

International Portfolios: An Incomplete Markets General Equilibrium Approach

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September 2010

Abstract

We study an international portfolio choice in a general equilibrium incomplete markets model. We solve for a wealth-recursive equilibrium which is shown to exist in our setting. We calibrate the model to the U.S. versus OECD economies and solve the model nonlinearly. We depart from a symmetric two-country setting by assuming that a) debt market is dominated by dollar assets and b) the U.S. economy is less volatile than a typical OECD economy. In equilibrium the U.S. accumulates a negative position in debt and has a significant negative net foreign asset position.

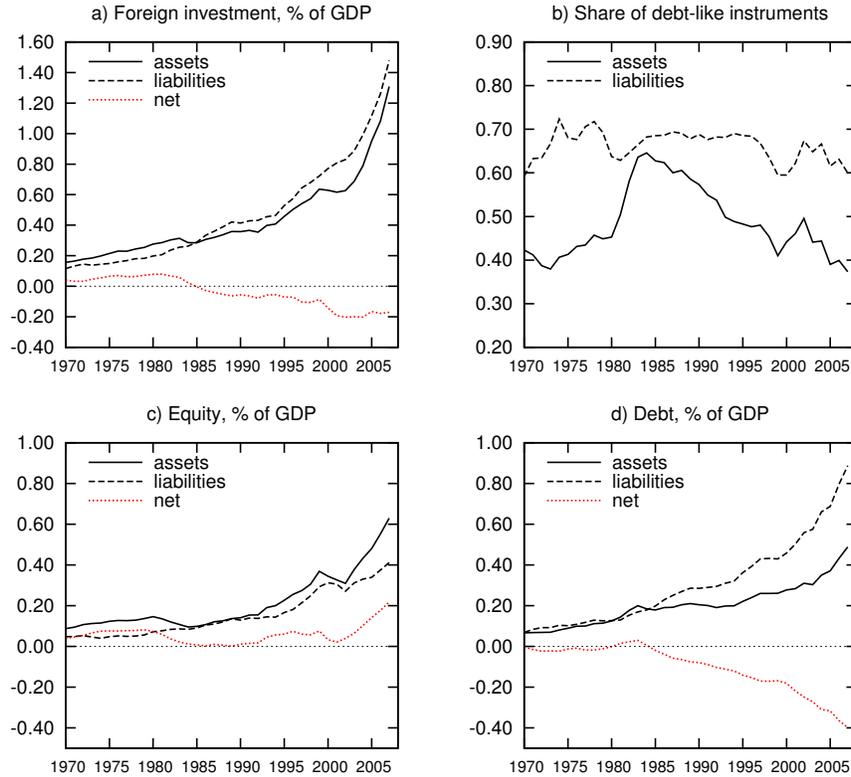
1 Introduction

During the past several decades we witnessed a growing financial integration between the world economies. There was a spectacular increase in both the number of internationally-traded assets and the magnitude of the cross-border gross capital flows.¹ According to the dataset compiled by Lane & Milesi-Ferretti (2007b) the total gross foreign assets of the U.S. were 16% of GDP in 1970, stayed below 32% until 1984 and grew to 131% of GDP in 2007. The total gross foreign liabilities were 12% of GDP in 1970, stayed below 30% until 1984 and later grew to 148% of GDP in 2007.

The increase of the net and gross international capital flows led to the “global imbalances” and, in particular, to a significant deterioration of the the U.S. net foreign asset (NFA) position. As can be seen from figure 1 panel

¹For a detailed account of these developments see Gourinchas & Rey (2007) and Lane & Milesi-Ferretti (2008).

Figure 1: International investment position of the U.S.



(c) and (d) the NFA deterioration in the U.S. was driven by accumulation of debt liabilities while the NFA in equity improved. Gourinchas & Rey (2007) conclude that “as financial globalization accelerated its pace, the U.S. transformed itself from a world banker into a world venture capitalist, investing greater amounts into high yield assets such as equity and FDI”, while “its liabilities have remained dominated by bank loans, trade credit and debt, i.e. low yield safe assets”. This evidence is supported by Obstfeld (2004), who states that for the U.S., “the striking change since the early 1980s is the sharp growth in foreign portfolio equity holdings”, while on the liabilities’ side, “the most dramatic percentage increase has been in the share of U.S. bonds held by foreigners”. Similar observations were made by Mendoza, Quadrini & Rios-Rull (2008) and Obstfeld & Rogoff (2000).

To address the questions related to the dynamics, direction (and/or sus-

tainability of capital flows) one needs to understand the factors that drive the international portfolio choice, and thus, develop and solve a model with multiple internationally traded assets. Obstfeld (2004) writes:

[the early intertemporal models of the current account] now look manifestly inadequate to describe the dynamics of net foreign assets in the brave new world of huge two-way diversification flows.

A model that would be capable of capturing the developments described above must have the following three ingredients. First, it must be an incomplete markets economy. In complete markets models with time-separable preferences portfolios are typically constant and, therefore, capital flows are absent.² Second, it must have multiple assets. Two assets are needed to model gross asset position and three assets are needed if one wants to distinguish between equity and debt flows. Third, the model must be capable of modeling asymmetric economies. In a symmetric model any imbalances are necessarily transitory unless there are multiple equilibria that naturally pose problems for a quantitative analysis.

We obtain a “global” solution to the international portfolio choice problem in a two-country two-good model by adapting the projection method developed in a series of papers by Judd, Kubler and Schmedders.³ In order to generate NFA imbalances, we introduce into the model two features that make model economies asymmetric.

1. International debt markets are dominated by the assets denominated in only a few “global” currencies.⁴ The U.S. dollar played a dominant role here until recently, when the introduction of euro led to the increased share of euro-denominated internationally traded debt assets. The rise of euro can be partially accounted for by increased financial flows within the Eurozone. But trade in euro assets with non-euro economies may be limited. Finally, the U.S. dollar dominates trades in commodity markets.⁵

We model the role of the U.S. dollar as the leading global currency by assuming that there is only one internationally-traded bond that pays

²For a proof of this statement see Judd, Kubler & Schmedders (2000).

³See Judd et al. (2000) and Kubler & Schmedders (2003).

⁴For a review see Eichengreen & Hausmann (2005).

