in discrete innovation, and continuous improvement on the existing technology. In general, discrete innovation involves significant uncertainty. According to Mansfield et al. (1971), a survey of 120 large companies doing a substantial amount of R&D indicated that, in half of these firms, at least 60% of the R&D projects never resulted in a commercially used product or process.¹ We capture the uncertainty by assuming that the investment in discrete innovation turns into a success with a certain probability. On the other hand, investment in continuous improvement involves no uncertainty in our model. Note that, in this class of model, competitive pressure is captured by transport cost. That is, a reduction in transport cost increases the substitutability between the two firm’s products, which in turn intensifies the competition between them.

Our model captures a key interplay between discrete innovation and continuous improve- ¹See also, for example, Schmookler (1966), Aghion and Howitt (1992), and Grossman and Helpman (1991).
ment by assuming that the improvement made upon the existing technology is not effective for the new technology introduced by a successful discrete innovation. That is, the very success of discrete innovation makes continuous improvement obsolete, and hence reduces the payoff of continuous improvement. Redding (2002) recently made a distinction between “fundamental innovations” and “secondary innovations” in his model of endogenous innovation and growth, where the secondary knowledge acquired for one fundamental technology has limited relevance for the next. This distinction is similar to our distinction between discrete innovation and continuous improvement. Redding’s analysis, however, focuses on path dependence and technological lock-in of technological progress, and does not incorporate competitive pressure which is a crucial element of our analysis. His analysis is therefore fundamentally different from ours (see Section 2 for details).

Does competitive pressure foster innovation? Our analysis offers a new perspective on this important question, which goes back at least to Schumpeter (1943) and Arrow (1962). Effects of competitive pressure on firms’ innovation incentives have been previously explored in the theoretical industrial organization literature, where innovative activity is often modelled as deterministic investment in cost reduction (which is continuous improvement in our terminology). Recently, Vives (2006) made an important contribution by investigating this question under general functional specifications of demand system. Vives found, among other things, that an increase in competitive pressure measured by product substitutability increases (although perhaps weakly) R&D effort per firm, and this finding is consistent with the results obtained by previous studies under particular functional specifications (see Section 2 for details).

We contribute to the literature by demonstrating that the result can be overturned when the interplay between continuous improvement and discrete innovation is explicitly taken into account. In particular, our model predicts that an increase in competitive pressure measured by product substitutability decreases firms’ incentives to conduct continuous improvement in a broad range of parameterizations. To the best of our knowledge, no previous papers in the literature made an explicit distinction between discrete innovation and continuous improvement, which is the driving force of our result.

How do changes in the nature of discrete innovation affect firms’ incentives to conduct continuous improvement? The interplay between discrete innovation and continuous improvement yields a new prediction on this question. In particular, our model predicts that an increase in the size of discrete innovation decreases firms’ incentives to conduct continuous
improvement in a broad range of parameterizations.

In Section 5, we explore the real-world relevance and usefulness of the model through field research. Continuous improvement had been regarded as an important source of strength in Japanese manufacturing until the 1980s. However, several studies have found that levels of continuous improvement have recently decreased in a number of Japanese manufacturing firms. To understand the causes of the declining focus on continuous improvement in Japan, we conducted detailed field research at two Japanese manufacturing firms. By applying the model to the findings from our field research, we demonstrate that the model offers fresh insights on possible mechanisms behind the changing nature of innovation that we observed at these firms.

The rest of the paper is organized as follows: Section 2 discusses the related literature. Section 3 presents a Hotelling style duopoly model that incorporates the interplay between discrete innovation and continuous improvement. Section 4 analyzes the model, and presents comparative statics results concerning the equilibrium level of continuous improvement. Section 5 presents findings from our field research, and discuss how our model can be applied to shed light on what we observed. Section 6 offers concluding remarks.

2 Relationship to the literature

The present paper contributes to the industrial organization literature that theoretically investigates relationships between competition or market structure and firms’ innovation incentives. In models that analyze the extent of innovation, innovative activity is typically modeled as deterministic investment in cost reduction (see, for example, Dasgupta and Stiglitz, 1980; Spence, 1984; Tandon, 1984; Boone, 2000; Vives, 2006). Vives (2006) analyzed the effects of competition on cost-reducing R&D effort under general functional specifications and a variety of market structures, and found that increasing the number of firms tends to reduce R&D effort, whereas increasing the degree of product substitutability, with or without free entry, increases R&D effort. These findings are consistent with the results obtained by previous studies under particular functional specifications. On the other hand, in models that analyze the timing of innovation (i.e. “patent race” type models), R&D investment either stochastically or deterministically affects the eventual date at which an innovation is successfully introduced, where higher level of investment results in faster innovation. See Reinganum (1989) for a survey on the literature.
In the theoretical industrial organization literature mentioned above, to the best of our knowledge, no papers made a distinction between discrete innovation and continuous improvement and investigated the interplay between them. Boone (2000) analyzed the effects of competitive pressure on a firm’s incentives to invest in product and process innovations. In his model, each agent decides whether to enter the market with a new product and, if he/she enters, how much to invest to improve its production efficiency. That is, product innovation consists of entry of agents in this model, whereas we model discrete innovation by existing firms. Furthermore, our model captures the idea that continuous improvement is not effective for the new process or product introduced by a successful discrete innovation, which is not captured by Boone’s model. Also, the patent-design literature has addressed two-stage innovation, where a second innovation builds upon the first (see, for example, Green and Scotchmer, 1995; Chang, 1995). In these models, although the second innovation is an improvement upon the first, it is still discrete innovation rather than continuous improvement which consists of numerous minor improvements and modifications made upon the existing technology.

Similarly, relationship between competition or market structure and innovation have been investigated in the empirical industrial organization literature. Recent papers in this literature pointed to a positive correlation between product market competition and innovative activity (see e.g. Geroski, 1990; Nickell, 1996; Blundell, Griffith and Van Reenen, 1999). Geroski (1990) used data based on a study of 4378 major innovations in the UK, 1945-83, while in Blundell et al. (1999) innovation is a count of “technologically significant and commercially important” innovations commercialized by the firm. That is, these papers analyzed discrete innovation in our terminology. On the other hand, Nickell (1996) found that competition is associated with higher rates of total factor productivity growth. Again, to the best of our knowledge, no papers in this literature made a distinction between discrete innovation and continuous improvement as in the present paper.

