Evolutionary game theory abandons the assumption of ultra-rationality. Rather than trying to select a strategy rationally, each individual is thought of as being hard wired to play a particular (pure or mixed) strategy at any point in time. We can interpret this as the individual being committed to a rule of thumb at any given point in time. However, the population of individuals then evolves over time with relatively high payoff strategies proliferating within the population and relatively low payoff strategies being abandoned. We can think of this as a sluggish process of social learning in which relatively successful strategies used by some players are then adopted by others. However, the relative payoffs at a point in time will themselves typically depend on the relative mix of strategies actually in use in the population, so the set of strategies that are most successful can change over time as the population evolves.

For whatever game we wish to study (e.g., a one shot game of Chicken), we assume that there is a large number of players and that, over time, individuals play the game with other individuals with whom they are randomly matched.

Single Population Games: If the normal form game is symmetric (e.g., Prisoners' Dilemma, or Chicken), then the players do not have different preferences or roles in the game. We can then consider a single population of players within which individual players are randomly matched to play the game over time.

Multiple Population Games: If the normal form game is asymmetric, so that the players have different preferences or roles in the game (e.g., Battle of the Sexes, or Ultimatum), then we can consider a separate population for each player role. E.g., in an ultimatum game played between individual firms and workers, where firms make offers and workers take or leave them, it may be sensible to treat the population of proposers (firms) as distinct from the population of respondents (workers). Firms are then randomly matched with workers to play the game.

Symmetrized Games: If the normal form game is asymmetric, but individuals can play any of the roles (e.g., I might be a proposer sometimes and a respondent at other times), then we can treat this as a single population game. Players are randomly matched out of the single population and randomly assigned roles in the game. Players' strategies are then conditioned on which role they receive (e.g., a strategy in ultimatum must indicate what to do if I am a proposer and what to do if I am a respondent). The resulting normal form game is symmetric (i.e., we have symmetrized the asymmetric game by adding a stage in which roles are randomly assigned). Skyrms takes this approach in his treatment of evolutionary dynamics in the ultimatum game.

For this handout, we will restrict our attention to single population games.

The expected payoff of a player currently hardwired to play a particular strategy σ prior to being matched with a partner depends on the mix of types (strategies) of players in the population. Call this expected payoff the **fitness** of strategy σ.

Suppose for example that there are only two strategies (σ and σ') currently being played in the population, and the proportion of individuals in the population playing the strategy σ is x (so that the proportion playing the alternative strategy σ' is $(1-x)$). Then the fitness of strategy σ is $F_\sigma = x \cdot u(\sigma, \sigma) + (1-x) \cdot u(\sigma, \sigma')$.[1]

[1] An individual playing strategy σ has probability x of playing against another σ type and probability $(1-x)$ of playing against a σ' type, given that matches are made randomly. Note then that playing σ against a randomly assigned player is equivalent (in expected payoff) to playing σ against a single known player who is playing the mixed strategy (x,1-x).

We typically assume that the proportion of relatively high fitness strategies will increase over time, while the proportion of relatively low fitness strategies will decrease over time. These *evolutionary dynamics* can take a wide variety of forms, but all should have this general property. The *replicator dynamic* discussed by Mailath (1998) is a particular (and fairly simple) model of evolutionary dynamics.

In this context, an equilibrium should be a rest point of whatever evolutionary dynamics we have specified for the evolutionary game that we are looking at. An *evolutionary stable strategy* (ESS) corresponds to a rest point of the general evolutionary dynamics that is stable in the sense that if we are kicked away from the rest point a little bit, the dynamics push us back to it.

Specifically, a strategy σ is an ESS if, starting with a population of all σ players, a small group of 'mutants' playing *any* alternative strategy σ' would have lower fitness than the σ players. Thus a small 'invasion' of σ' type players would be repelled (the σ' types would die out).

