Dynamic Scoring in a Romer-style Economy

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Abstract

This paper analyzes how changes in tax rates affect government revenue in a Romer-style endogenous growth model. I show that in this environment lowering taxes on capital income may not have significant long-run effects on tax revenue and the tax base, contrary to other studies of the dynamic response of revenue to tax rates. The appropriate conclusions to draw about tax policy depend on the values of some key parameters in the idea production function.

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1. Introduction

Policymakers who want to change the tax rates that apply to capital income and wages must consider the effects that this will have on overall revenues. Traditional approaches to estimating the revenue effects of tax rate changes focused on static behavioral responses – essentially the short-run response of labor and capital income to a change in the tax rate.[1] Fullerton (1982) discusses whether labor income tax reductions might induce expansions of tax revenue, arguing that such Laffer curve effects (higher government revenue at lower tax rates) are unlikely due to low labor supply elasticities. Malcomson (1986) studies the same question and emphasizes the relevance of general equilibrium effects.

In spite of skepticism about the short-run revenue enhancing effects of tax cuts in the 1980s, recent studies have considered the possibility that the long-run effect of a tax cut is to expand the government's tax collection. Economic research has generally been skeptical of large short-run behavioral responses to tax rate changes. By contrast, more economists believe that long-run responses of labor supply and especially of capital supply may be large, potentially justifying lower tax rates.[2] As Mankiw and Weinzierl (2006) point out in the context of the Ramsey model, the accumulation of capital means that a lower tax rate on capital will ultimately increase the tax base, limiting the long-run reduction in revenues from a tax rate reduction. Auerbach (1996) gives a general presentation of issues related to dynamic scoring. See also Auerbach and Kotlikoff (1987) who study a wide range of issues related to dynamic aspects of fiscal policy.

A number of other studies have considered the dynamic effects of taxes on government revenue in endogenous growth models. Those who have used AK models to explore the effects of taxes include Stokey and Rebelo (1995); Agell and Persson (2001); Ireland (1994); Bruce and Turnovsky (1999). The results of the AK model are fairly straightforward to develop. Production is proportional to capital, $Y = AK$, though individuals may perceive this production function

[1] This issue gained prominence in the early 1980s when some claimed that the United States had tax rates so high that lower tax rates would *increase* tax revenue. This hypothetical situation was known as being on the wrong side of the Laffer Curve.

[2] A notable counterexample is Goolsbee (2000) who argues that a reduction in high-income tax rates had large short-run effects but small long-run effects.

to be $Y = \tilde{A}K^{\alpha}L^{1-\alpha}$. Absent depreciation, the real interest rate is $r = \alpha A$. The growth rate of the economy is determined from the consumption Euler equation: $\dot{C}/C = \sigma((1-\tau_v)\alpha A - \rho)$, where σ is the intertemporal elasticity of substitution, ρ is the discount rate, and τ_v is the tax rate on capital income. As the tax rate on capital income τ_v falls the steady-state growth rate of the economy increases. Tax cuts therefore tradeoff current revenue losses for future revenue gains.

In the AK model above, the optimal growth rate exceeds the growth rate in the decentralized allocation. As such, the appropriate policy is to subsidize capital income, rather than taxing it.[3] It is natural to think that lower taxes (at least when such taxes are positive) would expand the tax base. In the model I discuss, there are several distortions that make it uncertain, a priori, whether a decentralized allocation will result in too much or too little investment in research and development.

Others have used models in which growth is driven by the accumulation of human capital, as in Lucas (1988). Examples include Novales and Ruiz (2002); Pecorino (1995); De Hek (2006); Milesi-Ferretti and Roubini (1998a,b); Hendricks (1999). While some find that lower tax rates provide extensive stimulus to the economy, others report more modest responses. For example, Hendricks (1999) presents a life-cycle model with human capital accumulation. In his model human capital accumulation drives the economy in the long run, but lower tax rates do not generate large increases in the scale of the economy.

Another strand of the literature on the effects of taxation discusses the uses of government revenue. For example, Jones et al. (1993, 1997) modify the classic Chamley (1986) and Judd (1985) result that the optimal tax rate on capital income is zero. In their model the government uses tax revenue to provide productive public goods. Cutting taxes means having to cut public services, which may reduce output. Ferede (2008) follows a similar approach. In these papers, a reduction in the tax rate may not be followed by large increases in the tax base since the government has to reduce its investments in public infrastructure. For example, while Mankiw and Weinzierl find that 50% of a tax cut on capital income is self-financing, Ferede concludes that only 6% of the tax cut would be

[3]When there is no population growth or depreciation, the optimal subsidy to capital income, financed by lump sum taxes, would be $(1-\alpha)/\alpha$.

self-financing if the tax cut meant the government had to cut back on productive spending.

The AK model as described above relies on large spillovers from using capital to generate endogenous growth through capital accumulation as well as being consistent with facts about the share of income paid to capital. Furthermore, empirical evidence on the effect of the size of government on the economy's growth rate does not come down strongly in favor of such strong scale effects (Easterly and Rebelo (1993); Mendoza et al. (1997); Jones (1995b)). While this may be consistent with appropriately parametrized AK models, as in Stokey and Rebelo (1995), it is also consistent with the model I present in which tax rates do not affect the steady-state growth rate of the economy, but may affect the steady-state level of activity.

This paper presents new results on the short-run and long-run effects of taxes in an endogenous growth model. I focus on the issue of how a tax rate reduction affects government revenue. The model is a version of Romer (1990), with growth driven by the production of new designs for capital goods. The taxation of returns to accumulated factors does not distinguish between physical capital and knowledge. The model is designed to be consistent with long-run balanced growth with population growth. It is a second generation endogenous growth model in which scale effects are present in the level of activity rather than the growth rate of activity, as emphasized by Jones (1995b). Government policies, such as the tax rate on capital income, do not affect the growth rate, but do affect the level of output and tax revenue.

This paper adds to the growing literature on the dynamic revenue response to tax rate changes by embedding the policymaker in an endogenously growing economy as modelled by Romer (1990) and Jones (1995a). In this kind of model, the long-run growth rate and level of economic activity are determined in part by the deliberate actions of entrepreneurs and engineers who develop new products and techniques. The extent of innovation is driven by the returns to innovation, and these may be influenced by the supply of capital and labor. Therefore, taxes on these factors may have additional effects on economic activity through this innovation channel.

The paper proceeds as follows. Section 2 presents the Romer-style model with

taxes on capital income and discusses the steady-state and transition dynamics in this model. Section 3 presents comparative dynamic responses of tax revenue to tax rates. Section 4 concludes.

2. The Romer Model with Capital Income Taxes

2.1. The Economic Environment and Agents

The economic environment consists of three production sectors. Final goods are produced using durable intermediate goods and labor. The intermediate goods are produced using final output (in the form of capital) and designs. These designs come from the research and development sector, which uses labor and previously developed designs in production, though existing designs used in the R&D sector are not compensated in the decentralized allocation considered here.

The production of new designs used for making intermediate goods proceeds according to

$$\dot{A}_t = \nu A_t^{\phi} L_{At}^{\lambda}, \quad \phi < 1, \lambda > 0, A_0 > 0, \nu > 0 \tag{1}$$

The intermediate goods sector uses capital together with designs to produce differentiated intermediate inputs. One unit of capital produces one unit of the intermediate good. Each intermediate goods producer owns the design used in production. The measure of designs is A_t. Total production of intermediate goods is determined by the size of the capital stock:

$$\int_0^{A_t} x_{it} di = K_t \tag{2}$$

Final output, which can be consumed or transformed into capital, is produced with intermediate inputs and labor

$$Y_t = \left(\int_0^{A_t} x_{it}^{\theta} di \right)^{\alpha/\theta} L_{Yt}^{1-\alpha} \tag{3}$$

The decentralized equilibrium in this economy features solutions to the following problems.