Distinctions between different types of technological change have been explored in several endogenous growth models. For example, Jovanovic and Rob (1990) formalized the distinction between extensive and intensive search, where extensive search seeks major breakthroughs while intensive search attempts to refine such breakthroughs. Also, Young (1993) developed a model that incorporates an interaction between invention and learning by doing, and Aghion and Howitt (1996) introduced the distinction between research and development into Schumpeterian growth model. Redding (2002) recently proposed a model of endogenous
innovation and growth, in which technological progress is the result of a combination of “fundamental innovations” (which opens up whole new areas for technological development) and “secondary innovations” (which are the incremental improvements that realize the potential in each fundamental innovation). As in our model, Redding’s model incorporates the idea that the secondary knowledge acquired for one fundamental technology has often limited relevance for the next, and hence his distinction is perhaps closest to our distinction between discrete innovation and continuous improvement. However, none of these models incorporate competitive pressure, which is a crucial element of our analysis. Our model, therefore, is fundamentally different from models in this literature.

In summary, relationships between competitive pressure and firms’ innovation incentives have been investigated in the industrial organization literature, but the interplay between discrete innovation and continuous improvement have not been explored in this literature. On the other hand, distinctions between different types of technological change have been explored in endogenous growth models, but they did not incorporate competitive pressure. We combine the two literatures and provide fresh insight on the interplay between competitive pressure and firms’ incentives to invest in two types of innovation activities, discrete innovation and continuous improvement.

3 Model

Assume that a unit mass of consumers are uniformly distributed on the line segment [0, 1]. Each consumer is indexed by her location \( y \in [0, 1] \) on the line, which represents her ideal point in the product characteristic space. Each consumer buys at most one unit of exactly one of the two varieties sold in the market. The price and location of variety \( i (=A, B) \) on the line are denoted by \( p_i \) and \( z_i \) respectively where \( z_i \in [0, 1] \) for all \( i (=A, B) \). The indirect utility for consumer \( y \in [0, 1] \) of purchasing one unit of variety \( i \) is given by \( V_i(y) = R - p_i - t|z_i - y| \), where \( R \) is the gross utility from consuming one unit of any variety, \( |z_i - y| \) denotes the distance between \( z_i \) and \( y \), and \( t > 0 \) denotes per unit transport cost. The utility from not purchasing any variety is normalized to zero.

There are two firms, denoted \( A \) and \( B \), located respectively at 0 and 1. Firm \( i (=A, B) \) sells variety \( i \) at price \( p_i \), and hence \( z_A = 0 \) and \( z_B = 1 \). Each firm \( i \) has a constant marginal cost \( c_i \) and no fixed costs of production. Each firm can invest in discrete innovation (DI) and continuous improvement (CI) to reduce its cost. If firm \( i \) makes no investments in
DI and CI, then \( c_i = c (> 0) \). By investing a fixed amount \( F (> 0) \) in DI, each firm \( i \) can reduce its constant marginal cost from \( c \) to \( c - \Delta \) (\( \Delta \in (0, c) \)) with probability \( s \in (0, 1) \). Here we interpret \( \Delta \) to be the size of cost reduction due to the new technology introduced by the success in the discrete innovation. As pointed out in Introduction, in general discrete innovation involves significant uncertainty, which is incorporated in our model by the success probability \( s \). Assume that the two firms’ successes in DI are mutually independent. This assumption is for simplifying the algebra, and not crucial for our results. Note that results will be unchanged under an alternative specification that the success in DI increases the product’s gross utility from \( R \) to \( R + \Delta \). Under this alternative specification, a success in DI can be interpreted as a successful introduction of a new product.

Regarding CI, each firm \( i \) can reduce its constant marginal cost from \( c \) to \( c - x_i \) by investing \( d(x_i) \) in CI, where \( d(\cdot) \) is a convex function and \( x_i \in [0, X] \) (\( X \in (0, \Delta) \)). To obtain closed form solutions in the analysis, let \( d(x) = \frac{\gamma x^2}{2} \) (\( \gamma > 0 \)). The return from investment in CI is certain, and we interpret \( x_i \) to be the size of cost reduction due to the continuous improvement made by firm \( i \) upon the existing technology. If firm \( i \)’s investment in DI turns out to be successful, then firm \( i \) chooses the new technology (\( c_i = c - \Delta \)) or the old technology with improvement (\( c_i = c - x_i \)) where we assume that CI is not effective for the new technology introduced by the successful DI. The firm chooses the new technology under our assumption of \( X \in (0, \Delta) \). That is, we assume that the successful DI is more cost effective than the highest possible level of CI made upon the existing technology.

Following previous analyses in the industrial organization literature (see, for example, Raith, 2003; Aghion and Schankerman, 2004; Vives, 2006), we interpret that the per unit transport cost, \( t \), captures the degree of competitive pressure between firms. That is, a reduction in \( t \) increases the substitutability between the products of firms A and B, which in turn intensifies the competition between them.

We consider the two-stage game described below:

**Stage 1 [Investment]:** Each firm \( i \) simultaneously and non-cooperatively decides whether or not to invest in DI, and chooses \( x_i \in [0, X] \), the level of investment in CI.

**Stage 2 [Bertrand competition]:** The outcomes of DI are realized and become common knowledge. Each firm \( i \)’s constant marginal cost of production is \( c_i = c - \Delta \) if its investment in DI turns out to be successful, and \( c_i = c - x_i \) otherwise. Given \((c_A, c_B)\), each firm \( i \) simultaneously and non-cooperatively choose \( p_i \) to maximize its profit.
4 Analysis

4.1 Symmetric pure-strategy equilibrium

We derive Subgame Perfect Nash Equilibria (SPNE) of the model described above, focusing on symmetric equilibria. In this subsection we will focus our analysis on the case in which \( F \) (the fixed cost for \( DI \)) is sufficiently small so that both firms \( A \) and \( B \) invest in \( DI \) in pure-strategy equilibria. In the next subsection we will discuss the robustness of our results when \( F \) takes larger values. Given that competitive pressure is a critical element in our analysis, we assume that the gross utility from consuming a variety (captured by \( R \)) is high enough so that the two firms compete over all consumers in the equilibrium.\(^2\) We also assume that \( \gamma \) (the cost parameter for investment in \( CI \)) is large enough to ensure an interior solution for the equilibrium level of investment in \( CI \).\(^3\)