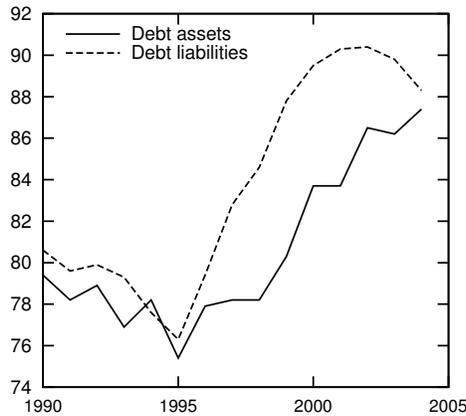
⁵See Tesar & Tille (2009) for the role of the U.S. dollar in international trade.

off in the good of one of the two countries. This asymmetry allows us to generate a negative net debt position for the economy that has a privilege of issuing debt denominated in its domestic good.

2. The U.S. economy is half as volatile as an average OECD economy (see Data section for details about this statement). This difference was less stark before the great moderation that started around 1984.⁶

By assuming that output shocks in one economy are smaller we are able to obtain a realistic portfolio of foreign assets. A country with higher volatility for precautionary reasons demands safe assets (bonds) even if denominated in foreign currency. The country also accumulates more wealth driving the economy with lower volatility into a state of essentially a permanent debtor.

Figure 2: Percentage of the U.S. debt assets and liabilities in domestic currency



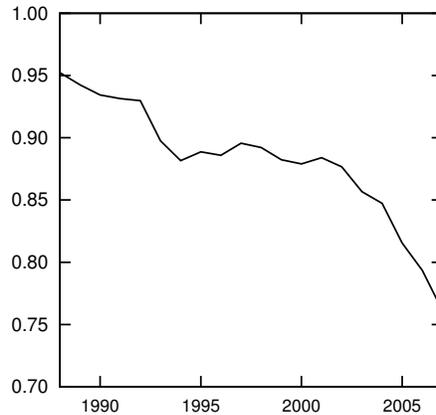
Our benchmark economy is constructed to be consistent with the data in two important dimensions. Our specification of the labor income and dividend processes endogenously generates equity home bias (see figure 3). We introduce preference shocks to resolve the consumption–real exchange rate disconnect (commonly referred to as Backus-Smith puzzle⁷). Equity

⁶See ? who argue that volatility in the U.S. decreased by more than in other OECD economies.

⁷See also Kollmann (1994).

premium is an equally important dimension as the two considered here. We analyze an extended model with risk premium in a separate project.⁸

Figure 3: Home equity bias in the U.S.



The main contributions of this paper is to explain how economic asymmetries can generate plausible foreign asset positions. We also demonstrate how to solve an international portfolio problem by explicitly modeling the wealth distribution. Existing solution methods either assume symmetric economies or a fixed wealth distribution.⁹

2 Model

2.1 Environment

Time is discrete and indexed by $t = 0, 1, 2, \dots$. The exogenous state of the economy is fully summarized by z_t . The state is a first-order Markov process

⁸In this way we are considered in this work with a direction of financial flows. Adding equity premium should increase financial flows and NFA imbalances as investing in foreign assets by incurring more debt would have an additional merit of generating a high investment return.

⁹Many papers were written about solving international portfolio problems in general equilibrium. These include Devereux & Sutherland (2009), Tille & van Wincoop (2010), Evans & Hnatkowska (2008). The first two need a steady state wealth share. In a symmetric case it is 0.5, but this bounds these methods to models with symmetric economies. The third paper allows for an arbitrary wealth share but it is assumed to be fixed in equilibrium. Pavlova & Rigobon (2007) is a continuous time model that allows for analytic solutions. But their setting is effectively a one-good model.

with finitely many states, $\mathcal{Z} = \{\bar{z}_1, \dots, \bar{z}_S\}$, and a probability transition matrix Π . A partial history of the state realizations (z_0, \dots, z_t) is denoted by z^t and its probability by $\pi(z^t)$.

There are two countries, each populated by a representative household.

There are two perishable goods traded every period.

Household in country i receives income $w_i(z_t)$ units of good i . The latter represents wages and profit of privately held companies.

Financial markets trade three financial assets: home and foreign equity and a bond. Home stock is a claim to a stream of dividend $d_1(z_t) > 0$ units of good 1 and foreign stock pays dividend $d_2(z_t) > 0$ units of good 2. Supply of each stocks is fixed and normalized to 1. We assume further that there is no short-selling. The two equity claims are traded at the ex-dividend prices $q_1(z^t)$ and $q_2(z^t)$. Bond's payoff consists of α units of good 1 and $1 - \alpha$ units of good 2. Such a payoff is valued at $\alpha p_1(z^t) + (1 - \alpha)p_2(z^t) \equiv p_b(z^t)$. Assuming $\alpha = 1$ corresponds to a situation in which country 1 has a privilege of issuing debt in its own good. Negative bond positions are subject to a "finite borrowing limit" described below. Bond is in net zero supply. The bond is traded at $q_b(z^t)$.

Household in country i trades in financial and goods markets to achieve maximum expected life-time utility given by:

$$U(c) = E \left[\sum_{t=0}^{\infty} \beta^t u(g^i(c_1^i(z^t), c_2^i(z^t)), z_t) \middle| z_0 \right], \quad \beta \in [0, 1), u' > 0, u'' < 0. \quad (2.1)$$

Function g^i is a constant return to scale consumption aggregator. We assume that households' preferences display *consumption home bias*. That is consumption aggregate is largely composed of the domestic good. In the case of the CES aggregator this assumption is isomorphic to assuming trade cost/tax if the latter is rebated back to the households.

We introduce *preference shocks* by making the one-period utility function dependent on z_t . These shocks are pure demand shifters and are uncorrelated with other exogenous processes in the model. As in Stockman & Tesar (1995) preference shocks are a "reduced form" way of fixing the Backus-Smith puzzle. Pavlova & Rigobon (2007) provide empirical support to the existence and magnitude of these preference shocks.