Household Problem. The household problem is to choose time paths of c_t (consumption) and v_t (financial assets) that maximize

$$\int_0^\infty e^{-(\rho-n)t}\frac{c_t^{1-1/\sigma}-1}{1-1/\sigma}dt$$

taking the full time series of prices and taxes as given, and subject to the following constraints

$$\dot{v}_t = ((1-\tau_v)r_t - n)v_t + w_t - c_t + tr_t, \quad v_0 > 0 \tag{4}$$

$$\lim_{t\to\infty} v_t \exp\left\{-\int_0^t ((1-\tau_v)r_s - n)ds\right\} \geq 0 \quad NPG \tag{5}$$

where v is assets per person, c is consumption per person, w is the wage rate, r is the pre-tax return on assets, ρ discounts future utility, n is the growth rate of population, and τ_v is the tax rate for asset income.[4]

Final Goods Problem. The final goods sector is perfectly competitive. At each point in time, firms demand labor and intermediate goods, taking wages and intermediate goods prices as given, to maximize

$$\left(\int_0^{A_t} x_{it}^\theta di\right)^{\alpha/\theta} L_{Yt}^{1-\alpha} - w_t L_{Yt} - \int_0^{A_t} p_{it}x_{it}di \tag{6}$$

Intermediate Goods Problem. Patent-holding firms in the intermediate goods sector choose a price p_{it} and quantity to produce to maximize profits

$$x(p_{it})(p_{it} - r_t - \delta) \tag{7}$$

Research and Development Problem. Firms in the R&D sector produce new designs that intermediate goods firms use to produce new intermediate inputs. There is free entry in this sector, but there are externalities. Firms perceive a constant returns to scale production function, ignoring diminishing returns to labor

[4] I abstract from a menu of taxes that includes taxes on labor incomes and consumption. Without a labor-leisure choice, these taxes do not distort allocations. Altering capital income taxes may affect tax revenues gathered through the labor income and consumption taxes since different levels of capital income tax imply different degrees of capital accumulation and production. My analysis neglects these possible feedback effects.

at the aggregate level in this sector. Increases in activity (L_A) generate something akin to congestion effects, lowering the marginal product of labor. Firms sell their patented designs for price P_{At}. They demand labor, paid at the economy-wide wage rate w_t, maximizing profits

$$P_{At}\bar{\nu}_t L_{At} - w_t L_{At} \tag{8}$$

where $\bar{\nu} = A^{\phi} L_A^{\lambda-1}$.

Government Budget. The government simply collects taxes and returns them to households lump sum:

$$tr_t = \tau_v r_t v_t \tag{9}$$

In this model there is neither government consumption nor public goods provision.[5] Households are Ricardian, so the timing of tax rebates is irrelevant to the households' decisions. Assuming the government rebates all revenues immediately means that we do not have to keep track of the government's asset position.

2.2. Definition of Equilibrium

The *decentralized equilibrium* in this Romer economy with taxes is a time path for quantities $\{c_t, L_{Yt}, L_{At}, L_t, A_t, K_t, Y_t, v_t, \{\pi_{it}\}_{i=0}^{A_t}, \{x_{it}\}_{i=0}^{A_t}, \bar{\nu}_t, tr_t\}_{t=0}^{\infty}$ and prices $\{P_{At}, \{p_{it}\}_{i=0}^{A_t}, w_t, r_t\}_{t=0}^{\infty}$ such that for all t:

1. c_t, v_t solve the household problem
2. $\{x_{it}\}_{i=0}^{A_t}$ and L_{Yt} solve the final goods firm problem
3. $\{p_{it}\}_{i=0}^{A_t}$ and $\{\pi_{it}\}_{i=0}^{A_t}$ solve the intermediate goods firm problem
4. L_{At} solves the research and development firm problem
5. $Y_t = \left(\int_0^{A_t} x_{it}^{\theta} di\right)^{\alpha/\theta} L_{Yt}^{1-\alpha}$
6. A_t follows from equation (1)
7. K_t satisfies $\int_0^{A_t} x_{it} di = K_t$

[5]See Barro (1990); Jones et al. (1993); Ferede (2008) and others for models where the government can provide productive public goods.

8. $\bar{\nu}_t$ satisfies the ideas production function: $\bar{\nu}_t = A_t^{\phi} L_{At}^{\lambda-1}$

9. Asset arbitrage: $r_t = \frac{\pi_{it}}{P_{At}} + \frac{\dot{P}_{At}}{P_{At}}$

10. r_t clears the capital market: $v_t L_t = K_t + P_{At} A_t$

11. w_t clears the labor market: $L_{Yt} + L_{At} = L_t$

12. $L_t = L_0 e^{nt}$

13. tr_t satisfies the government budget constraint: $tr_t = \tau_v r_t v_t$

Note that households are taxed on their capital income. Capital income is derived from either physical capital or intellectual property (A). Asset arbitrage implies that the returns to investing a dollar in each asset class be the same. This is condition (9) in the definition of equilibrium above. In the presence of taxes, this condition implies that capital gains from appreciating prices of intellectual property are taxed. If only profits were taxed, the arbitrage equation would be:

$$(1-\tau_v) r_t = (1-\tau_v) \frac{\pi_{it}}{P_{At}} + \frac{\dot{P}_{At}}{P_{At}},$$

and this would have different implications for the steady-state price of patented ideas. In fact, it is possible to show that in the absence of depreciation, or if income from physical capital is taxed without allowing for depreciation, a policy that taxes only dividend payments and not capital gains of patented technologies makes the composition of the capital stock (i.e., the share of the overall capital stock that is physical capital distinct from intellectual property) invariant to the capital income tax rate.

2.3. Balanced Growth Path

This section presents some properties of the balanced growth path for the economy. Consider first static aspects of the equilibrium allocation. Each intermediate goods producer faces the same problem, so they will produce the same quantity x and sell it for the same price p. The profit π for each patent holder will be the

same and all patents will trade at the same price P_A. Since the entire capital stock is divided among the intermediate goods producers, we find that

$$x_{it} = x_t = \frac{K_t}{A_t}$$

and the price charged is a markup over marginal cost

$$p_{it} = p_t = \frac{1}{\theta}(r_t + \delta)$$

so that the profit for each firm is

$$\pi_{it} = \pi_t = \frac{1-\theta}{\theta}(r_t + \delta)\frac{K_t}{A_t} = \alpha(1-\theta)\frac{Y_t}{A_t}.$$

Note that θ relates to the profit share. Specifically, the share of final output (note: not of total income) paid out as pure profits is $\alpha(1-\theta)$. If profits actually represent 10% of final output and α is one-third, then the appropriate value for θ would be about 0.7. The gross markup is $1/\theta$. So in order to match net markups of 10%, θ should be around 0.9.

As in Jones (1995a), the steady-state growth rate of A is given by[6]

$$g_A = \frac{\lambda n}{1-\phi}.$$

Since output is equal to

$$Y_t = A_t^{\alpha\frac{1-\theta}{\theta}} K_t^{\alpha} L_{Yt}^{1-\alpha}$$

the growth rate of output in steady state is given by

$$g_Y = g_K = n + \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}g_A = \left(1 + \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{\lambda}{1-\phi}\right)n$$

so that the growth rate of output and capital in steady state depends only on structural parameters, not on investment rates or tax rates.[7] From the capital ac-

[6] I use g_z to denote the balanced growth path growth rate of the variable z.

[7] This is the distinguishing feature of semi-endogenous growth models. By contrast, first generation endogenous growth models (Romer (1990); Grossman and Helpman (1991); Aghion and Howitt (1992)) have strong scale effects so that the growth rate may be influenced by tax rates.

cumulation equation ($\dot{K}_t$) we know that consumption grows at the same rate as output and capital in steady state. Therefore, the consumption euler equation determines the steady-state interest rate: from the household problem, the growth rate of consumption is

$$\begin{aligned} \dot{c}_t/c_t &= \sigma((1-\tau_v)r_t - \rho) && (10) \\ &\rightarrow g_Y - n && (11) \\ \Rightarrow r^* &= \frac{\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{g_A}{\sigma} + \rho}{1-\tau_v} && (12) \end{aligned}$$

Capital income taxes do not affect the balanced growth rate of consumption. Higher tax rates raise the steady-state return to assets the household owns. Since the marginal product of capital is decreasing in the amount of capital, this means that the steady-state capital stock is lower. It turns out that the stock of knowledge is also lower in a steady state with higher capital income taxes, though it is less straightforward to demonstrate.

The fraction of labor allocated to the research and development sector is consistent with integrated labor markets. The wage paid to researchers is equal to the wage received by laborers producing final output. Therefore,

$$P_{At}\frac{\dot{A}_t}{L_{At}} = (1-\alpha)\frac{Y_t}{L_{Yt}} \tag{13}$$

which implies that

$$\frac{s_{At}}{1-s_{At}} = \frac{P_{At}\dot{A}_t}{(1-\alpha)Y_t}. \tag{14}$$

On the balanced growth path, asset arbitrage requires $P_{At} = \frac{\pi_t}{r^* - g_{P_A}}$, where $g_{P_A} = g_\pi = g_Y - g_A$. Consequently

$$\frac{s_A^*}{1-s_A^*} = \frac{\alpha(1-\theta)g_A}{(1-\alpha)(r^* - (g_Y - g_A))} \equiv \psi^*. \tag{15}$$

See Jones (1999) and Jones (2005) for more on this point.