Technically, this means that, for a population of almost all σ types, we would have $F_\sigma > F_{\sigma'}$. I.e., for a very small proportion ϵ of σ' players in the population, we would have

A. $$(1-\epsilon)u(\sigma,\sigma) + \epsilon u(\sigma,\sigma') > (1-\epsilon)u(\sigma',\sigma) + \epsilon u(\sigma',\sigma')$$

Since ϵ is very small, this condition will hold iff:[2]

1. $u(\sigma,\sigma) \geq u(\sigma',\sigma)$ (i.e., (σ,σ) is a NE of the normal form game), and
2. If $u(\sigma,\sigma) = u(\sigma',\sigma)$ (i.e., if there is more than one BR to σ), then $u(\sigma,\sigma') > u(\sigma',\sigma')$ (the mutants do poorly against themselves).

Notice that condition 1. says that an ESS must be a NE. However, not all NE will be ESS. To see this, let's consider a specific game.

Chicken:

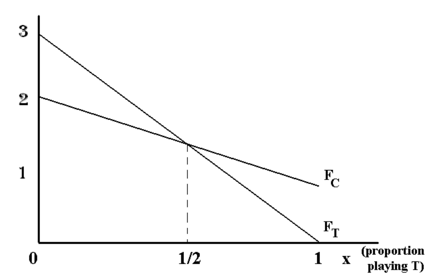
Recall our game of Chicken with payoff structure:

		Player 2	
		Chicken	Tough
Player 1	Chicken	2,2	1,3
	Tough	3,1	0,0

Now call x the proportion of individuals in the population that are hardwired to play Tough (T) at a given point in time, and suppose that the rest of the population is hardwired to play Chicken at that time (C). Then the fitnesses (expected payoffs) of each of these pure strategies are:

$$F_T = x \cdot 0 + (1-x) \cdot 3 = 3 - 3\,x$$
$$F_C = x \cdot 1 + (1-x) \cdot 2 = 2 - x$$

Notice then that if $x = \frac{1}{2}$ then $F_T = F_C$,
while if $x > \frac{1}{2}$ then $F_C > F_T$,
and if $x < \frac{1}{2}$ then $F_T > F_C$.



[2] Conditions (1. and 2.) are equivalent to condition A., and so either can be used to describe an ESS. Notice that ESS is defined in such a way as to capture the notion of stability under evolutionary dynamics generically, without having to specify the exact dynamics.

Thus, if we start with less than half of the population playing Tough, then Tough has higher fitness (higher expected payoff) than does Chicken. Consequently, over time, the proportion x of individuals playing Tough should increase. Similarly, if we start with more than half of the population playing Tough, then Chicken has higher fitness than does Tough, and x should fall over time. We see that the evolutionary dynamics lead the population to a *polymorphic* equilibrium in which half of the population plays Tough and half plays Chicken.

What are the ESS of this game? It turns out that the only ESS in this game is the mixed strategy $\sigma = (\frac{1}{2}, \frac{1}{2})$. You should demonstrate that σ is an ESS by confirming condition A for this strategy σ, first for an invasion of C mutants and then for an invasion of T mutants.[3] Similarly, you should demonstrate that neither pure strategy is an ESS. For example, while it is true that a population of all Tough players ($x = 1$) would all have the same fitness, and so would remain unchanged in the absence of mutation (i.e., $x = 1$ is a rest point of a standard evolutionary dynamic such as the replicator dynamic), a mutant Chicken player in that population would have higher fitness and so would replicate. In terms of the conditions for ESS above, we have, for very small ϵ (i.e., for a small proportion playing C), that $F_T < F_C$, since $(1-\epsilon)\cdot 0 + \epsilon \cdot 3 < (1-\epsilon)\cdot 1 + \epsilon \cdot 2$, and so condition A. is violated (equivalently, we could note that condition 1. is violated, since $1 > 0$).

Note that the unique ESS in this game is the mixed strategy played in the mixed strategy NE of the strategic form game. We can thus say that the criterion of evolutionary stability selects the mixed strategy NE of Chicken and does not support the pure strategy equilibria. Note however that we can interpret this ESS as describing the stable configuration of either a monomorphic population (with every individual mixing 50/50) or a polymorphic population (with half of the individuals playing C and half playing T). Under either interpretation, the expected payoff for each individual is 1.5.

[3] I.e., a *monomorphic* population playing mixed strategy σ is resistant to invasion.