Budget constraint of a household living in country i after history z^t is:

$$p_1(z^t)c_1^i(z^t) + p_2(z^t)c_2^i(z^t) + q_1(z^t)\theta_1^i(z^t) + q_2(z^t)\theta_2^i(z^t) + q_b(z^t)b^i(z^t) = I^i(z^t), \quad (2.2)$$

where $(\theta_1^i, \theta_2^i, b^i) \in [0, 1]^2 \times [-B(z^t), \infty)$ is the portfolio of the domestic equity trader that consists of his positions in home equity, foreign equity and bond. $I^i(z^t)$ is “cash-in-hand” that consists of the market value of his non-financial income w^i and the income that he receives from his financial portfolio (including dividends):

$$I^i(z^t) \equiv p_i(z^t)w^i(z^t) + \sum_{j=1}^2 (q_j(z^t) + p_j(z^t)d_j(z^t))\theta_j^i(z^{t-1}) + p_b(z^t)b^i(z^{t-1}). \quad (2.3)$$

2.2 Borrowing limit

In a finite-horizon model the requirement that debt is fully repaid in the final period, together with the budget constraints, puts explicit restrictions on the borrowing behavior of the agents in the earlier periods. These restrictions are absent in the infinite horizon case. However, if no borrowing limits are imposed, the problem of the agent is not well-defined. He may achieve unbounded consumption by accumulating an infinite debt. The problem is more severe with incomplete markets because the present value of an agent’s future stream of income is not defined.¹⁰

There are several ways to solve this problem. Levine & Zame (1996) propose an implicit debt constraint, which is a limit of the debt constraint in the finite-horizon case. But, as Kubler and Schmedders (2007) point out, with such a constraint in a model with long-lived assets an equilibrium may not exist.

Another way to limit debt is to impose collateral constraints as in Geanakoplos & Zame (2000). Agents can pledge either durable goods or equity as collateral if they want to borrow. Agents are allowed to default on their debt and have their collateral seized. However, when agents are interpreted as countries it is unclear how should we think of collateral.

The constraint that we use is closest in spirit to the one in Magill & Quinzii (1994) who require that it should be possible for agents to repay

¹⁰If every agent’s income were in the span of the financial asset payoffs then the market’s would be effectively complete.

their obligations over a finite period of time. We require that the agents should be able to do so in one period:

$$B^i(z^t) \equiv \min_{z_{t+1}} \frac{p_i(z^{t+1})w_i(z_{t+1}) + \sum_{j=1}^2 (q_j(z^{t+1}) + p_j(z^{t+1})d_j(z_{t+1}))\theta_j^i(z^t)}{p_b(z^{t+1})}. \quad (2.4)$$

Note that this constraint is still fairly generous since households can borrow against their financial income.

3 Equilibrium

A competitive equilibrium is a price system $\mathcal{P} = \{p_1(z^t), p_2(z^t), q_1(z^t), q_2(z^t), q_b(z^t) : \forall z^t\}$, an allocation $\mathcal{C} = \{(c_1^i(z^t), c_2^i(z^t))_{i=1}^2 : \forall z^t\}$ and asset positions $\mathcal{A} = \{(\theta_1^i(z^t), \theta_2^i(z^t), b^i(z^t))_{i=1}^2, \forall z^t\}$ such that:

1. given the price system \mathcal{P} , the allocation and asset positions solve each household's optimization problem;
2. financial and goods markets clear: $\forall z^t, j = 1, 2$,

$$c_j^1(z^t) + c_j^2(z^t) = w_j(z^t) + d_j(z^t) \quad (3.1a)$$

$$\theta_j^1(z^t) + \theta_j^2(z^t) = 1 \quad (3.1b)$$

$$b^1(z^t) + b^2(z^t) = 0. \quad (3.1c)$$

Walras' law requires that we need to add a price normalization: $p_1(z^t) + p_2(z^t) = 1, \forall z^t$.

3.1 Wealth recursive equilibrium

In general it is not feasible to compute the competitive equilibrium as defined above. The usual reason is the curse of dimensionality: a "natural" state vector for this model includes portfolio holdings of each household. To sidestep this problem we restrict our attention to *wealth-recursive equilibria*. Duffie, Geanakoplos, Mas-Colell & McLennan (1994) call recursive equilibria dynamically simple. However, the equilibrium may fail to exist if the state space is not sufficiently rich. Duffie et al. (1994) show that the equilibrium is guaranteed to exist if the state space includes all the equilibrium variables. A wealth recursive equilibrium is a Markov competitive

equilibrium in which the distribution of wealth is a sufficient statistic for all the equilibrium variables. Kubler & Schmedders (2003) derive conditions under which there exists a wealth recursive equilibrium. They show that there exists a map from the wealth distribution and the current exogenous state to all the endogenous variables in the model.¹¹ In what follows we consider only wealth recursive equilibria.

3.2 State

Let the wealth share of country 1 be denoted by ω :

$$\omega(z^t) \equiv I^1(z^t)/[I^1(z^t) + I^2(z^t)]. \quad (3.2)$$

Under the borrowing limit (2.4) the wealth share always lies in the unit interval. Note also that the total wealth depends only on the prices and the exogenous labor income and dividend processes:

$$I^1(z^t) + I^2(z^t) = \sum_{j=1}^2 [p_j(z^t)(w_j(z^t) + d_j(z^t)) + q_j(z^t)]. \quad (3.3)$$

Note also that the portfolio in the definition of $\omega(z^t)$ was chosen after history z^{t-1} . Then (3.2) implicitly defined a law of motion for the endogenous state:

$$\omega(z^{t+1}) = \Omega(\omega(z^t), z^t, z_{t+1}), \quad \forall z^t, z_{t+1}. \quad (3.4)$$

In a wealth-recursive competitive equilibrium all the equilibrium variables are functions of (ω, z) – current wealth share of country 1 and current exogenous state. Let x denote the set of all current endogenous variables in the system. Then $x = \rho(\omega, z)$. Our task is to solve the following system for x :

$$\Phi(x, \rho(\Omega(\omega, z, z'), z'), z) = 0, \quad \forall \omega, z, \quad (3.5)$$

where Φ is the system of equilibrium conditions. This system for our model is provided in the Appendix.

¹¹However, this map may not be unique. We do not consider this to be a deficiency of our approach since multiplicity plagues even models that have analytic solutions.

4 Solution

We solve the model numerically using the projection method.¹² That is for each $z \in \mathcal{Z}$ we approximate the policy functions $\rho(\omega, z)$ by cubic splines on $[0, 1]$. We use Chebyshev nodes as our grid for ω . The update for the policy functions is obtained by solving the system (3.5). We iterate on policy functions until the change in the price system is less than 10^{-4} . This tolerance level is chosen so that the change in the price system between two consecutive iterations is less than 1 basis point of the average price.

Accuracy of the solution. We test our solution method on a model in which there is no labor income. In such a case a “linear sharing rule” applies. That is trade in the two stocks is enough to achieve maximum consumption insurance. So, the markets are effectively complete even with a number of assets. This insight was used by Baxter & Jermann (1997) and recently by Heathcote & Perri (2010). In this case our solution performs extremely well.¹³ But this is the only type of models for which we have an analytic solution.