The steady-state share of labor allocated to research and development is

$$s_A^* = \frac{\psi^*}{1+\psi^*} = \frac{\alpha(1-\theta)g_A}{(1-\alpha)(r^* - (g_Y - g_A)) + \alpha(1-\theta)g_A}. \tag{16}$$

Of the terms in this equation, only the steady-state interest rate depends on the tax rate applied to capital income. Intuitively, since higher interest rates lower the present value of future profits resulting from innovation, they reduce the price of a patented idea. Lower prices for patents discourage the research and development required to develop new ideas.[8]

The production function for new ideas shows that on the balanced growth path

$$A_t^* = \left[\frac{\nu L_t^\lambda s_A^{*\lambda}}{g_A}\right]^{\frac{1}{1-\phi}}. \tag{17}$$

Higher capital income taxes raise the interest rate and lower the fraction of workers producing new ideas. Therefore the balanced growth path stock of ideas is lower when tax rates are higher.

Along a balanced growth path, capital and output are equal to

$$\left(\frac{K}{Y}\right)^* = \frac{\alpha\theta}{r^*+\delta} \tag{18}$$

$$Y_t^* = A_t^{*\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}} \left(\frac{K}{Y}\right)^{*\frac{\alpha}{1-\alpha}} (1-s_A^*)L_t \tag{19}$$

$$K_t^* = \left(\frac{K}{Y}\right)^* Y_t \tag{20}$$

$$= A_t^{*\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}} \left(\frac{K}{Y}\right)^{*\frac{1}{1-\alpha}} (1-s_A^*)L_t. \tag{21}$$

Increases in the capital income tax rate lower A and K/Y but increase the fraction of workers producing physical output, so there are competing effects of capital income taxes on output. This mirrors the relationship between the optimal and equilibrium allocations in the Romer model. For some parametrizations the equi-

[8] Alternatively, higher tax rates cause less capital to be accumulated, raising its marginal product and therefore the interest rate. It follows that the share of labor working in R&D is lower the higher is the tax rate τ_v. As a result, the stock of knowledge is affected by τ_v.

librium involves overinvestment in R&D, while in others there is too little R&D (s_A is too small).[9] If their effect through labor market channels is strong enough, higher capital income taxes could actually increase the size of the stock of physical capital.

The stock of assets includes both physical capital and patented ideas. The total value of these assets on the balanced growth path is

$$V_t^* = K_t^* + P_{At}^* A_t^* \tag{22}$$

$$= \left(\frac{\alpha\theta}{r^* + \delta} + \frac{\alpha(1-\theta)}{r^* - g_{P_A}} \right) Y_t^* \tag{23}$$

so that the share of assets in the form of physical capital (versus patented ideas) depends on the steady state interest rate, which in turn depends on the tax rate on capital income.[10]

Since the value of innovations in the R&D sector are paid out to researchers as wages, changes in the allocation of labor and of the price of new ideas can affect the labor share of income. Note that total income in this model is $Y + P_A A$. Payments to labor are $wL = (1-\alpha)Y + P_A A$. A reduction in the tax rate on capital income lowers the real interest rate and raises the value of output in the R&D sector relative to the final goods sector. This in turn means the the labor share of income rises.

Tax revenue for the government is $\tau_v r^* V^*$. Of this, the tax base is $r^* V^*$. The Laffer conjecture in this context is that a reduction in the tax rate will cause the tax base to increase so much that the product of the two increases. Since a reduction

[9]For more, see Jones and Williams (1998, 2000) for a discussion of the social returns to R&D. Those papers discuss a related model that also includes a creative destruction distortion. Jones (2005) shows how the socially optimal rates of investment relate to the decentralized allocation's rates of investment in a model that does not have the creative destruction distortion.

[10]If $\delta = 0$ and $\alpha = \theta$, then $V_t^* = \alpha \frac{Y_t^*}{r^*} \left(\frac{r^* - \alpha n}{r^* - n} \right)$. In that case the asset structure of the economy depends on the growth rate of population and on the steady-state interest rate, which may respond to capital income taxes. Assuming there is no population growth, the share of assets that are physical capital is α, independent of τ_v. More generally, the effect of the population growth rate on the composition of assets depends on other parameters in the model. If $\alpha = \theta$, then higher n causes the growth rate of the price of an idea to be higher. This lowers the current price of a new idea and means more of the stock of assets will be physical capital. If $\alpha < \theta < 1$ and $\lambda > 1 - \phi$, entirely plausible values, it is possible for this effect to be reversed. For some such combinations of parameters higher population growth lowers the growth rate of the price of an idea, increasing its current price and the extent of investment in R&D.

in the tax rate causes the real interest rate to be lower in steady state, this would require a very large increase in the stock of assets.

2.4. Transition Dynamics

This section discusses transition dynamics for the economy. My approach is to log-linearize the key equations in the model. This forms the basis for the comparative dynamics exercises in section 3. More details are provided in the appendix. I log-linearize the model as follows. Define the vector γ as

$$\gamma_t = \begin{pmatrix} \gamma_{1t} \\ \gamma_{2t} \\ \gamma_{3t} \\ \gamma_{4t} \end{pmatrix} \equiv \begin{pmatrix} \log(C_t/K_t) \\ \log(\bar{Y}_t/K_t) \\ \log(s_{At}) \\ \log(\bar{\dot{A}}_t/A_t) \end{pmatrix} \tag{24}$$

where $\bar{Y}$ is the maximum output that could be obtained at a point in time, based on setting s_A equal to zero, and $\bar{\dot{A}}$ is the maximum rate of change of A that is possible at a point in time, based on setting s_A equal to one.[11] Therefore,

$$Y_t = A_t^{\alpha\frac{1-\theta}{\theta}} K_t^{\alpha} L_t^{1-\alpha}(1-s_{At})^{1-\alpha} = \bar{Y}_t(1-s_{At})^{1-\alpha} \tag{25}$$

and

$$\bar{\dot{A}}_t = \nu A_t^{\phi} L_t^{\lambda} = \dot{A}_t s_{At}^{-\lambda} \tag{26}$$

The limiting values of these variables are determined as follows. Equation (16) determines the steady-state value γ_3^*. Then γ_4^* is equal to $\log(g_A(s_A^*)^{-\lambda})$. The steady-state interest rate in equation (12) determines the steady-state capital out-

[11] Arnold (2006) analyzes the dynamics of this model with $\lambda = 1$. He reduces the model to a similar set of variables as I do here. The main differences are that one of his variables corresponds roughly to P_A rather than s_A and the variable that represents the output-capital ratio in his paper uses actual output rather than maximum output. I prefer to use maximum output so that this variable is a genuine state variable and is unable to jump. With my set-up there are two obvious state variables and two control variables that correspond to the two key allocation decisions in the model: to consume or invest, and to produce final output or to produce ideas. See also Schmidt (2003) for another discussion of transition dynamics in the Romer model.

put ratio, which combined with s_A^* determines γ_2^*. Finally, $C/K = Y/K - \dot{K}/K - \delta$ which determines γ_1^*.

It is convenient to work with these four variables since they are each constant on a balanced growth path. Two correspond roughly to the state variables in the model (γ_4 relates to the stock of knowledge, γ_2 to the capital stock), and do not jump. By contrast, the other two correspond to control variables (γ_1 to the investment rate, and γ_3 to the intensity of research and development efforts) and can jump. The dynamics of the four variables are determined by two initial conditions (K_0 and A_0) and two endpoint conditions (the limiting behavior of C and s_A).

The rate of change of γ is given by

$$\dot{\gamma}_t = \begin{pmatrix} \frac{\dot{C}_t}{C_t} - \frac{\dot{K}_t}{K_t} \\ \frac{\dot{Y}_t}{Y_t} - \frac{\dot{K}_t}{K_t} \\ \frac{\dot{s_{At}}}{s_{At}} \\ \lambda \frac{\dot{L}_t}{L_t} - (1-\phi)\frac{\dot{A}_t}{A_t} \end{pmatrix} \tag{27}$$

Equations for three elements of this vector are straightforward. The growth rate of consumption is given by the household's Euler equation. The growth rate of the capital stock comes from the capital accumulation equation. The growth rate of the maximum growth rate of A is determined by the growth rate of A and of population. The growth rate of s_{At} is rather more complicated, and is based on the dynamics of the labor market equilibrium condition in equation (14). This equation implies that the rate of change of s_A is influenced by the rate of change of P_A, A, K, and L. If the price of patented ideas is rising over time then, all else equal, the fraction of labor allocated to R&D will also be rising. The derivation of each equation is covered in the appendix.