First consider stage 3 subgames. At stage 3, given a cost vector \((c_A, c_B)\), each firm \( i \) simultaneously and non-cooperatively chooses \( p_i \) to maximize its profit. Let \((p_A, p_B)\) be given, and suppose that \( R \) is sufficiently large so that all consumers purchase one unit of a variety. Then, consumer \( y \in [0, 1] \) purchases variety \( A \) from firm \( A \) if \( p_A + ty \leq p_B + t(1 - y) \Leftrightarrow y \leq \frac{1}{2} + \frac{p_B - p_A}{2t} \). We then find that demand for variety \( i \), denoted \( q_i(p_i, p_j) \), is given by \( q_i(p_i, p_j) = \max\{0, \frac{1}{2} + \frac{p_i - p_j}{2t}\} \) if \( \frac{1}{2} + \frac{p_i - p_j}{2t} \leq 1 \), and 1 otherwise, where \( i, j = A, B, i \neq j \). Each firm \( i \) chooses \( p_i \) to maximize \((p_i - c_i)q_i(p_i, p_j)\), where \( c_i = c - \Delta \) if firm \( i \) succeeds in \( DI \) and \( c_i = c - x_i \) otherwise. If \(|c_A - c_B| \leq 3t\), the SPNE outcome of the stage 3 subgame is characterized as follows:

\[
\tilde{p}_i(c_i, c_j) \equiv t + \frac{2c_i + c_j}{3}, \quad \tilde{q}_i(c_i, c_j) \equiv \frac{1}{2} + \frac{c_j - c_i}{6t} \tag{1}
\]

\[
\tilde{\pi}_i(c_i, c_j) \equiv (\tilde{p}_i(c_i, c_j) - c_i)\tilde{q}_i(c_i, c_j) = 2t\tilde{q}_i(c_i, c_j)^2, \tag{2}
\]

where \( i, j = A, B, i \neq j \), and \( \tilde{p}_i(c_i, c_j), \tilde{q}_i(c_i, c_j) \) and \( \tilde{\pi}_i(c_i, c_j) \) denote firm \( i \)'s equilibrium price, quantity and profit, respectively. Else, if \(|c_A - c_B| > 3t\) then

\[
\tilde{p}_i(c_i, c_j) = I(c_j - t) + (1 - I)c_i, \quad \tilde{q}_i(c_i, c_j) = I, \tag{3}
\]

\[
\tilde{\pi}_i(c_i, c_j) = I(c_j - c_i - t), \tag{4}
\]

\(^2\)More precisely, we assume that the value of \( R \) is high enough so that the following property holds: Every consumer who purchases a product from firm \( i (= A, B) \) in the equilibrium could enjoy a positive indirect utility by purchasing a product from firm \( j \neq i \), instead, at its equilibrium price.

\(^3\)In particular, we assume that \( \gamma > \max\left\{\frac{1}{2t}, \frac{1}{3X}\right\} \).
where the indicator variable $I = 1(0)$ if and only if $c_i < (>) c_j$. As mentioned earlier, we assume for now that $F$ is sufficiently small so that both firms invest in $DI$ at stage 1. In the subsequent stage 2 subgame, each firm $i$ chooses $x_i$ to maximize its expected overall profit, which is given by

$$s\pi^S_i(x_i, x_j) + (1-s)\pi^F_i(x_i, x_j) - \frac{\gamma x_i^2}{2} - F,$$

(5)

where $i, j = A, B$ ($i \neq j$), $\pi^S_i(x_i, x_j)$ denotes each firm $i$’s expected stage 3 profit conditional upon its success in $DI$, and $\pi^F_i(x_i, x_j)$ is analogously defined conditional upon its failure in $DI$. Recall that $c_i = c - \Delta$ upon firm $i$’s success in $DI$, while $c_i = c - x_i$ upon its failure. Hence we have

$$\pi^S_i(x_i, x_j) = s\tilde{\pi}_i(c - \Delta, c - \Delta) + (1-s)\tilde{\pi}_i(c - \Delta, c - x_j),$$

(6)

$$\pi^F_i(x_i, x_j) = s\tilde{\pi}_i(c - x_i, c - \Delta) + (1-s)\tilde{\pi}_i(c - x_i, c - x_j).$$

(7)

Given this, we find that the symmetric pure-strategy equilibrium of the entire game is unique, and in the equilibrium each firm $i$ chooses $x_i = x^*$, where

$$x^* \equiv \max\left\{\frac{(1-s)(3t - s\Delta)}{9t\gamma - s(1-s)}, \frac{(1-s)^2}{3\gamma}\right\}.$$

(8)

We are now ready to present comparative statics results concerning $x^*$, the equilibrium level of $CI$.

Let us start from the effect of competitive pressure on $x^*$. As mentioned earlier, effects of competitive pressure on firms’ innovation incentives have been previously explored in the theoretical industrial organization literature, where innovative activity is typically modelled as deterministic investment in cost reduction (which is $CI$ in our model). Recently, Vives (2006) found that an increase in competitive pressure measured by product substitutability increases (although perhaps weakly) R&D effort per firm under general functional specifications, and this finding is consistent with the results obtained by previous studies under particular functional specifications.

We contribute to the literature by demonstrating that the result can be overturned when the interplay between $CI$ and $DI$ is explicitly taken into account in the presence of competitive pressure.

**Proposition 1:** The equilibrium level of continuous improvement, $x^*$, declines as the degree of competitive pressure increases. More precisely, there exists a threshold value $\bar{\Delta} \equiv 3t + \frac{(1-s)^2}{3\gamma} > 0$ such that $\frac{dx^*}{d\Delta} > 0$ if $\Delta < \bar{\Delta}$ while $\frac{dx^*}{d\Delta} = 0$ if $\Delta > \bar{\Delta}$. 
Recall that in our model, competitive pressure is captured by per unit transport cost $t$: As $t$ decreases, product substitutability increases, which in turn increases competitive pressure. Hence, $\frac{dx^*}{dt} > 0$ means that the equilibrium level of $CI$ decreases as the degree of competitive pressure increases.