4.1 Home equity bias and exchange rate anomaly

Since we are interested in current account and valuation adjustments it is crucial for us to have model that is consistent with salient features of the data like home equity bias and the consumption – real exchange rate disconnect. The home equity bias is important because portfolios directly enter the calculation of NFA and current account. The second anomaly is important because exchange rate fluctuations account for 47% of the changes in the U.S. NFA.¹⁴

Both of these “anomalies” are resolved by introducing preference shocks. Our preference shocks are pure demand shocks rather than terms-of-trade shocks. An alternative interpretation of the preference shocks is that these represent heterogeneous beliefs of the households in the two countries. We also assume that the preference shocks are uncorrelated with other processes

¹²See Judd (1998, chapter 11).

¹³Results from this experiment are available in the extended version of the paper online at <http://www.arts.cornell.edu/econ/tsyrennikov/files/capflows-ext.pdf>.

¹⁴According to the BEA data over the period 1989-2008 changes in the U.S. NFA were on average 27.2% of the initial position size, while exchange rate related adjustments constituted on average 12.7%

in the model.¹⁵

These shocks affect the economy as follows. When a positive shock hits country 1 it demands more of each good. Because preferences of country 1 are tilted towards good 1 price of the latter must rise. Hence, consumption of country 1 increases with its exchange rate as in the data. Since the domestic stock pays more in this situation it is a good hedge against preference shocks leading to home equity bias.

4.2 Calibration

We assume a CRRA utility function and a CES aggregator:

$$u(c, z) = \delta(z)c^{1-\gamma}/(1-\gamma), \quad \gamma > 0, \quad (4.1a)$$

$$g^1(c_1, c_2) = (sc_1^\rho + (1-s)c_2^\rho)^{1/\rho}, \quad s \in [0.5, 1], \rho \leq 1 \quad (4.1b)$$

$$g^2(c_1, c_2) = ((1-s)c_1^\rho + sc_2^\rho)^{1/\rho}. \quad (4.1c)$$

We choose $\gamma = 2$, a standard value in macroeconomics. We set $\rho = -0.11$ which is equivalent to assuming that the elasticity of substitution between goods is 0.90. This number is the upper bound of available estimates. With a lower ES (a smaller ρ) our results strengthen. We choose $s = 0.8560$ to match the average trade/GDP ratio in the U.S. of 16.8%.

We set σ_e and ρ_e to match the standard deviation and autocorrelation of log-output in the U.S. data. The stochastic process for endowments is assumed to be a 9-state first-order Markov process.¹⁶ We construct the stochastic processes for wages and dividends in the following way:

$$w_i = (1 - \Pi)E(e_i), \quad (4.2a)$$

$$d_i = e_i - w_i, \quad (4.2b)$$

where Π is the average share of corporate profits in the U.S. GDP. We choose this specification for the reason that dividends are more than 5 time more

¹⁵Note that in the “heterogeneous beliefs” interpretation we take a neutral position in the following sense. First, beliefs switch between optimism and pessimism as opposed to being either one permanently. Second, beliefs change in a way that is not related to other exogenous shocks in the model.

¹⁶The 9 states are formed by a tensor product of the same set with 3 states. The states and the transition matrices were constructed as in Rouwenhorst (1995). This algorithm is superior to the commonly use Hussey-Tauchen procedure as it perfectly matches persistence of the process even for a low number of states. We would also like to point out that the model with 4 states gives nearly identical results but it is easier to replicate.

volatile than labor income.¹⁷

We specify preference shocks to be an *i.i.d.* process and uncorrelated with other stochastic processes in the model. We let $\delta(z)$ take only 2 values:

$$\delta(z) = (\delta^1(z), \delta^2(z)) \in \{(1 - \sigma_\delta, 1 + \sigma_\delta), (1 + \sigma_\delta, 1 - \sigma_\delta)\}. \quad (4.3)$$

Both states are equiprobable. We set $\sigma_\delta = 0.043$ so that the Backus-Smith correlation is zero in our simulations.¹⁸

Table 1: Benchmark parameter values

	Value	Moment/Source
β	0.9615	Return on bond = 4%
γ	2.0000	Common benchmark value
ρ	-.1100	Heathcote & Perri (2002)
s	0.8570	Trade/GDP = $0.5(X+M)/(C+NX)$ = 16.9%
Π	0.1290	Corporate profits/GDP ratio
σ_δ	0.0440	Corr(rel.consumption,real exchange rate) = 0.0
σ_e^1	0.0099	Volatility of income in the U.S.
σ_e^2	0.0200	Volatility of income abroad.
ρ_e^1	0.6500	Persistence of income in the U.S.
ρ_e^2	0.6500	Persistence of income abroad.

4.3 Defying multiplicity

Multiplicity is a plague for numerical analysis. There may be multiple equilibria in our model and we know of no work that would provide conditions under which a competitive equilibrium is unique.¹⁹ Yet, we would like at least some indication that our equilibrium is unique. Our idea stems from

¹⁷Recall that our measure of output does not include investment and government spending. So, there are many possibilities to calibrate the labor income and dividend processes. One alternative is to choose the three dividend states to match three moments of the process – mean, volatility and correlation with output. Such calibration strengthens our results but it requires a counterfactual assumption that output and labor income are (slightly) negatively correlated.

¹⁸The Backus-Smith correlation in our sample of countries is -0.42 with standard deviation of 0.43. So, the data estimate is negative but statistically insignificant. Since this correlation is fairly sensitive to the period and the set of countries we decided to choose a “neutral” value which is zero.

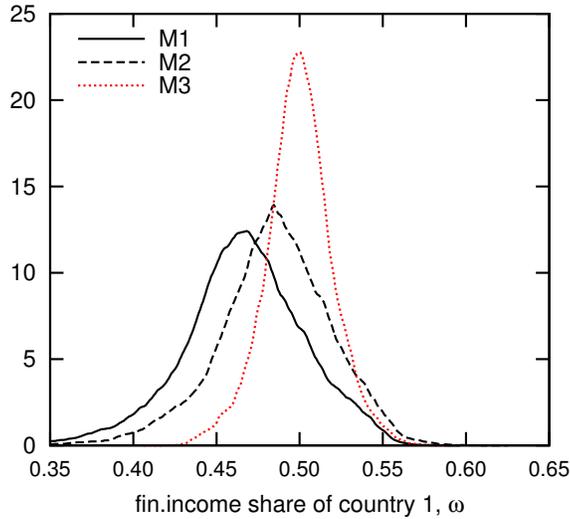
¹⁹Note that is also a problem in models for which an analytic solution is available.