For all the calibrations I applied, the log-linearized system of equations was characterized by two negative and two positive eigenvalues. This is consistent with there being two state variables and two jump variables (C and s_A). Arnold (2006) shows that a slightly simpler version of this model without taxes must have two negative and two positive eigenvalues.

3. Comparative Dynamics: Response to τ_v Changes

This section discusses the response of the economy in general and tax revenues in particular when there is a change in the capital income tax rate. It shows the long-run response of tax revenues to tax rates as well as transition paths for a range of variables when there is a change in the capital income tax rate.

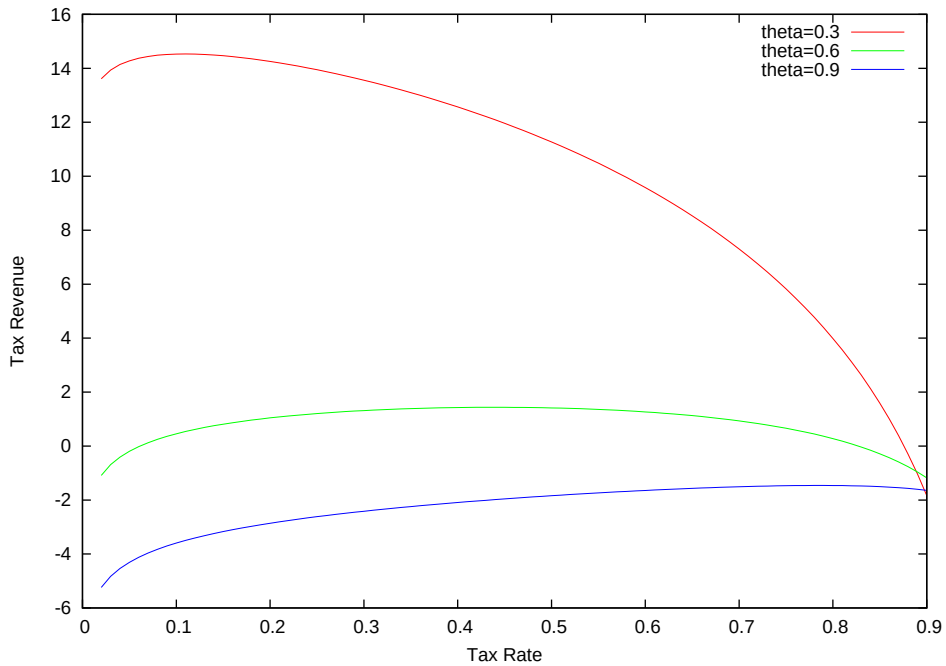
3.1. Static Response of Tax Revenue

In the Ramsey model, the interest rate at a point in time is determined by the capital stock. Factor supplies are inelastic in the short-run, so the marginal product of capital is a given. The elasticity of tax revenue with respect to the capital income tax rate is equal to one in the Ramsey model in the short run (Mankiw and Weinzierl (2006)). This is because the marginal product of capital, which determines the real interest rate, is pinned down by the capital stock, and the capital stock cannot jump. In the Romer model, the marginal product of capital depends on the allocation of labor between the two sectors. And even if the real interest rate were not to jump in the R&D model, if the price of a patented idea jumps, then the stock of assets whose income streams are taxed also jumps so that the elasticity of tax revenue with respect to changes in the tax rate need not be one. For some parametrizations, tax revenue jumps less than the percentage of the change in the tax rate, while in other parametrizations it jumps more.

3.2. Long-run Response of Tax Revenue

In the long run, the response of tax revenue to the tax rate on capital income depends mainly on a small number of key parameters. First, θ, which governs the substitutability in production of different kinds of capital goods, has a particularly important role. For low values of θ, low tax rates are consistent with high tax revenues, so that a very relevant Laffer curve effect is present. Evidence on the share of income received as pure profits (returns to patents) and on markups suggest that such values of θ are implausible. For higher values of θ (closer to 0.9 so that markups are around 10%) suggest that tax revenues are maximized at tax

Figure 1: Steady-State Tax Revenue as a Function of the Tax Rate



rates closer to 85%.[12]

Figure 1 shows the (log of) steady-state tax revenue as a function of the tax rate for three different values of θ.[13] For high values of θ the long-run elasticity of output with respect to A is low. Therefore, lower tax rates, while they increase the stock of knowledge and hence output, do not raise government revenue except when starting from extremely high tax rates. For lower values of θ, tax revenue peaks as a function of the tax rate at relatively moderate tax rates. Such low values of θ imply that a large share of income is accrued as pure profits, which is inconsistent with evidence from Basu and Fernald (1997) and Broda and Weinstein (2006). Parameter values used to generate these figures are reported in Table 1.

The subsequent figures, 3(a) and 3(b), show steady-state output and con-

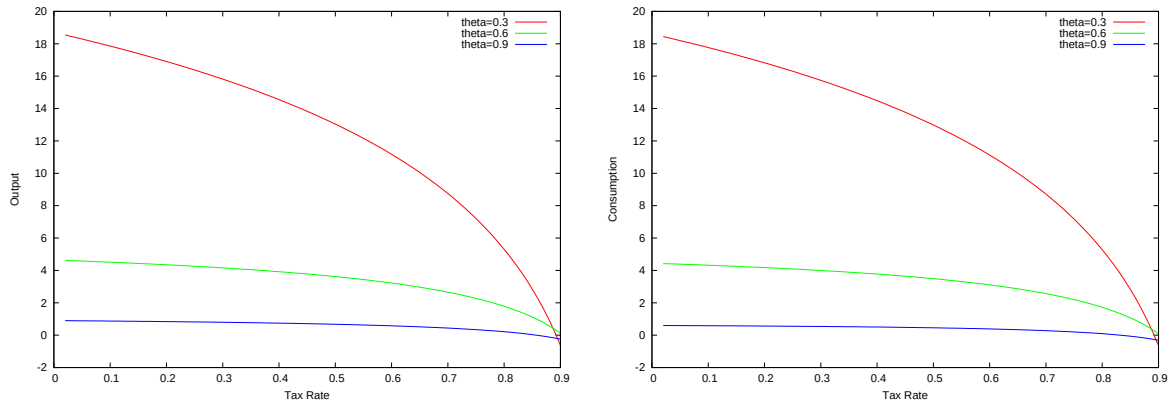
[12]By contrast, in the Ramsey model, tax revenues are maximized when $\tau_v = 1 - \alpha$ where α is the elasticity of ouptut with respect to capital.

[13]These are for a particular point in time. On the balanced growth path tax revenue grows at the same rate regardless of the tax rate, so choosing a different point in time would amount to shifting the curves in this graph up or down by some constant amount.

Table 1: Calibrated Values of Model Parameters

Parameter	Meaning	Calibrated Value
α	Elasticity of Y w.r.t. K	1/3
θ	Related to Elasticity of Substitution between Varieties of Capital	0.7
δ	Depreciation rate for physical capital	0.05
ν	Productivity in R&D	1
λ	Elasticity of $\dot{A}$ w.r.t. L_A	0.7
ϕ	Elasticity of $\dot{A}$ w.r.t. A	0.9
ρ	Discount rate	0.02
σ	Intertemporal elasticity of substitution	1
n	Population growth rate	0.01

Figure 2: Steady-State Output and Consumption



(a) Steady-State Output as a Function of the Tax Rate

(b) Steady-State Consumption as a Function of the Tax Rate

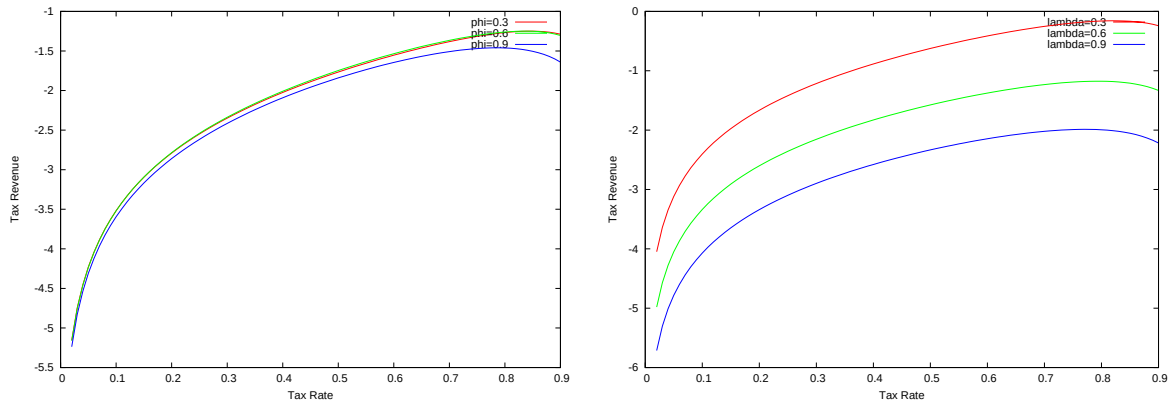
sumption as functions of the tax rate, again for different values of θ. Since taxing capital income does not correct the underlying distortions in this economy, higher taxes are always associated with lower output and consumption in the model.