The logic behind this result can be explained as follows: Let $s \pi^S_i(x_i, x_j) + (1-s) \pi^F_i(x_i, x_j) \equiv \Pi_i(x_i, x_j)$, which is firm $i$’s expected stage 3 profit. Note that firm $i$’s investment in $CI$ turns out to be useful only if it fails in $DI$. Hence firm $i$’s expected marginal benefit from its investment in $CI$ is

$$\frac{\partial}{\partial x_i} \Pi_i(x_i, x_j) = (1-s) \frac{\partial}{\partial x_i} \pi^F_i(x_i, x_j) = s(1-s) \frac{\partial}{\partial x_i} \tilde{\pi}_i(c-x_i, c-\Delta) + (1-s)^2 \frac{\partial}{\partial x_i} \tilde{\pi}_i(c-x_i, c-x_j).$$

(9)

In the equilibrium, the marginal benefit of investment in $CI$, $\frac{\partial}{\partial x_i} \Pi_i(x^*, x^*)$, is equal to its marginal cost $\gamma x^*$. How does an increase in product substitutability (i.e., a decrease in $t$) affect the marginal benefit? To answer this question, we need to find the sign of

$$\frac{\partial^2}{\partial t \partial x_i} \Pi_i(x^*, x^*) = s(1-s) \frac{\partial^2}{\partial t \partial x_i} \tilde{\pi}_i(c-x^*, c-\Delta) + (1-s)^2 \frac{\partial^2}{\partial t \partial x_i} \tilde{\pi}_i(c-x_i, c-x^*).$$

(10)

First consider $\frac{\partial^2}{\partial t \partial x_i} \tilde{\pi}_i(c-x^*, c-x^*)$, which appears in the second term of the RHS of equation (8) and corresponds to the case in which both firms fail in $DI$. This term captures the effect of competitive pressure on $CI$ in absence of $DI$, which has been investigated by previous studies as mentioned above. We find, consistent with previous studies, that $\frac{\partial^2}{\partial t \partial x_i} \tilde{\pi}_i(c-x^*, c-x^*) = 0$. Following Raith (2003), the logic behind this result can be explained as follows: There are two effects which work in opposite directions. First, there is a business stealing effect: A lower value of $t$ makes demand more elastic, and, with more elastic demand, a firm with a cost advantage can more easily attract business from its rival. Hence, for a given price set by its rival, greater competition increases a firm’s marginal benefit of reducing its cost. Second, there is a scale effect: A firm whose rivals charge lower prices loses market share and therefore has less to gain from reducing its costs. Although the net outcome of these two effects is in general ambiguous, they exactly cancel each other in the location model, resulting in $\frac{\partial^2}{\partial t \partial x_i} \tilde{\pi}_i(c-x^*, c-x^*) = 0$.

Novelty of our analysis is captured by $\frac{\partial^2}{\partial x_i \partial x_j} \tilde{\pi}_i(c-x^*, c-\Delta)$, the first term of the RHS of equation (8), that corresponds to the case in which firm $j$ succeeds in $DI$ and firm $i$ fails in it. This term is unique to our analysis, capturing the interaction between $CI$ and $DI$ in the presence of competitive pressure. We find that $\frac{\partial^2}{\partial x_i \partial x_j} \tilde{\pi}_i(c-x^*, c-\Delta) > 0$, and the logic

9
is as follows: As product substitutability increases (i.e., as \( t \) decreases), demand becomes more elastic. Note that firm \( j \) has a cost advantage in this case because of its success in DI. Then, with more elastic demand, firm \( j \) can more easily attract business from its rival. This decreases firm \( i \)'s market share, and the smaller market share in turn reduces firm \( i \)'s marginal benefit of investment in CI. This results in
\[
\frac{\partial^2}{\partial t \partial x_i} \tilde{\pi}_i(c - x^*, c - \Delta) > 0.
\]

Combining \( \frac{\partial^2}{\partial t \partial x_i} \tilde{\pi}_i(c - x^*, c - x^*) = 0 \) and \( \frac{\partial^2}{\partial t \partial x_i} \tilde{\pi}_i(c - x^*, c - \Delta) > 0 \) yields \( \frac{\partial^2}{\partial t \partial x_i} \Pi_i(x^*, x^*) > 0 \). In absence of DI, an increase in product substitutability does not affect the equilibrium level of CI under the location models. Under general functional specifications, Vives (2006) showed that an increase in competitive pressure measured by product substitutability increases (perhaps weakly) R&D effort. In contrast we demonstrate that, in presence of the interaction between DI and CI, an increase in product substitutability can reduce the marginal benefit of CI, resulting in the lower equilibrium level of CI under the Hotelling style location model.

How do changes in the nature of DI affect firms’ incentives to invest in CI? By making a distinction between CI and DI and capturing their connection in the presence of competitive pressure, our model provides novel answers to this question.

**Proposition 2:** The equilibrium level of continuous improvement, \( x^* \), declines as the size of discrete innovation increases. More precisely, \( \frac{dx^*}{d\Delta} < 0 \) if \( \Delta < 1 \Delta \) while \( \frac{dx^*}{d\Delta} = 0 \) if \( \Delta > 1 \Delta \), where \( 1 \Delta \) is as defined in Proposition 1.

Competitive pressure plays a crucial role in driving this result. To see this, first consider what happens without competitive pressure by supposing that firm \( i \) is a monopolist, investing in DI and CI. Then, since firm \( i \)'s investment in CI is useful only when its DI turns out to be unsuccessful, the size of DI does not affect firm \( i \)'s incentive to invest in CI.

The presence of competition significantly changes the scenario. As mentioned earlier, firm \( i \)'s expected marginal benefit from its investment in CI is \( \frac{\partial}{\partial x_i} \Pi_i(x^*, x^*) \) in the equilibrium (see equation (8) above). We have
\[
\frac{\partial^2}{\partial \Delta \partial x_i} \Pi_i(x^*, x^*) = s(1 - s) \frac{\partial^2}{\partial \Delta \partial x_i} \tilde{\pi}_i(c - x^*, c - \Delta).
\]
That is, a change in the size of DI affects firm \( i \)'s marginal benefit of investment in CI only when firm \( i \) fails and firm \( j \) succeeds in DI. In this case, firm \( j \) has a cost advantage because of its success in DI. The advantage increases as \( \Delta \) increases, which reduces firm \( i \)'s market share. The smaller market share in turn reduces firm \( i \)'s marginal benefit of investment in
This implies that the equilibrium level of CI is decreasing in the size of DI. In the presence of competition, it is the possibility of firm i’s rival’s success in DI that reduces firm i’s incentive to invest in CI.