Kubler & Schmedders (2002) who suggest that multiplicity of equilibria in models with incomplete financial markets may be related to multiplicity of efficient allocations. So, we solve for all Pareto allocations in our model for values of the Pareto weight on country 1 being on a dense grid $(0, 0.0001, 0.0002, \dots, 1)$. (More details are provided in the appendix.) In every case we found a unique solution.

4.4 Numerical Results

We consider several specifications. Specification/model M1 is the benchmark calibrated to the data. In this case country one has advantage in the bond market as debt is denominated in good 1 (dollarization) and country 1's income is also less volatile than abroad. Specification M2 imposes that volatility is the same in the two countries: $\sigma_{e1} = \sigma_{e2}$. Specification M3 additionally imposes that the bond market is symmetric: $\alpha = 0.5$. In what follows results are presented for the model M1 until specified otherwise.

Figure 4: Stationary distribution of the fin. income share. (Solid line: asymmetric bond market (dollarization); dashed line: symmetric bond market.)



Result 1. Under the stationary wealth distribution country 1's wealth is skewed towards 0.

Consider first the stationary distribution of the financial income share, ω . It is positively skewed with mean below 0.5 as can be seen from figure 4. Expected financial wealth of country 2 is 16.4% smaller than that of country 1 (see table 2). In the symmetric specification M3 wealths are on average equal. With an asymmetric bond market (M2) financial wealth of country 1 is on average 7.9% smaller. Note that this does *not* imply that welfare of country 1 is smaller because financial markets are incomplete. Even though country 1 has lower financial wealth price of good 1 is smaller than price of good 2 (see figure 5, “Terms of trade” panel). In the benchmark calibration the price effect is strong enough to make country 1 better off despite lower wealth. Here a) financial markets are structured to the advantage of country 1 and b) supply of good 1 (that comprises the major portion of country 1’s consumption) is less volatile.

Table 2: Moments in the data and in the model*

Moment	Data	M1	M2	M3
<i>International investment</i>				
Equity A / output	0.274	0.141	0.145	0.161
Equity L / output	0.218	0.349	0.241	0.161
Net Debt / output	-0.169	-0.053	-0.017	-0.001
$E(\frac{1-\omega}{\omega})$	na	1.161	1.079	1.000
<i>Business cycle moments</i>				
std(RER)	0.035	0.026	0.019	0.017
cor(RER, Y/Y*)	-.190	-0.814	-0.696	-0.653
<i>Calibrated moments</i>				
Rate of return on debt, %	4.00	4.00	4.00	4.00
Trade / output	0.169	0.168	0.167	0.165
Home equity bias	0.934	0.956	0.958	0.960
Backus-Smith corr	0.000	0.000	0.330	0.352

Specifications M1-M3 are explained above in text.

Result 2. Domestic stocks dominate countries’ portfolios.

Equity home bias obtains because labor income and domestic stock payoff are negatively correlated. When dividends, and hence output, increase in country 1, price of good 1 decreases. Country 1’s labor income declines

while value of domestic dividends goes up.²⁰ Preference shocks on the other hand attenuate equity home bias. When country 1 experiences a positive preference shock then it increases demand for both goods proportionally. Because of consumption home bias the relative price of good 1 increases. The final result is that demand for domestic good 1 increases by a smaller percentage than demand for foreign good 2.²¹ So, an increased position in foreign equity is required to hedge preference shocks. Yet, quantitatively the effect of preference shocks on home equity bias is small.

Result 3. A country that can issue bonds accumulates a negative bond position and increases its portfolio share in foreign equity.

With an asymmetric bond market country 1 accumulates debt and invests borrowed funds in foreign equity. This happens because of the perfect positive correlation between the bond payoff and labor income in country 1. For country 2 bond payoff and labor income are negatively correlated; so, country 2 is willing to buy bonds. Country 1 can assure a constant stream of good 1 for consumption by trading domestic bonds only. Trade in bonds allows to achieve a desired average level of consumption.²² In our parametrization country 1's income of good 1 is too high relative to the desired average consumption of good 1. This leads country 1 to take a short position in bonds.

With the set of financial markets assumed in our model we expect the allocation to be close to the efficient one. Yet, when country 1 takes a negative position in bond it effectively becomes poorer. Lower wealth in country 1 reduces relative demand for good 1 and price of good 1 declines; and so does the price of stock 1. Country 1 then sells a larger share of the domestic equity just to hold to its foreign equity position. As a result share of foreign equity increases in country 1's portfolio and the net foreign asset position in equity in country 1 becomes negative. The latter prediction is counterfactual but to fix this aspect of the model we need to generate significant differences between returns on debt and equity.

If the bond payoff were denominated in a 50-50 combination of the two goods (specification M1) then bond positions would be zero on average.

²⁰Because dividends are only a small fraction of output income effect plays no role here. So, "quantity" effect dominates "value" effect.

²¹The demand for domestic good may even decrease.

²²Trade in equity allows countries to hedge fluctuations in the relative price.

Result 4. A country with lower volatility accumulates a negative bond position and increases its portfolio share in foreign equity.

The intuition for this result is similar to the intuition for result 3. However, the driving force for the result 4 is a higher precautionary demand for saving in country 1. For the result 3 it is a change in the correlation of asset payoffs.

5 Conclusions

We build an incomplete markets general equilibrium model of international portfolio choice. We show that a wealth recursive equilibrium exists in our model and show how to compute it using a projection method. Our calibrated model features two important asymmetries. 1) only country 1 is able to sell debt in its own currency. 2) country 1 is less volatile than country 2. A calibrated model allows us explaining why the U.S. accumulated a negative net foreign asset position and why its liabilities are dominated by debt liabilities. Our model also generates home bias, resolves the relative consumption-exchange rate disconnect puzzle and generates a volatile real exchange rate.