One feature of the graphs above that may not be obvious from casual inspection is that the long-run response of tax revenue to a change in the tax rate may be greater than the short-run response. Typically, the dynamic scoring perspective on the effects of tax rate changes is that the short-run changes in tax revenue are followed by movements in the tax base that mute the initial effects on the government budget. A tax cut is followed by an expansion of the tax base, making up some of the lost revenue. A tax increase is followed by a contraction of the tax base, damping the revenue gains (Mankiw and Weinzierl (2006)). In this model, the tax base actually contracts in response to a tax cut, at least for certain parameter values. Figure 4 makes this somewhat clearer.

Figures 4(a) and 4(b) show how steady-state tax revenue responds to the tax rate for different values of ϕ and λ. Most significant about these graphs is that variation in ϕ is less important than variation in the tax rate, since the lines in figure 4(a) are close together.

Figure 4 shows that as θ increases, tax cuts are less likely to generate large rev-

Figure 3: Steady-State Tax Revenue as a Function of the Tax Rate



(a) For values of ϕ (b) For values of λ

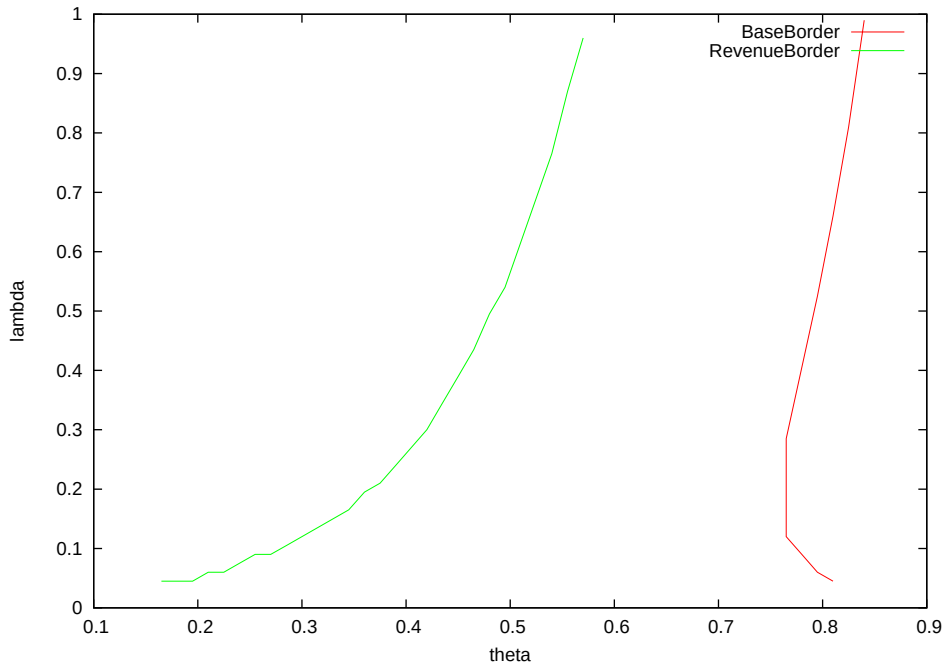
enue gains, or even large offsetting tax base expansions. As λ increases, revenue gains are more likely, at least for lower values of θ. While there is some evidence that is relevant to the appropriate value of θ, it is less clear how to calibrate λ in conjunction with ϕ. Are the stepping on toes effects important or not? What is a reasonable guess at the long-run growth rate of the economy, since the ratio $\lambda/(1-\phi)$ is important in determining this growth rate.

3.3. Dynamic Response of Tax Revenue

This section illustrates the response of the economy to a reduction in tax rates from 40% to 35%. As in the set-up above, there are no taxes other than capital income taxes. The economy starts on its balanced growth path, then faces a new, permanently lower tax rate. The dynamic response of the economy is computed using the log-linearized version of the model.

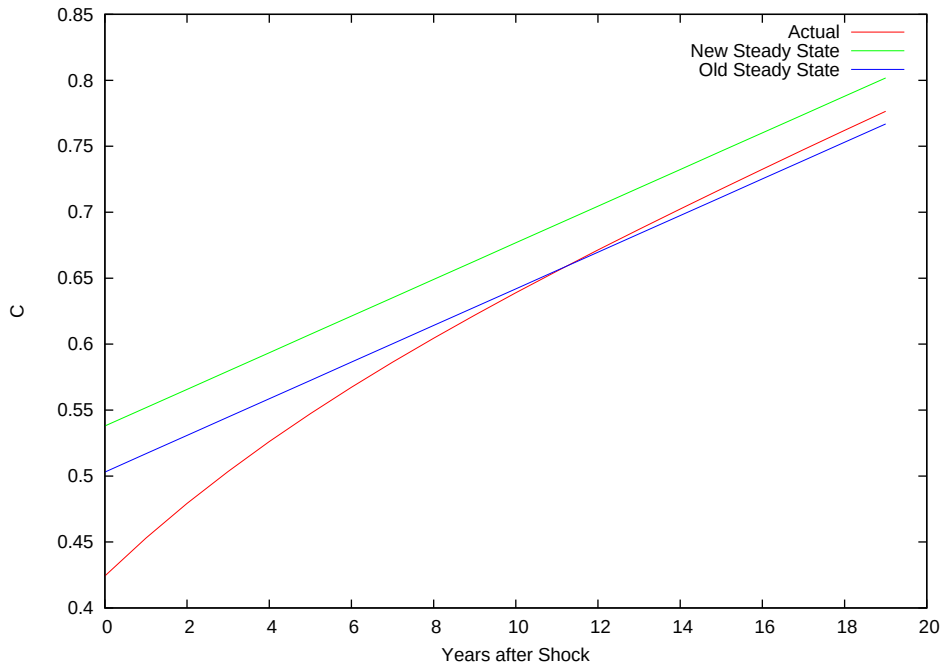
Figures 5 and 6 show the responses of the two key allocation choices in the economy, consumption and the allocation of labor between the two sectors. Initially consumption drops around 5%, but quickly rises to be above the previous balanced growth path, eventually converging to the new steady-state with consumption around 2% higher than it would have been without the tax reform. This cut in current consumption is a response to the suddenly higher after-tax re-

Figure 4: Thresholds for Long-run Revenue Neutrality and Tax Base Neutrality of a Tax Cut



Notes: the Base Border curve indicates combinations of θ and λ that are consistent with a tax cut inducing no change in the size of the tax base in the long run. The Revenue Border curve indications combinations of the parameters that are consistent with not long-run change in tax revenue in response to a tax cut. (That is, the economy is at the peak of the Laffer Curve.) These curves are constructed based on a change in the tax rate from 25% to 24%.

Figure 5: Consumption Response to a Lower Tax Rate



turns available. Consumers are willing to reduce current consumption because, more than at the higher tax rate, this generates a build-up in the capital stock that advances output.

The response of the labor share working in R&D is more subtle. Initially the share working in R&D jumps up toward the new steady state. But after the jump the share gradually falls before eventually converging to the new steady-state value. This non-monotonic convergence is due to the dynamics of the system being governed by two negative eigenvalues. For s_A the signs of the coefficients on the corresponding eigenvectors are opposite, hence the initial drift away from the steady state before convergence.

Figure 7 shows the price of patented inventions jumping up from its initial balanced growth path, though it eventually converges to a level below its prior trajectory.