Finally, in Proposition 3 we consider the effect of a change in the success probability of DI (denoted s).

**Proposition 3:** The equilibrium level of continuous improvement, \(x^*\), declines as the success probability of discrete innovation increases. That is, \(\frac{dx^*}{ds} < 0\).

Logic behind the result is simple, and it does not rely on competitive pressure. Given each firm i’s investment in CI is useful only when it fails in its DI, its marginal benefit of investing in CI decreases as the success probability of DI increases. This implies the result.

### 4.2 Robustness of the results

Thus far we have focused on the case of \(F \leq F'\), in which both firms A and B invest in DI in the symmetric pure-strategy equilibrium. In this subsection section, we discuss the robustness of our results when this assumption is relaxed. Note, details of the analysis are available upon request.

In order to focus our analysis on symmetric equilibria, we allow mixed strategies for firms’ investments in DI. We find that there exist symmetric equilibria in mixed strategies characterized by \((\sigma^*, x^*)\), where \(\sigma^* \in [0, 1]\) denotes the probability for each firm to invest in DI in the equilibrium, and \(x^* \geq 0\) denotes the level of each firm’s investment in CI in the equilibrium. If \(F \leq F'\), then \(\sigma^* = 1\) holds in the equilibrium; that is, both firms invest in DI with probability 1 in the equilibrium. This equilibrium is identical to the pure-strategy equilibrium derived above in this section. Also, we find that there exists a value \(F'' (> F')\) such that \(0 < \sigma^* < 1\) if \(F' < F < F''\) while \(\sigma^* = 0\) if \(F \geq F''\).

Suppose \(F' < F < F''\). Then there exist symmetric equilibria characterized by \((\sigma^*, x^*)\) that solves the following simultaneous equations:

\[
\sigma^* = \frac{6st(\Delta - x^*) + s(\Delta - x^*)^2 - 18Ft}{2s^2(\Delta - x^*)^2}, \tag{12}
\]

\[
x^* = \frac{3t(1 - \sigma^*s) - \sigma^*s(1 - \sigma^*s)}{9kt - \sigma^*s(1 - \sigma^*s)}. \tag{13}
\]

We have proved that there exists at least one solution to this system, but have not been able
to rule out the possibility of multiple solutions. If more than one solution exist, then each solution pair \((\sigma^*, x^*)\) corresponds to an equilibrium.

In what follows we discuss the robustness of the comparative statics results presented above in Propositions 1 - 3. Proposition 1 told us that, if \(F \leq F'\), the equilibrium level of continuous improvement, \(x^*\), declines as the degree of competitive pressure increases. If \(F' < F < F''\), we have found that this result continues to hold as long as \(\frac{d\sigma^*}{dt} < 0\). Holding \(\sigma^*\) fixed, \(x^*\) declines as the degree of competitive pressure increases (i.e. as \(t\) decreases) through the logic analogous to the one presented after Proposition 1. This effect is reinforced by the impact of \(t\) on \(\sigma^*\), if \(\frac{d\sigma^*}{dt} < 0\). That is, if \(\frac{d\sigma^*}{dt} < 0\), then the equilibrium probability of success in DI increases as \(t\) decreases, which further reduces each firm’s incentive to invest in CI. We have found that \(\frac{d\sigma^*}{dt} < 0\) holds for all \(s < \frac{1}{2}\). However, if \(s \geq \frac{1}{2}\), we were not able to rule out the possibility of \(\frac{d\sigma^*}{dt} > 0\), and in such a case the result might be opposite to Proposition 1 under certain parameterizations.

Proposition 2 told us that, if \(F \leq F'\), \(x^*\) declines as the size of discrete innovation (captured by \(\Delta\)) increases. If \(F' < F < F''\), we have found that this result continues to hold as long as \(\frac{d\sigma^*}{d\Delta} > 0\), which says that as the size of DI increases, each firm increases its probability to invest in DI in the mixed-strategy equilibrium. This makes an intuitive sense, and we found that \(\frac{d\sigma^*}{d\Delta} > 0\) holds under a range of parameterizations. We were however unable to rule out the possibility of \(\frac{d\sigma^*}{d\Delta} < 0\), and in such a case the result might be opposite to Proposition 2. The result presented in Proposition 3 is robust when \(F' < F < F''\). Holding \(\sigma^*\) fixed, \(x^*\) declines as the success probability in DI (captured by \(s\)) increases. This is reinforced by \(\frac{d\sigma^*}{ds} > 0\). That is, as \(s\) increases, firms invest in DI with higher probability, which further reduces firms’ incentives to invest in CI.

### 5 Field research at two manufacturing firms

This section explores the real-world relevance and usefulness of the model. In particular, we present the findings from our field research at two Japanese manufacturing firms, and demonstrate that our model offers fresh insights on possible mechanisms behind the changing nature of innovation that we observed at these firms. Note that the purpose of this section is not to conduct rigorous empirical tests of the model’s theoretical predictions. Given the difficulty of obtaining reliable data on innovation activities within the firm that make a precise distinction between discrete innovation and continuous improvement, rigorous empirical
tests are beyond the scope of this paper.

Continuous improvement was once heralded as the hallmark of Japanese manufacturing system; in particular, employees in typical Japanese firms had been strongly encouraged to improve their work methods by actively participating in SGAs (Small Group Activities) such as quality control (QC) circles, Zero Defects, and Kaizen in which small groups at the workplace level voluntarily set plans and goals concerning operations and work together toward accomplishing these plans and goals. However, several recent studies report that Japanese firms appear to have been downplaying the importance of continuous improvement lately.\(^4\) To identify possible causes of the declining focus on continuous improvement in Japan, we conducted detailed field research at two Japanese manufacturing firms, METAL and AUTOPARTS.\(^5\)

We first present findings from our field research in Subsection 5.1, and then discuss applications of our model in Subsection 5.2. We present all our main findings from field research, where some of them are not directly relevant to our model. Reality is quite complex, and we certainly do not claim that our model captures all important aspects of reality. Our model however does capture interconnections among several key field research findings through novel angles, suggesting possible mechanisms behind the declining focus of continuous improvement in these firms. In other words, our model enables us to see important connections of several key changes taking place at these two firms and hence ascertain powerful underlying forces behind each firm’s decision to weaken its investment in traditional continuous improvement activities. Thus, we demonstrate the usefulness of our model.