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A System of equilibrium conditions

The system of equilibrium conditions consists of first-order necessary conditions for the household optimization problems, market clearing conditions and the wealth share evolution equation. The set of equilibrium variables is:

$$x \equiv \{(c_1^i, c_2^i, \theta_1^i, \theta_2^i, b^i, \mu_1^i, \mu_2^i, \mu_b^i)_{i=1}^2, p_1, p_2, q_1, q_2, q_b\} = \rho(w, z).$$

The set of first-order optimality conditions (arguments (w, z) are suppressed):

$$\begin{aligned}
p_1 &= \frac{g_1^i(c_1^i, c_2^i)}{g_1^i(c_1^i, c_2^i) + g_2^i(c_1^i, c_2^i)} \\
p_2 &= \frac{g_2^i(c_1^i, c_2^i)}{g_1^i(c_1^i, c_2^i) + g_2^i(c_1^i, c_2^i)} \\
q_1 &= \frac{\beta E[u'(c^{i'})g_1^i(c_1^{i'}, c_2^{i'})(q_1' + p_1'd_1')/p_1'] + \mu_1^i + \mu_b^i(q_1 + p_1d_1)/p_b}{u'(c^i)g_1^i(c_1^i, c_2^i)/p_1}, \\
q_2 &= \frac{\beta E[u'(c^{i'})g_1^i(c_1^{i'}, c_2^{i'})(q_2' + p_2'd_2')/p_1'] + \mu_2^i + \mu_b^i(q_2 + p_2d_2)/p_b}{u'(c^i)g_1^i(c_1^i, c_2^i)/p_1}, \\
q_b &= \frac{\beta E[u'(c^{i'})g_1^i(c_1^{i'}, c_2^{i'})p_b'/p_1'] + \mu_b^i/p_b}{u'(c^i)g_1^i(c_1^i, c_2^i)/p_1}, \\
\omega I &= p_1c_1^1 + p_2c_1^1 + q_1\theta_1^1 + q_2\theta_2^1 + q_b b^1 \\
\mu_1^i &\geq 0, \mu_1^i\theta_1^i = 0, \\
\mu_2^i &\geq 0, \mu_2^i\theta_2^i = 0, \\
\mu_b^i &\geq 0, \mu_b^i(b^i + B^i) = 0.
\end{aligned}$$

We use policy functions to eliminate future variables: $x' = \rho(\Omega(w, z, z'), z')$. Next we eliminate the complementary slackness conditions using Garcia-Zangwill re-parametrization trick. Together with market clearing conditions this gives us a 15x15 system of equations in x .²³

B Uniqueness of efficient allocation

The efficient allocation is characterized by the following system of equations:

$$\begin{aligned}
\theta g_1^1(c_1^1, c_2^1) &= (1 - \theta)g_1^2(e_1 - c_1^1, e_2 - c_2^1), \\
\theta g_2^1(c_1^1, c_2^1) &= (1 - \theta)g_2^2(e_1 - c_1^1, e_2 - c_2^1),
\end{aligned}$$

where θ is the country 1's Pareto weight. In case of a CES aggregator the system reduces to the following equation:

$$G(c_1^1|\theta) \equiv \theta g_1^1(c_1^1, f(c_1^1)) - (1 - \theta)g_1^2(e_1 - c_1^1, e_2 - f(c_1^1)) = 0, \quad (\text{B.1})$$

where

$$c_2^1 = f(c_1^1) = e_2 \frac{s^{2/(1-\rho)} c_1^1}{(1-s)^{2/(1-\rho)} (e_1 - c_1^1) + s^{2/(1-\rho)} c_1^1}.$$

We plot $G(x|\theta)$ for $\theta \in \{0, 0.0001, 0.0002, \dots, 1\}$. If the system has a unique solution then $G(x|\theta)$ should intersect the x -axis only once. This is true for the parametrization used in this paper.

²³The system can be further reduced by eliminating current prices.

C Data

Figure 1 uses data from Lane & Milesi-Ferretti (2007b). In our computations we exclude financial derivatives. The latter have been included into official estimates only in 2005, but no prior data is available. Our measure of equity flows includes portfolio equity and FDI.

The sample of our countries includes the U.S., Western Europe,²⁴ Japan, Canada, Australia and New Zealand. Total number of countries is 23. The source of all NIA data is the OECD database. Sample period is 1985-2007. Our measure of output is private consumption plus net exports. Exchange rates used to compute Backus-Smith correlations are from the Penn World Tables database version 6.4. Consumption, output were “logged” and then linearly detrended.²⁵ Table 3 presents the data. All cross country statistics were computed against the U.S. (as opposed to the rest of the sample).

Figure 3 uses data from Lane & Milesi-Ferretti (2007a) and the World Bank stock market capitalization data.²⁶ We compute home equity bias according to:

$$\text{Home equity bias} = \frac{\text{Stock market capitalization} - \text{Foreign equity liabilities}}{\text{Stock market capitalization} - \text{Foreign equity liabilities} + \text{Foreign equity assets}}.$$

D Simulation

We start each simulation from ω_0 that is a “fixed point” of Ω map:

$$\omega_0 = \Omega(\omega_0, E(z), E(z)).$$

This is an analog of a steady state for the stochastic environment. Whenever needed the initial portfolio used is $\theta_0^1 = (1, 0, 0)$. Statistics reported in the paper are obtained from 1000 simulations of length 100000. The first 10000 observations in each simulation were discarded.

E Policy functions

Figure 5 reports policy functions for the benchmark calibration. It plots policy functions as a function of ω for each of the 18 values of z .

²⁴Austria, Belgium, Germany, Denmark, Spain, Finland, France, Great Britain, Greece, Ireland, Italy, Luxembourg, the Netherlands, New Zealand, Portugal, Sweden.

²⁵Detrending methods that we tried (Hodrick-Prescott filter, band-pass filter, quadratic detrending) has little effect on the statistics.

²⁶The latter is available at <http://xxx>.

Table 3: Selected business cycle statistics and Backus-Smith correlations

	Country	$cor(y, y_{us})$	$std(y), \%$	ρ_y	BS
1	AUS	0.24	2.82	1.30	-0.65
2	AUT	0.34	1.55	1.14	-0.57
3	BEL	0.28	1.49	0.93	-0.50
4	CAN	0.51	3.58	1.82	-0.83
5	CHE	0.30	2.09	1.21	-0.51
6	DEU	-0.16	1.42	0.92	-0.48
7	DNK	-0.09	1.33	2.43	-0.41
8	ESP	0.33	1.49	2.99	-0.78
9	FIN	0.77	3.26	4.34	-0.73
10	FRA	0.54	1.06	1.88	-0.67
11	GBR	0.66	1.69	3.09	-0.62
12	GRC	-0.28	2.88	2.32	-0.62
13	IRL	0.49	4.69	3.71	0.65
14	ISL	0.46	5.24	6.22	-0.37
15	ISR	0.20	3.03	3.08	-0.04
16	ITA	0.46	1.07	2.02	-0.73
17	JPN	-0.43	2.05	1.80	-0.21
18	LUX	-0.02	3.53	2.66	0.14
19	NLD	0.26	1.83	2.85	0.70
20	NZL	0.20	2.07	2.13	-0.81
21	PRT	0.19	2.01	2.22	-0.44
22	SWE	0.30	2.14	3.07	-0.71
Average		0.25	2.39	2.43	-0.40
Std.dev		(0.31)	(1.16)	(1.25)	(0.44)
USA		–	0.99	0.65	–