The tax revenue generated for the government falls initially, since the tax rate is reduced. Somewhat surprisingly, it continues to grow more slowly than its steady-state growth rate, even dipping below the new balanced growth path

Figure 6: Labor Allocation Response to a Lower Tax Rate

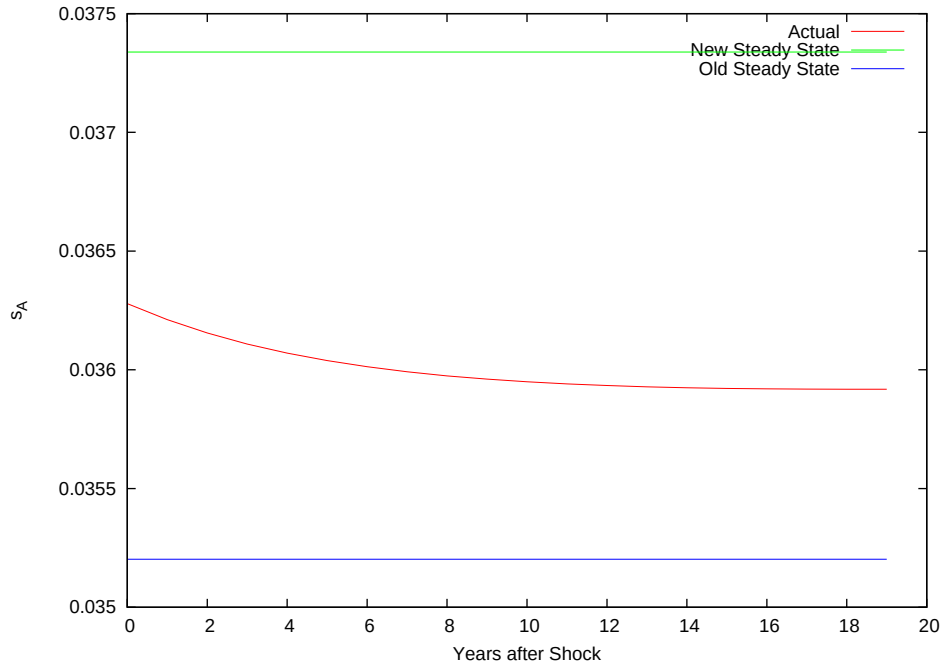


Figure 7: Response of the Price of a Patent to a Lower Tax Rate

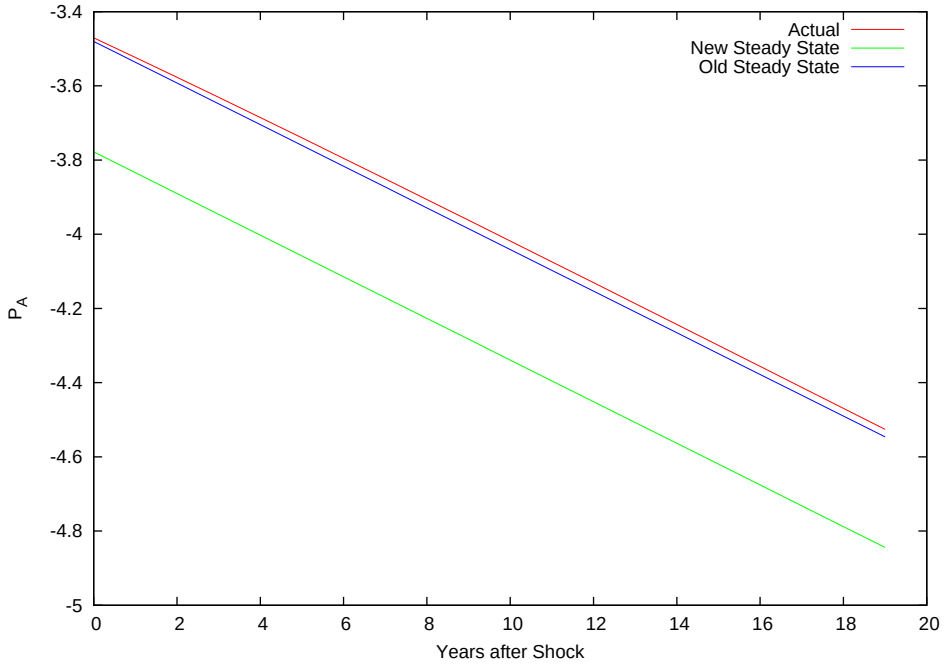


Figure 8: Response of Tax Revenue to a Lower Tax Rate

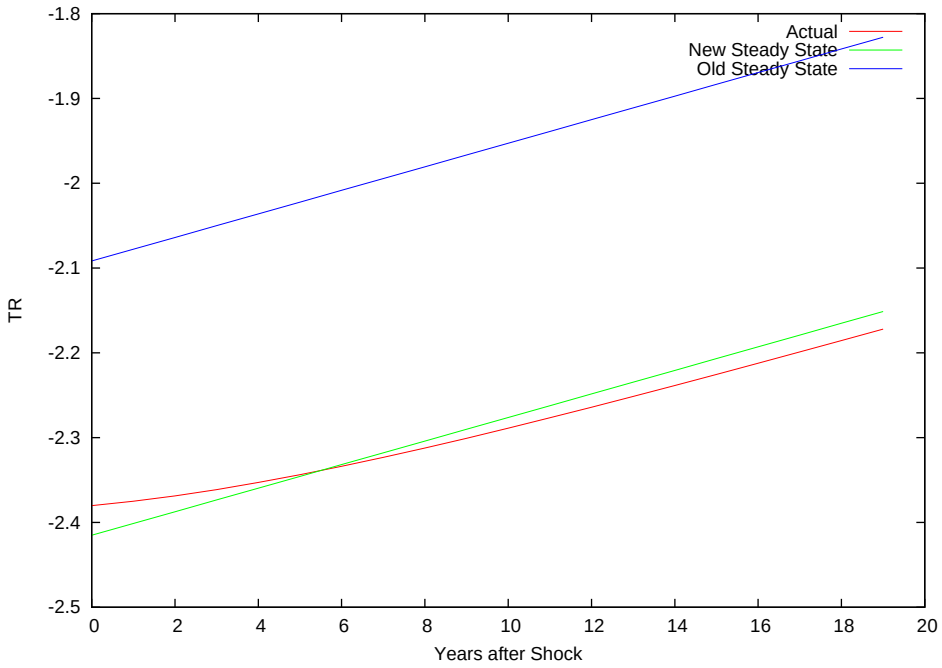
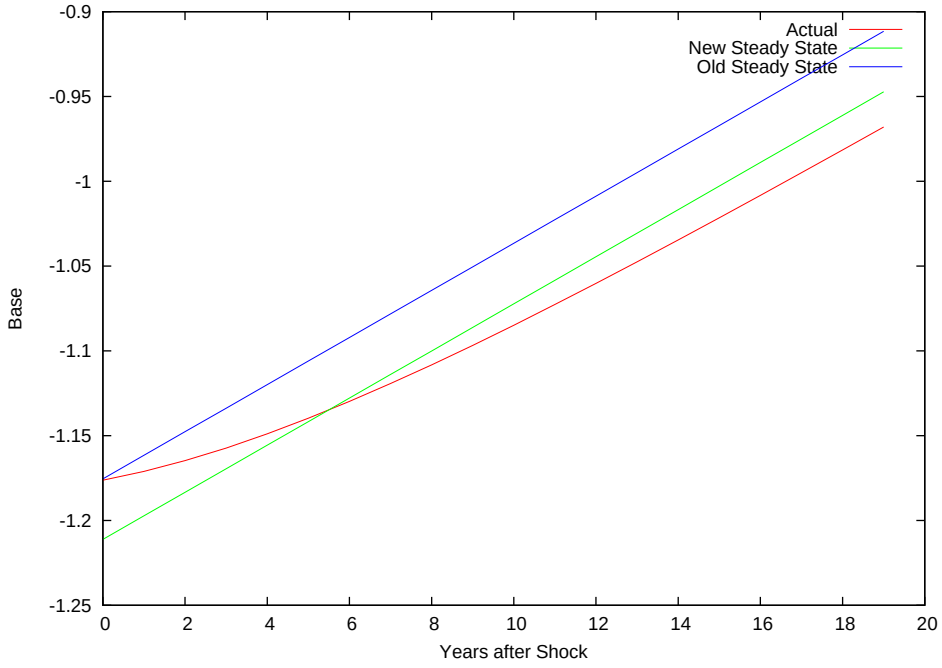


Figure 9: Response of Tax Base to a Lower Tax Rate



to which it eventually converges. What is most surprising about this is that the long-run response of tax revenues is larger than the short-run response. In Mankiw and Weinzierl (2006), the initial drop in tax revenue is compensated for partially in the long run, since the tax base expands. In this parametrization of the endogenous growth model the tax base is lower at the lower tax rate. Figure 9 shows the dynamic response of the tax base.