\(^4\)For instance, according to a recent survey conducted by Chuma, Ohashi and Kato (2005), nearly one in two SGA participants believe that SGAs are LESS active now than 10 years ago whereas only 17 percent think SGAs are MORE active now in the industry. Furthermore, the same survey reveals that 30 percent of workers experienced the termination of their small group activities in the last ten years. An extensive case study of the Japanese semi-conductor industry by Chuma (2002) also demonstrates vividly the declining focus on traditional small group activities by Japanese semi-conductor firms.

\(^5\)Our confidentiality agreements with METAL and AUTOPARTS prohibit us from revealing the actual names of these firms.
5.1 Findings from field research

5.1.1 METAL

METAL is a large unionized manufacturing firm with sales of over 400 billion yen and employment of close to 4,000 workers in 2005. It is listed in the first section of Tokyo Stock Exchange. The corporation has nine plants. There have been four important changes at METAL in the last two decades which are relevant to their activities to facilitate continuous improvement and promote discrete innovation. First, Chinese firms have been taking over the lower end of the product line of METAL. For example, METAL has been called “the department store of specialty metal” and well-known for its comprehensive product line supplying nearly all kinds of specialty metal to major users of such metal (such as auto manufacturers). As a result of increased competition mostly coming from Chinese firms, in the last three years, METAL has been shifting its strategy from “all-round utility player” to “specialty player” focusing on the high end of the product line. A key component of this new strategy is to develop “NO. 1 product” or “Only one product”. For example, METAL is currently working on developing a new high-quality, high-performance specialty metal used for jet engine while exiting from a line of more traditional low-cost metal.

Second, METAL has been experiencing a shortening cycle of their product in recent years. For example, METAL and a major auto manufacturer used to develop a new specialty metal (which a transmission will be made of) jointly under an implicit long-term (typically 2 years) contract which guarantees the eventual sale of the product to the auto manufacturer. In recent years, such long-term implicit contracts have been replaced with short-term (a few months) contracts with no guarantee for repeated transactions. As such, product cycles are now in months not in years. Another example is a new product area such as magnetic material and metal powders where METAL has been moving into lately. Users of such a new product area are closer to final consumers than traditional specialty metal users, and there are so many more competitors. As such, the market is closer to a short-term spot market than a relational long-term contract market.

Third, rapid retiring of seasoned operators who are fully capable of engaging in traditional bottom-up, self-directed problem solving activities, coupled with the recent downsizing of such operators, makes continuous improvement less effective.

Fourth, the nature of their small group activities, a hallmark of their grassroots innovation activities, has been changing from “bottom-up, operator-centered activities within
the workplace” to “more top-down, more engineer-centered, cross-functional offline activities across workplaces.” Specifically, METAL’s small group activities began in 1967. As in the case of many traditional small group activities in large Japanese firms, METAL’s small group activities started out as “voluntary” offline problem solving teams in which front-line workers meet normally after regular hours and “voluntarily” engage in problem solving activities with no or only token compensation. METAL called their small group activities “self-directed team activities” and stressed the importance of operator initiative in selecting themes, setting goals, scheduling meetings, writing up final reports and presenting them. It was clearly meant to be bottom-up, operator-initiated activities at the shopfloor level. It followed that their activities tended to focus on small, incremental problem solving within the shopfloor.

METAL made two major changes to their traditional “self-directed team activities” and increased the level of involvement of professional staff (engineers and managers) and changed the nature of problem solving from small, incremental improvements within the narrow workplace to larger and more discrete innovation involving multiple workplaces. First, in mid-1990s, METAL introduced a new type of small group activities, WANTED. Professional staff comes up with a specific theme (a problem to be solved), and ask a “self-directed team” to volunteer to take it up. Before the introduction of WANTED, all problems to be solved were set by operators. Within a few years after the introduction of WANTED, only about a half of all completed themes were set by operators and the rest were set by professional staff. Accordingly the nature of problem solving shifted from small and incremental improvements within the shopfloor to larger and discrete innovation involving multiple shopfloors. METAL estimated that the amount of productivity gain from problem solving by their self-directed team activities also doubled per activity a few years after the introduction of WANTED.

Second, in September 2004, METAL identified 7 workplaces out of 80 as target workplaces. These target workplaces were chosen mainly because of their known productivity problems. METAL then allocated considerable amount of money and professional staff to those target workplaces with a specific goal of 30 percent increase in productivity in 6 to 12 months. Most importantly METAL assigned key engineers from various parts of the firm to each of those seven target workplaces and such engineers initiated a variety of problem solving activities with operators in each target workplace. Due to the relatively short time span (6-12 months) and hefty goal (30 percent increase in productivity), those engineer-initiated problem solving activities were distinctly different from typical self-directed team activities.
They tended to go after bigger innovation by using more resources (money and professional staff) than traditional self-directed team activities. By the time of our more recent visit to METAL (June of 2005), 5 out of 7 target workplaces had already achieved their goal of 30 percent productivity increase.

5.1.2 AUTOPARTS

AUTOPARTS is a medium-size unionized manufacturing firm with sales of over 40 billion yen and employment of close to 1200 in 2004. It is a privately-held company with six plants. AUTOPARTS joined a supplier group of a major auto manufacturer, AUTOMAKER in 1949. The tie between the two firms continued to strengthen and by the end of 1980s, over 90 percent of sales of AUTOPARTS went to AUTOMAKER (a supplier group with a strong tie between a manufacturer and its suppliers is often called vertical *keiretsu* in Japan). Specifically AUTOMAKER used a unique type of engine parts which no other auto maker used, and AUTOPARTS was the only firm that supplies such a unique type of engine parts. As such, AUTOPARTS faced little competition in the market for their engine parts. In part due to the overall trend in weakening keiretsu and the increased global competition, however, at the beginning of the 1990s AUTOMAKER decided to weaken its tie to AUTOPARTS, declaring its decision to switch gradually from the unique type of engine parts to the universal type of engine parts which not only AUTOPARTS but also many other auto part suppliers produce. AUTOMAKER began telling AUTOPARTS that they may start buying engine parts from other suppliers and that AUTOPARTS is encouraged to sell its products to other auto manufacturers. As a result of the weakening tie between the two firms, in 2004, close to 30 percent of AUTOPARTS’ sales went to other auto makers (a considerable rise from less than 10 percent at the end of the 1980s).