F Existence proof

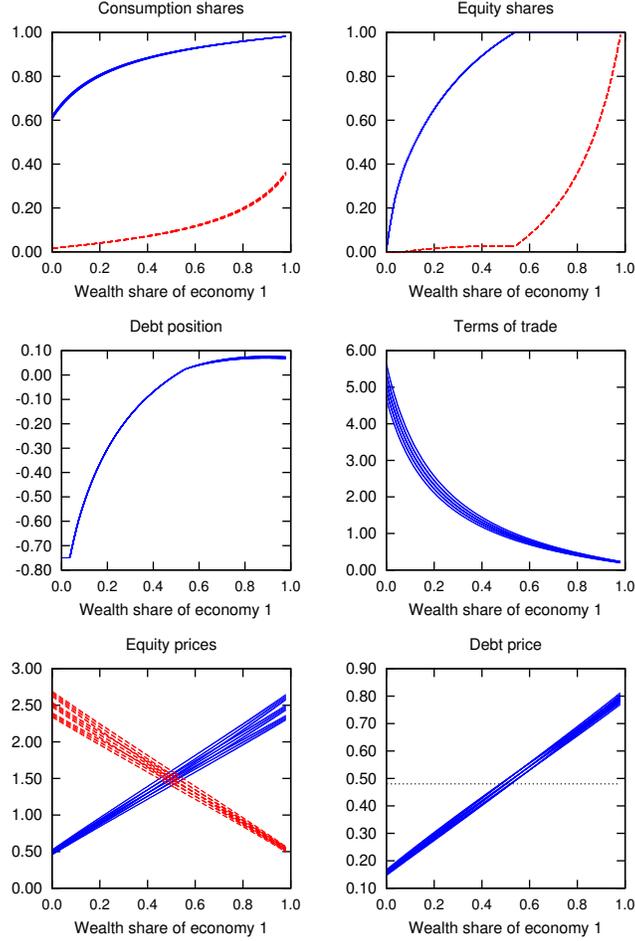
Let the superscript denote the country (and the consumer), and the subscript denote the good. The borrowing limit for each consumer i is of the form:

$$\min_{z_{t+1}} \left(kw_i^i(z^{t+1})p_i(z^{t+1}) + b^i(z^t)r_b(z^{t+1}) \right) \geq 0, \forall z^t, 1 > k > 0$$

Assume that endowments of each country of each good is strictly positive: $w_j^i > 0$.

Lemma F.1. *For all $T \geq 1$ there exists a financial markets equilibrium for the*

Figure 5: Policy functions for the benchmark calibration



truncated economy in which values of all prices, asset holdings, and consumption allocations lie in some compact set X^ , and $0 \notin X^*$.*

Proof. Since the budget set is homogenous of degree 0 we can normalize the prices so that for all z^t ,

$$p_1(z^t) + p_2(z^t) + q_1(z^t) + q_2(z^t) + q_b(z^t) = 1$$

Let $\xi = (p_1, p_2, q_1, q_2, q_b)$. After the normalization, we have $\xi \in \Delta$.

Define maximal the bounds on endowments: $\bar{e}_j = \max_z e_j(z)$. Then let the upper bound on consumption of good j be $\bar{c}_j = 2\bar{e}_j$. By the market clearing condition in equilibrium we must have $c_j^i < \bar{c}_j, \forall i, j$.

Next, let's establish the existence of the lower bounds on equilibrium consumption. Since the utility function is not bounded from below, we can find \underline{u}^i such that:

$$\begin{aligned} \underline{u}^i + \frac{\beta}{1-\beta} u^i(\max_z e_1(z), \max_z e_2(z)) \\ \leq u^i((1-k) \min_z w_i^i(z), \min_z w_j^i(z)) + \frac{\beta}{1-\beta} u^i(\min_z w_i^i(z), \min_z w_j^i(z)). \end{aligned}$$

Note that the above implies that

$$\underline{u}^i \leq u^i((1-k) \min_z w_i^i(z), \min_z w_j^i(z)).$$

Together these two inequalities imply existence of \underline{c}_j^i such that:

$$u^i(\underline{c}_1^i, \bar{e}_2) = \underline{u}^i, \quad u^i(\bar{e}_1, \underline{c}_2^i) = \underline{u}^i.$$

It can never be optimal for agent i to consume less than \underline{c}_j^i of good j . Let the lower bounds on consumption of good j be $\underline{c}_j = \min_i \underline{c}_j^i$.

Let $\bar{\theta} = 2$. By the market-clearing condition, we must have that in any equilibrium, $\theta_1^i < \bar{\theta}$ and $\theta_2^i < \bar{\theta}$.

Let $\underline{p} = \min_i \frac{\frac{\partial u^i}{\partial c_2^i}(\bar{c}_1, \underline{c}_2)}{\frac{\partial u^i}{\partial c_1^i}(\bar{c}_1, \underline{c}_2)} > 0$, $\bar{p} = \max_i \frac{\frac{\partial u^i}{\partial c_2^i}(\underline{c}_1, \bar{c}_2)}{\frac{\partial u^i}{\partial c_1^i}(\underline{c}_1, \bar{c}_2)}$. We will show later that in equilibrium, $\underline{p} \leq \frac{p_2}{p_1} \leq \bar{p}$. Since all consumption bounds are finite and strictly positive, we have $\underline{p} > 0$ and $\bar{p} < \infty$, which also implies that with our price normalization ($\xi \in \Delta$), there are \underline{p}_i and \bar{p}_i such that $0 < \underline{p}_i \leq p_i \leq \bar{p}_i < 1$ for $i = 1, 2$.

Let $\bar{b}^i = \frac{\bar{p}^i(\max_z w_i(z))}{(\alpha p_1 + (1-\alpha)\underline{p}_2)}$, and let $\bar{b} = \max_i \bar{b}^i$. If agent i stays within his borrowing limit, then $b^i > -\bar{b}^i$.

Let $C_1 = [\underline{c}_1, \bar{c}_1]$, $C_2 = [\underline{c}_2, \bar{c}_2]$. Let $\Theta_1 = \Theta_2 = [0, \bar{\theta}]$. Finally, let $B = [-\bar{b}, 2\bar{b}]$. Let's augment both agents' budget constraints by the requirement that $(c_1^i, c_2^i, \theta_1^i, \theta_2^i, b^i) \in C_1 \times C_2 \times \Theta_1 \times \Theta_2 \times B$.