For low values of θ, tax revenue is a concave function of the tax rate for most values of τ_v. For values that are more in line with evidence on the extent of profits in the economy, this function is convex for low values of τ_v. Mechanically, this is the origin of the result that the long-run response of tax revenue is larger than the short-run response.

4. Conclusion

This paper has investigated the dynamic response of tax revenue to changes in the tax rate applied to capital income in a model of endogenous growth due to research and development. The model modifies Romer (1990) and Jones (1995a) to incorporate a tax on capital income, without distinguishing between income derived from physical capital and pure profits that accrue to patent holders. The model is log-linearized and the dynamic response of the economy to a tax cut is presented.

The paper highlights an unexpected result. For plausible parameter values and tax rates, steady-state tax revenue from the capital income tax is a convex function of the tax rate. This is surprising in part because tax revenues are usually thought of as concave functions of the tax rate. What it means in this instance is that a reduction in the tax rate sees tax revenue fall by a larger fraction in the long-run. This contrasts with other studies of dynamic scoring that emphasize the long-run consequences of a tax reduction for an expanded tax base. More work is required to fully understand this result and to determine whether it is sensitive to the specification of the model here.

A Appendix: Log-Linearizing the Model

A1. Rate of Change

From the household's Euler Equation, we know that

$$\frac{\dot{C}_t}{C_t} = \sigma((1-\tau_v)r_t - \rho) + n \tag{28}$$

where

$$\begin{aligned} r_t &= \alpha\theta\frac{Y_t}{K_t} - \delta && (29)\\ &= \alpha\theta\frac{\bar{Y}_t}{K_t}(1-s_{At})^{1-\alpha} - \delta && (30)\\ &= \alpha\theta e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - \delta && (31) \end{aligned}$$

The capital accumulation equation is standard and gives

$$\begin{aligned} \frac{\dot{K}_t}{K_t} &= \frac{Y_t}{K_t} - \frac{C_t}{K_t} - \delta && (32)\\ &= e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - e^{\gamma_{1t}} - \delta. && (33) \end{aligned}$$

Therefore,

$$\dot{\gamma_{1t}} = \sigma((1-\tau_v)(\alpha\theta e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - \delta) - \rho) + n - e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} + e^{\gamma_{1t}} + \delta \tag{34}$$

The second element of γ changes according to the growth rates of $\bar{Y}$ and K. Note that maximum output can be written as

$$\bar{Y}_t = A_t^{\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}}\left(\frac{K_t}{\bar{Y}_t}\right)^{\frac{\alpha}{1-\alpha}} L_t \tag{35}$$

so the growth rate of $\bar{Y}$ is

$$\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{\dot{A}_t}{A_t} + \frac{\alpha}{1-\alpha}\left(\frac{\dot{K}_t}{K_t} - \frac{\dot{\bar{Y}}_t}{\bar{Y}_t}\right) + n \tag{36}$$

This implies that

$$\dot{\gamma_{2t}} = \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{\dot{A}_t}{A_t} - \frac{\alpha}{1-\alpha}\left(\frac{\dot{K}_t}{K_t} - \frac{\dot{\bar{Y}}_t}{\bar{Y}_t}\right) + n - \frac{Y_t}{K_t} + \frac{C_t}{K_t} + \delta \tag{37}$$

$$= \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}} - \frac{\alpha}{1-\alpha}\dot{\gamma_{2t}} + n - e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} + e^{\gamma_{1t}} + \delta \tag{38}$$

$$= \alpha\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}} + (1-\alpha)(n - e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} + e^{\gamma_{1t}} + \delta) \tag{39}$$

The rate of change of γ_4 is straightforward also.

$$\dot{\gamma_{4t}} = (\phi-1)\frac{\dot{A}_t}{A_t} + \lambda\frac{\dot{L}_t}{L_t} \tag{40}$$

$$= -(1-\phi)e^{\lambda\gamma_{3t}+\gamma_{4t}} + \lambda n \tag{41}$$

The rate of change of γ_{3t} is equal to

$$\dot{\gamma_{3t}} = \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}((1-\tau_v)(\alpha\theta e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - \delta) - (1-\alpha-\lambda)n \tag{42}$$

$$-\alpha(e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - e^{\gamma_{1t}} - \delta) \tag{43}$$

$$+e^{\lambda\gamma_{3t}+\gamma_{4t}}(\phi - \alpha\frac{1-\theta}{\theta} + (1-\theta)\frac{\alpha}{1-\alpha}\frac{1-e^{\gamma_{3t}}}{e^{\gamma_{3t}}})) \tag{44}$$

A2. Linearization

Linearize the transition equations above. Evaluate the Jacobian at the steady-state values.

$$\frac{\partial\dot{\gamma_{1t}}}{\partial\gamma_{1t}} = e^{\gamma_{1t}} \tag{45}$$

$$\frac{\partial\dot{\gamma_{1t}}}{\partial\gamma_{2t}} = e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\left(\alpha\theta\sigma(1-\tau_v)-1\right) \tag{46}$$

$$\frac{\partial\dot{\gamma_{1t}}}{\partial\gamma_{3t}} = -(1-\alpha)e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\left(\alpha\theta\sigma(1-\tau_v)-1\right)\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}} \tag{47}$$

$$\frac{\partial\dot{\gamma_{1t}}}{\partial\gamma_{4t}} = 0 \tag{48}$$

$$\frac{\partial \dot{\gamma_{2t}}}{\partial \gamma_{1t}} = (1-\alpha)e^{\gamma_{1t}} \tag{49}$$

$$\frac{\partial \dot{\gamma_{2t}}}{\partial \gamma_{2t}} = -(1-\alpha)e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} \tag{50}$$

$$\frac{\partial \dot{\gamma_{2t}}}{\partial \gamma_{3t}} = \alpha\lambda\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}} + (1-\alpha)^2 e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}} \tag{51}$$

$$\frac{\partial \dot{\gamma_{2t}}}{\partial \gamma_{4t}} = \alpha\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}} \tag{52}$$

$$\frac{\partial \dot{\gamma_{3t}}}{\partial \gamma_{1t}} = \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}\alpha e^{\gamma_{1t}} \tag{53}$$

$$\frac{\partial \dot{\gamma_{3t}}}{\partial \gamma_{2t}} = \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}\alpha e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}((1-\tau_v)\theta-1) \tag{54}$$

$$\frac{\partial \dot{\gamma_{3t}}}{\partial \gamma_{3t}} = \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}\Big(\quad \alpha(1-\alpha)e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\frac{e^{\gamma_{3t}}}{1-e_{3t}^{\gamma}}(1-(1-\tau_v)\theta) \tag{55}$$

$$+e^{\lambda\gamma_{3t}+\gamma_{4t}}\lambda(\phi-\alpha\frac{1-\theta}{\theta}+(1-\theta)\frac{\alpha}{1-\alpha}\frac{1-e^{\gamma_{3t}}}{e^{\gamma_{3t}}}) - e^{\lambda\gamma_{3t}+\gamma_{4t}}\frac{\alpha}{1-\alpha}\frac{1-\theta}{e^{\gamma_{3t}}} \tag{56}$$

$$-\frac{\alpha}{1-e^{\gamma_{3t}}}\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}\frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}\dot{\gamma_{3t}} \tag{57}$$

$$\frac{\partial \dot{\gamma_{3t}}}{\partial \gamma_{4t}} = \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}e^{\lambda\gamma_{3t}+\gamma_{4t}}\left(\phi-\alpha\frac{1-\theta}{\theta}+(1-\theta)\frac{\alpha}{1-\alpha}\frac{1-e^{\gamma_{3t}}}{e^{\gamma^{3t}}}\right) \tag{58}$$

$$\frac{\partial \dot{\gamma_{4t}}}{\partial \gamma_{1t}} = 0 \tag{59}$$

$$\frac{\partial \dot{\gamma_{4t}}}{\partial \gamma_{2t}} = 0 \tag{60}$$

$$\frac{\partial \dot{\gamma_{4t}}}{\partial \gamma_{3t}} = -(1-\phi)\lambda e^{\lambda\gamma_{3t}+\gamma_{4t}} \tag{61}$$

$$\frac{\partial \dot{\gamma_{4t}}}{\partial \gamma_{4t}} = -(1-\phi)e^{\lambda\gamma_{3t}+\gamma_{4t}} \tag{62}$$