While leaving a cocoon of keiretsu in the 1990s and facing more competition, the nature of innovation in AUTOPARTS changed considerably. AUTOPARTS used to have effective small group activities of operators with small, incremental process improvements. In the 1990s, such small group activities became less effective and active. Specifically, as competition intensified, AUTOPARTS introduced more sophisticated, advanced and expensive technologies. Some of these technologies are exceedingly sophisticated and expensive that even experienced engineers of AUTOPARTS are discouraged to attempt to tinker with them. As such, there is no room for onsite incremental improvements on such technologies and hence
no small group activities of operators are used in their workplaces with such technologies. In fact, AUTOPARTS filled most new openings for operator positions in their workplaces using such advanced technologies with migrant workers from Brazil. Since nearly all of these migrant workers from Brazil speak only Portuguese, even if AUTOPARTS decide to introduce small group activities to these workplaces, it will be prohibitively costly to run such bilingual small group activities. At the time of our most recent visit to AUTOPARTS (July of 2005), there are around 900 regular employees and about 300 temporary employees with fixed-term contracts. Almost all of these 300 temporary employees are migrant workers from Brazil.\footnote{These migrant workers from Brazil are Brazilians of Japanese decent, and since the 1989 revision of the Immigration Control and Refugee recognition Act, such foreigners of Japanese decent have been exempt from regular restrictions imposed on foreign visitors (Ogawa, 2005).}

Many traditional operator-oriented small group activities have been replaced with “technology groups.” Such technology groups are comprised of professional staff (such as engineers) and they specialize on process innovation. This represents yet another example of a shift from bottom-up, operator-initiated, voluntary, self-directed problem solving team activities to top-down, engineer-initiated, involuntary problem solving group activities of process innovation specialists.

On the other hand, AUTOPARTS has been faced with an increased need for developing attractive products. Traditionally AUTOPARTS receives from AUTOMAKER detailed specifications for specific engine parts used by AUTOMAKER, and sales of such parts to AUTOMAKER are guaranteed. In recent years, however, with the weakening role of keiretsu in Japan, AUTOMAKER demands AUTOPARTS to develop attractive products for them, and sales of their products to AUTOMAKER are no longer guaranteed. To respond to the enhanced need for product development, AUTOPARTS has been actively recruiting engineers with 4-year degrees in the last two decades. The number of engineers working in product development has increased from 20 to 53 in the last decade.

5.2 An application of the model

Consistent with the overall trend in Japanese manufacturing firms, we have observed that the level of continuous improvement has been declining at both firms. The declining trend was clear at AUTOPARTS, while it was more subtle at METAL. That is, METAL’s small
group activity itself continues to be active, but the nature of problem solving undertaken by small groups has shifted its focus to larger and more discrete innovation involving multiple workplace from small, incremental improvements within the narrow workplace. Note that it could have been difficult to identify such a subtle change without detailed field research.

Why have the levels of continuous improvement declined at these firms? Below we will explore this question by applying our model to field research findings. Let us start from AUTOPARTS. The level of continuous improvement in AUTOPARTS has drastically declined in the 1990s. The reason for this drastic decline, as we were told in our interview, was AUTOPARTS’s introduction of more sophisticated and advanced production system for which continuous improvement is virtually impossible. However, we have also found that this type of production system had been available for a number of years before the company actually introduced them. Why, then, did AUTOPARTS introduce the new system at that particular timing?

Recall that AUTOPARTS faced a higher degree of competition in the beginning of the 1990s due to the change of AUTOMAKER’s procurement policy. In particular, AUTOMAKER switched gradually from the unique type of engine parts supplied only by AUTOPARTS to the universal type of engine parts which not only AUTOPARTS but also many other auto part suppliers produce. The switch exposed AUTOPARTS to a tougher competition by reducing product substitutability.

As mentioned earlier, a robust finding in previous theoretical analyses is that an increase in competitive pressure measured by product substitutability increases deterministic investment in cost reduction (i.e., continuous improvement). In contrast, our model indicates that the interplay between discrete innovation and continuous improvement can reverse the result, predicting that an increase in the degree of competitive pressure decreases the equilibrium level of $CI$ when $F \leq F'$ (Proposition 1). Also, when $F > F'$, the model yields the same prediction as long as an increase in competitive pressure also increases $\sigma^*$, the equilibrium probability for each firm to invest in $DI$. And the recent drastic increase in the number of engineers working in product development suggests that AUTOPART has increased its investment in $DI$, resulting in higher $\sigma^*$.

Our model therefore indicates that the increase in competitive pressure can be a root cause of the drastic decline in the level of AUTOPART’s continuous improvement. That is, a possible scenario is that the higher degree of competition introduced by AUTOMAKER reduced AUTOPARTS’s return from continuous improvement on the existing production
system, which in turn induced AUTOPARTS to introduce the new production system upon which continuous improvement is virtually impossible.

Next we turn to METAL, which has changed the focus of its small group activity from traditional self-directed kaizen activities aimed at small incremental improvement within the narrow workplace to large-scale, engineer-initiated activities involving multiple workplaces with a clear objective of more discrete innovation. Our model suggests that the declining focus of METAL’s continuous improvement can be related to two other changes that we observed in this firm. First, METAL has been shifting its strategy from “all-round utility player” to “specialty player” focusing on the high end of the product line. In general, the nature of discrete innovation for high-end products tends to be “higher-risk higher-return” than that for low-end products. That is, in the context of our model, ∆ (the return of DI) tends to be higher and s (the success probability of DI) tends to be lower for high-end products. Noting that in steel manufacturing processes, a variety of different products ranging from high-end products to low-end products are produced in the same production facility, increasing weight of high-end products increases ∆ and reduces s at aggregate levels. Given this, we hypothesize that ∆ has recently increased while s has declined in METAL.

Second, METAL has been experiencing a shortening cycle of their product in recent years. In the context of our model, the product-life cycle becomes shorter, in an expected sense, as the overall success probability of DI (that is, sσ*) increases. Given this, we hypothesize that sσ* has recently become higher in METAL. Even though we hypothesize that s has recently declined in METAL, sσ* can still be higher. For example, an increase in ∆ increases σ* in a broad range of parameterizations in our model, which can in turn increase sσ*.

Our model predicts that an increase in ∆ reduces the equilibrium level of CI, holding else constant. Also, even though s has recently declined in METAL, an increase in ∆ reduces the equilibrium level of CI as long as sσ* has increased. Our model therefore indicates that the shift of METAL’s strategy to the high-end products, along with its shortening product-life cycle, can be a driving force of METAL’s declining focus on continuous improvement.