The augmented budget set of an agent is a non-empty ("endowment" always belongs to the budget set), compact-valued, convex-valued and continuous correspondence of the price vector. Hence, by the Berge's maximum theorem, the agent's demand correspondence for both goods and the assets is non-empty, upper-hemicontinuous, compact-valued and convex-valued.

Now define the total excess goods and assets demand correspondence:

$$\begin{aligned} D(\xi) &= (D_{c_1}(\xi), D_{c_2}(\xi), D_{\theta_1}(\xi), D_{\theta_2}(\xi), D_b(\xi)) \\ &= \left(\sum_{i=1}^2 (c_1^i - w_1^i - \theta_1^i d_1), \sum_{i=1}^2 (c_2^i - w_2^i - \theta_2^i d_2), \sum_{i=1}^2 \theta_1^i - 1, \sum_{i=1}^2 \theta_2^i - 1, \sum_{i=1}^2 b^i \right). \end{aligned}$$

For each $\delta \in D(\xi)$ and each $z^t \in Z^T$, define the following optimization problem:

$$\begin{aligned} \max_{p'_1, p'_2, q'_1, q'_2, q'_b} \sum_{i=1}^2 (p'_i \delta_{c_i} + q'_i \delta_{\theta_i}) + q'_b \delta_b & \quad (\text{OP}(z^t)) \\ \text{s.t. } (p'_1, p'_2, q'_1, q'_2, q'_b) & \in \Delta. \end{aligned}$$

The set of optimal solutions to (OP) is a non-empty, compact- and convex-valued and uhc correspondence. Let's denote this correspondence by $\text{PP}_{z^t}(\delta)$. Let the product of these correspondences for all $z^t \in Z^T$ be $\text{PP}(Z^T)$.

The correspondence $\text{PP}(Z^T) \times D(\xi)$ maps the set $(\tilde{C}_1 \times \tilde{C}_2 \times \tilde{\Theta}_1 \times \tilde{\Theta}_2 \times \tilde{B})^{|Z^T|} \times \Delta^{|Z^T|}$ into itself²⁷. This correspondence is non-empty, convex- and compact-valued and uhc. Kakutani's fixed-point theorem guarantees that there is a fixed point (ξ^*, δ^*) . Next, let's show that $\delta^* = 0$ and $\xi \gg 0$ (so that the fixed point is indeed an equilibrium). The proof is by contradiction.

First, note that if we sum the budget sets of both agents, we get the following version of the Walras law:

$$\sum_{j=1}^2 p_j \left(\sum_{i=1}^2 [c_j^i - w_j^i - \theta_j^i d_j] \right) + \sum_{j=1}^2 q_j \left(\sum_{i=1}^2 [\theta_j^i - \theta_i^i] \right) + q_b \sum_{i=1}^2 b^i - r_b \sum_{i=1}^2 b^i = 0$$

Suppose that at $t = 0$, at least one of the components of δ^* is strictly positive. Suppose that the largest positive excess demand is for good j . Then the optimal solution to $(\text{OP}(z_0))$ would be to set $p_j = 1$ and all other prices to 0. But then the above Walras law equation would be violated. The same argument works for the all assets (because we assume that $\sum_{i=1}^2 \theta_j^i(z_0) = 1$ and $\sum_{i=1}^2 b^i(z_0) = 0$).

Next, suppose that there is a negative excess demand in one of the markets at $t = 0$. Suppose that the smallest excess demand is for good j . Then it is optimal (from $(\text{OP}(z_0))$ to set $p_j = 0$. But then both agents would demand \bar{c}_j , meaning that the excess demand is positive – a contradiction. The same argument works for both assets.

By induction, it follows that $\delta = 0$ and $\xi \gg 0$ for all $z^t \in Z^T$.

Next, let's establish that $\frac{p_2}{p_1}$, $\frac{q_1}{p_1}$, $\frac{q_2}{p_1}$ and $\frac{q_b}{p_1}$ stay within certain positive bounds.

Bounds on goods' prices are as above. Lower bounds on stock prices are:

$$\underline{q}_j = \beta \min_i \left[\frac{\partial u^i(\underline{c}_1, \underline{c}_2)}{\partial c_1} / \frac{\partial u^i(\bar{c}_1, \bar{c}_2)}{\partial c_1} \right] \left[p_j \min_z d_j(z) \right].$$

Upper bounds on stock prices are:

$$\bar{q}_j = 2(\bar{p}_1 \bar{c}_1 + \bar{p}_2 \bar{c}_2).$$

The stock price cannot exceed \bar{q}_j , since in that case, the consumer that owns at least $\frac{1}{2}$ of that tree could sell it and receive the utility

$$u^i(\bar{c}_1, \bar{c}_2) > \frac{1}{1-\beta} u^i(\max_z e_1(z), \max_z e_2(z)),$$

²⁷ $\tilde{C}_i = [2\underline{c}_i - \max_z e_i(z), 2\bar{c}_i - \min_z e_i(z)]$, $\tilde{\Theta}_i = [-1, 2\bar{\theta} - 1]$, $\tilde{B} = [-2\bar{b}, 4\bar{b}]$

which clearly cannot happen in equilibrium.

$$u^i(\bar{c}_1, \bar{c}_2) + \beta \frac{1}{1-\beta} u(\min_z e_1(z), \min_z e_2(z)) > \frac{1}{1-\beta} u^i(\max_z e_1(z), \max_z e_2(z))$$

Finally, the bounds on the bond price are:

$$\begin{aligned} \underline{q}_b &= \beta \min_i \left[\frac{\partial u^i(\underline{c}_1, \underline{c}_2)}{\partial c_1} / \frac{\partial u^i(\bar{c}_1, \bar{c}_2)}{\partial c_1} \right] (\alpha \underline{p}_1 + (1-\alpha) \underline{p}_2), \\ \bar{q}_b &= \beta \max_i \left[\frac{\partial u^i(\bar{c}_1, \bar{c}_2)}{\partial c_1} / \frac{\partial u^i(\underline{c}_1, \underline{c}_2)}{\partial c_1} \right] (\alpha \bar{p}_1 + (1-\alpha) \bar{p}_2). \end{aligned}$$

The last step – the additional constraints that we assumed cannot be affecting the individual decision problem in equilibrium. \square

Lemma F.2. *The graph of g is closed.*

Theorem F.3. *Markov equilibrium exists.*

Proof. By lemma 1, each V^n is non-empty. By lemma 2, it is also closed. It is easy to show that $V^{n+1} \subseteq V^n$. Thus, $\bigcap_{n=1}^{\infty} V^n$ is non-empty. \square