We can write the linearized system as

$$(\gamma_t \dot{-} \gamma^*) \approx \Gamma(\gamma_t - \gamma^*) \tag{63}$$

$$\Gamma = \begin{pmatrix} \frac{r^*+\delta}{\alpha\theta} - g - \delta & \frac{r^*+\delta}{\alpha\theta}(\alpha\theta\sigma(1-\tau_v) - 1) & -\frac{r^*+\delta}{\theta}(1-\theta)\frac{\lambda n}{1-\phi}\frac{\alpha\theta\sigma(1-\tau_v)-1}{\rho+\frac{g-n}{\sigma}-(g-g_A)} & 0 \\ (1-\alpha)(\frac{r^*+\delta}{\alpha\theta} - g - \delta) & -(1-\alpha)\frac{r^*+\delta}{\alpha\theta} & \frac{\lambda n}{1-\phi}\frac{1-\theta}{\theta}(\alpha\lambda + (1-\alpha)(r^*+\delta)) & \alpha\frac{1-\theta}{\theta}\frac{\lambda n}{1-\phi} \\ x\alpha(\frac{r^*+\delta}{\alpha\theta} - g - \delta) & x\alpha\frac{r^*+\delta}{\alpha\theta}((1-\tau_v)\theta - 1) & \Gamma_{3,3} & \Gamma_{4,4} \\ 0 & 0 & -\lambda^2 n & -\lambda n \end{pmatrix} \tag{64}$$

where r^* is the steady-state interest rate from equation (12), and g is the steady-state growth rate of output, capital and consumption; $x = (1-\lambda+\frac{\alpha^2}{1-\alpha}(1-\theta)\frac{\lambda n}{1-\phi}\frac{1}{(1-\tau_v)r^*-(g-g_A)})^{-1}$, and

$$\begin{aligned} \Gamma_{3,3} &= x(\alpha(r^*+\delta)\frac{1-\theta}{\theta}\frac{\lambda n}{1-\phi}\frac{1-(1-\tau_v)\theta}{(1-\tau_v)r^*-(g-g_A)} + \frac{\lambda^2\phi n}{1-\phi} - \\ &\quad \frac{\alpha\lambda^2 n}{1-\phi}\frac{1-\theta}{\theta} + \lambda((1-\tau_v)r^* - (g-g_A))) \end{aligned}$$

and

$$\Gamma_{4,4} = x\left(\frac{\lambda^2\phi n}{1-\phi} - \frac{\alpha\lambda^2 n}{1-\phi}\frac{1-\theta}{\theta} + \lambda((1-\tau_v)r^* - (g-g_A))\right)$$

A3. Solutions of the Linearized System

The linearized system of equations is solved using the standard eigenvalue decomposition. Initial conditions for K and A generate the required boundary conditions to obtain the particular solution.

References

Agell, J. and M. Persson, "On the Analytics of the Dynamic Laffer Curve," *Journal of Monetary Economics*, 2001, *48* (2), 397–414.

Aghion, Philippe and Peter Howitt, "A Model of Growth through Creative Destruction," *Econometrica*, March 1992, *60* (2), 323–351.

Arnold, L. G., "The Dynamics of the Jones R&D growth Model," *Review of Economic Dynamics*, 2006, *9* (1), 143–152.

Auerbach, A. J., "Dynamic Revenue Estimation," *Journal of Economic Perspectives*, 1996, *10*, 141–158.

— and L. J. Kotlikoff, *Dynamic Fiscal Policy*, Cambridge University Press, 1987.

Barro, Robert J., "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, October 1990, *98*, S103–S125.

Basu, Susanto and John G. Fernald, "Returns to Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy*, April 1997, *105*, 249–283.

Broda, C. and D. E. Weinstein, "Globalization and the Gains from Variety," *Quarterly Journal of Economics*, 2006, *121* (2), 541–585.

Bruce, N. and S. J. Turnovsky, "Budget Balance, Welfare, and the Growth Rate: "Dynamic Scoring" of the Long-Run Government Budget.," *Journal of Money, Credit & Banking*, 1999, *31* (2), 162–163.

Chamley, C., "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 1986, pp. 607–622.

De Hek, P. A., "On Taxation in a Two-sector Endogenous Growth Model with Endogenous Labor Supply," *Journal of Economic Dynamics and Control*, 2006, *30* (4), 655–685.

Easterly, W. and S. Rebelo, "Fiscal Policy and Economic Growth," *Journal of Monetary Economics*, 1993, *32*, 417–458.

Ferede, E., "Dynamic Scoring in the Ramsey Growth Model," *The B. E. Journal of Economic Analysis & Policy*, 2008, *8* (1), 1948.

Fullerton, D., "On the Possibility of an Inverse Relationship Between Tax Rates and Government Revenues," *Journal of Public Economics*, 1982, *19* (1), 3–22.

Goolsbee, A., "What Happens When You Tax the Rich? Evidence from Executive Compensation," *Journal of Political Economy*, 2000, *108* (2), 352–378.

Grossman, Gene M. and Elhanan Helpman, *Innovation and Growth in the Global Economy*, Cambridge, MA: MIT Press, 1991.

Hendricks, L., "Taxation and Long-run Growth," *Journal of Monetary Economics*, 1999, *43* (2), 411–434.

Ireland, P. N., "Supply-side Economics and Endogenous Growth," *Journal of Monetary Economics*, 1994, *33* (3), 559–571.

Jones, Charles I., "R&D-Based Models of Economic Growth," *Journal of Political Economy*, August 1995, *103* (4), 759–784.

— , "Time Series Tests of Endogenous Growth Models," *Quarterly Journal of Economics*, May 1995, *110* (441), 495–525.

— , "Growth: With or Without Scale Effects?," *American Economic Association Papers and Proceedings*, May 1999, *89*, 139–144.

— , "Growth and Ideas," in Philippe Aghion and Steven N. Durlauf, eds., *Handbook of Economic Growth*, Vol. 1B 2005, chapter 16, pp. 1063–1114.

— and John C. Williams, "Measuring the Social Return to R&D," *Quarterly Journal of Economics*, November 1998, *113* (455), 1119–1135.

— and — , "Too Much of a Good Thing? The Economics of Investment in R&D," *Journal of Economic Growth*, March 2000, *5* (1), 65–85.

Jones, L. E., R. E. Manuelli, and P. E. Rossi, "Optimal Taxation in Models of Endogenous Growth," *Journal of Political economy*, 1993, pp. 485–517.

— , — , and — , "On the Optimal Taxation of Capital Income," *Journal of Economic Theory*, 1997, *73* (1), 93–117.

Judd, K., "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 1985, *28*, 59–83.

Lucas, Robert E., "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 1988, *22* (1), 3–42.

Malcomson, James M., "Some Analytics of the Laffer Curve," *Journal of Public Economics*, April 1986, *29* (3), 263–279.

Mankiw, N. G. and M. Weinzierl, "Dynamic Scoring: A Back-of-the-Envelope Guide," *Journal of Public Economics*, 2006, *90* (8-9), 1415–1433.

Mendoza, E. G., G. M. Milesi-Ferretti, and P. Asea, "On the Ineffectiveness of Tax Policy in Altering Long-run Growth: Harberger's Superneutrality Conjecture," *Journal of Public Economics*, 1997, *66* (1), 99–126.

Milesi-Ferretti, G. M. and N. Roubini, "Growth Effects of Income and Consumption Taxes.," *Journal of Money, Credit & Banking*, 1998, pp. 721–722.

__ and __ , "On the Taxation of Human and Physical Capital in Models of Endogenous Growth," *Journal of Public Economics*, 1998, *70* (2), 237–254.

Novales, A. and J. Ruiz, "Dynamic Laffer Curves," *Journal of Economic Dynamics and Control*, 2002, *27* (2), 181–206.

Pecorino, P., "Tax Rates and Tax Revenues in a Model of Growth Through Human Capital Accumulation," *Journal of Monetary Economics*, 1995, *36* (3), 527–539.

Romer, Paul M., "Endogenous Technological Change," *Journal of Political Economy*, October 1990, *98* (5), S71–S102.

Schmidt, G. W., *Dynamics of Endogenous Economic Growth: A Case Study of the "Romer Model"*, North Holland, 2003.

Stokey, N. L. and S. Rebelo, "Growth Effects of Flat-tax Rates," *Journal of Political Economy*, 1995, *103* (3), 519–50.