Consider a market (e.g., for water from a stream) in which sellers (e.g., owners of land through which the stream runs) face a downward sloping linear market demand curve and zero costs of production (consumers pay to retrieve water from the stream). Assume for convenience that market demand is given by $P = 1 - Q$. Assume also that the product (the water) is homogenous (consumers don't see any difference between different sellers).

Perfect Competition: Under "perfect competition" price P will be driven to zero (marginal cost) and quantity supplied Q in the market will be one.

> *Details:* The competitive supply curve in this case is horizontal at a price of zero. As a price taker, each firm wants to increase it's output indefinitely if the price is greater than zero.

Monopoly: If there is only one seller in this market, then she will maximize profits by setting her price P at 0.5 and selling $Q = 1/2$.

> *Details:* To see this, note that profit is $TR - TC$ and that for the current example $TC = 0$ and $TR = P \cdot Q = (1 - Q) \cdot Q$. Thus the monopolist's profit (or payoff) function is $u(Q) = Q - Q^2$. To find the profit maximizing level of Q to supply on the market, set $\frac{d}{dQ}u(Q) = 0$. This yields $Q = 0.5$. Plugging that back into the demand curve yields $P = 0.5$.
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> Equivalently, we could see this by noting that the monopolist's $MR = 1 - 2 \cdot Q$ so that $MR = MC$ implies $Q = 1/2$, or by recognizing geometrically that with a linear demand curve and zero marginal cost, the profit maximizing price and quantity will lie at the midpoint of the demand curve.

Cournot Duopoly: If there are two sellers, the profit maximizing decision for each depends on the actions taken by the other. Thus, individual profit maximization is now a strategic problem. Augustin Cournot (1838) investigated the outcome for the case in which each player makes a decision about the *quantity* that she wishes to supply. Price is then determined by market demand given the quantities supplied by the two firms.

Note that market demand can now be written $P = 1 - q_1 - q_2$, where P is the market price and q_1 and q_2 are the quantities supplied by each firm. Each firm now essentially faces a residual demand curve given the quantity supplied to the market by the other firm. E.g., given q_2, firm 1 faces demand $P = (1 - q_2) - q_1$ and thus $MR_1 = (1 - q_2) - 2q_1$, and it's profit (payoff) function is $u_1(q_1, q_2) = (1 - q_2) \cdot q_1 - q_1^2$.

At the *Nash equilibrium*, each firm is selecting a best response to the other's actual quantity. For example, given firm 2's output, firm 1 is playing a BR (i.e., maximizing it's profit)if it sets its quantity at $q_1 = \frac{1}{2}(1-q_2)$.

> *Details:* Again, you can calculate this profit maximizing quantity by differentiating the profit function with respect to q_1 and setting this derivative equal to zero or by recognizing that the profit maximizing price and quantity lie at the midpoint of the residual demand left over by seller 2 or by setting $MR_1 = MC_1$.

Thus, the best response of firm 1 to the quantity selected by firm 2 is $\mathrm{BR}_1(q_2) = \frac{1}{2}(1 - q_2)$. Similarly $\mathrm{BR}_2(q_1) = \frac{1}{2}(1 - q_1)$. At a NE, we must have $q_1 = \mathrm{BR}_1(q_2)$ and $q_2 = \mathrm{BR}_2(q_1)$. So Nash equilibrium in this game is characterized by two equations:

$$q_1 = \frac{1}{2}(1 - q_2)$$
$$q_2 = \frac{1}{2}(1 - q_1)$$

Solving these simultaneously gives us the unique NE of this game: $s^* = (q_1^*, q_2^*) = (1/3, 1/3)$. Thus market supply will be $Q = q_1^* + q_2^* = 2/3$ and market price will be $P = 1/3$. Price is lower and quantity higher than in the monopoly case.

Short cut: Since the two firms are identical in our example and the NE is symmetric, we could have taken a short cut above. Rather than solving the two equations simultaneously, we could recognize that the each seller will have the same quantity q in equilibrium, where $q = \frac{1}{2}(1-q)$, and so $q = 1/3$. I.e., we could have used either of the two equations above and set $q_1 = q_2$ to get our answer above.

Cournot Competition with N sellers: Suppose that there are N sellers. Then seller 1 will profit maximize (given the residual demand left over by the other sellers) by setting $q_1 = \frac{1}{2}(1 - \sum_{j=2}^{N} q_j)$. In equilibrium, each seller sells the same quantity $q = \frac{1}{2}(1 - (n-1)q) = \frac{1}{n+1}$. Thus market supply is $Q = nq = \frac{n}{n+1}$, and the market price is $P = \frac{1}{n+1}$.

Notice that for $N = 2$ this formula gives us the duopoly result $Q = 2/3$ that we got above (as it should). As N rises, market supply rises and market price falls. In the limit, as $N \to \infty$, we have $Q \to 1$ and $P \to 0$, which is the competitive equilibrium (price equal to cost of production).

Bertrand Duopoly: Bertrand (1883) investigated the outcome for the case in which each player makes a decision about the *price* that she wishes to supply, and the seller with the lowest price takes the entire market demand (unless they charge the same price in which case they split the market demand). Then each seller's best response (i.e., profit maximizing response) is to set its price just below that of its competitor, as long as the latter is greater than zero, thus stealing the entire market. Thus, the Nash equilibrium has both firms pricing at zero. Here, we get the competitive equilibrium with just two sellers - i.e., a little competition goes a long way in Bertrand's game.