It is important to note that the purpose of this section is not to offer the explanation for the decline in continuous improvement observed at the two firms, but to illustrate possible ways in which our model can be applied to real-world contexts and provide fresh insights on the causes of the diminishing focus on continuous improvement that was once heralded as the hallmark of the Japanese enterprise system. Specifically, in our model, an increase in competitive pressure reduces the equilibrium level of CI, and an increase in the return
from DI also reduces the equilibrium level of CI. These theoretical predictions suggest possible reasons for the decline in continuous improvement at AUTOPARTS and METAL, respectively. The interplay between discrete innovation and continuous improvement in the presence of competitive pressure suggests novel underlying mechanisms behind the changing nature of innovation at these firms.

6 Conclusion

In studying a possible linkage between firms’ innovation incentives and competitive pressure, prior studies ignored a distinction between discrete innovation aiming at entirely new technology and continuous improvement consisting of numerous incremental improvements and modifications made upon the existing technology. In this paper, we have demonstrated theoretically and empirically that distinguishing between these two types of innovation will lead to a much richer understanding of the interplay between firms’ incentives to innovate and competitive pressure. As such, we have provided novel insights on the sources and nature of technical progress.

Specifically, we have considered a Hotelling style duopoly model in which firms’ locations are fixed. Each firm makes decisions concerning its investment in discrete innovation, and continuous improvement on the existing technology. Discrete innovation generally involves more significant uncertainty than continuous improvement. There is, however, an important risk with continuous improvement. Various improvements and modifications made on the existing technology will be nullified by the very success of discrete innovation. In other words, when deciding on its innovation strategy, the firm will take into consideration the negative consequence on continuous improvement of the very success of discrete innovation.

Our model has yielded several new predictions. We have found that the equilibrium level of continuous improvement declines as the degree of competitive pressure increases. This is in contrast to the previous results in the theoretical industrial organization literature: The robust findings have been that an increase in competitive pressure measured by product substitutability increases firms’ investment in continuous improvement. The difference arises due to the aforementioned interplay between continuous improvement and discrete innovation in the presence of competitive pressure, which is uniquely captured by our model. Another new theoretical prediction is that firms’ incentives to conduct continuous improvement declines as the size of discrete innovation increases.
To demonstrate the relevance and usefulness of the model, we have applied these theoretical predictions to the findings from our field studies conducted at two Japanese manufacturing firms, METAL and AUTOPARTS. Continuous improvement was once heralded as the hallmark of Japanese manufacturing system. However, several recent studies report that Japanese firms appear to have been downplaying the importance of continuous improvement lately. Consistent with the overall trend, we have observed that the level of continuous improvement has been declining at both firms.

Through capturing the interplay between discrete innovation and continuous improvement in the presence of competitive pressure, our model has suggested novel underlying mechanisms behind the changing nature of innovation at these firms. At METAL, we have observed that the firm has been shifting its strategy from all-round utility player to specialty player focusing on the high end of the product line. Given that return from investment in discrete innovation tends to be higher for high-end products, our model indicates that this trend can be a driving force of METAL’s declining incentive to invest in continuous improvement. At AUTOPARTS, we have observed that the firm has been exposed to much tougher competition with its rivals, and our model indicates that the tougher competition can be a root cause of the drastic decline in AUTOPART’s continuous improvement.

Our framework can be applied to broader and more general real-world contexts. For example, competitive pressure has increased in a number of Japanese industries in the recent trend of globalization and deregulation. Our model predicts that the increase in competitive pressure can be an important cause of the recent decline in the level of Japanese firms’ continuous improvement. In a future work, we plan to conduct intensive data collection for rigorous econometric tests of this prediction.

7 Proofs

Proof of Proposition 1: Expanding (8) gives \( x^* = \frac{(1-s)(3t-s\Delta)}{9\gamma-\gamma(1-s)} \) if \( \Delta < 3t + \frac{(1-s)^2}{3\gamma} \) and \( x^* = \frac{(1-s)^2}{3\gamma} \) if \( \Delta > 3t + \frac{(1-s)^2}{3\gamma} \). Define \( \bar{\Delta} = 3t + \frac{(1-s)^2}{3\gamma} \). For \( \Delta < \bar{\Delta} \), we have that \( \frac{dx^*}{dt} = \frac{9\gamma s (1-s) \Delta}{9\gamma-\gamma(1-s)} \) since \( X - \frac{1}{3\gamma} > 0 \) (by footnote 3) and \( \Delta > X \) we have that \( \Delta - \frac{1-s}{3\gamma} > 0 \) which in turn implies that \( \frac{dx^*}{dt} > 0 \) for \( \Delta < \bar{\Delta} \). If \( \Delta > \bar{\Delta} \), \( \frac{dx^*}{dt} = 0 \) since \( x^* = \frac{(1-s)^2}{3\gamma} \) is independent of \( t \). \( Q.E.D. \)

Proof of Proposition 2: If \( \Delta < \bar{\Delta} \), \( \frac{dx^*}{d\Delta} = -\frac{s(1-s)}{9\gamma-\gamma(1-s)} < 0 \). If \( \Delta > \bar{\Delta} \), \( \frac{dx^*}{d\Delta} = 0 \) since
\( x^* = \frac{(1-s)^2}{3\gamma} \) is independent of \( \Delta \). Q.E.D.

**Proof of Proposition 3:** If \( \Delta < \bar{\Delta} \), \( \frac{dx^*}{ds} = -\frac{(3t-s\Delta)(9t\gamma-(1-s)^2)+\Delta(1-s)(9t\gamma-s(1-s))}{(9t\gamma-s(1-s))^2} \). By footnote 3, \( 9t\gamma - 1 > 0 \). Hence \( 9t\gamma - (1-s)^2 > 0 \) and \( 9t\gamma - s(1-s) > 0 \). Finally, since \( 3t - s\Delta > 3t - s\bar{\Delta} (\equiv \frac{(1-s)(9t\gamma-s(1-s))}{3\gamma}) > 0 \) we have that \( \frac{dx^*}{ds} < 0 \). If \( \Delta > \bar{\Delta} \), \( \frac{dx^*}{ds} = -\frac{2(1-s)}{3\gamma} < 0 \). Q.E.D.
References


