Econ 460
Game Theory
Assignment 1
Representing Games (Ch. 2,3), Mixed Strategies (Ch 4), Dominance and Rationalizability (Ch 6,7) , Nash Equilibrium (Ch. 9)

1. The Black Knight: The Black Knight stops Arthur at a crossroads in the woods. At this point he can let Arthur pass (L) or challenge Arthur to a fight (C). If the Black Knight lets Arthur pass, his payoff is 0 and Arthur's is 10. If he challenges Arthur to fight, Arthur can either fight (F) or turn back (T). If Arthur fights, the payoffs are -5 for the Black Knight and 5 for Arthur, and if Arthur turns back the payoffs are 10 for the Black Knight and -5 for Arthur.
    a. Write down the extensive and normal form representations for this game (i.e., write down a game tree and a game matrix).
    b. What strategies are *rationalizable* in this game? Can the game be solved by iterated removal of strictly dominated strategies (iterated dominance)?
    c. What are the *Nash equlibria* (in pure strategies) of this game.
    d. Now suppose that the Black Knights was a better fighter. Suppose that the game is as above, except that now, if the Black Knight challenges Arthur and Arthur fights, the payoffs are 5 for the Black Knight and -10 for Arthur. Repeat parts a, b, and c for this new game.

2. Mixed Strategies: Consider the Matching Pennies game.
    a. Suppose that Player I plays the pure strategy Heads and Player II plays the mixed strategy (0.5,0.5). Calculate the *expected payoffs* for each player given the strategies being played.
    b. Now suppose that both players play the mixed strategy (0.8,0.2). Calculate the *expected payoffs* for each player in this case given the strategies being played. Who's expected payoff is greater? Why (intuitively)?
    c. In either case above, would either player be better off changing her strategy, given the strategy of the other player?

3. Consider the eight games on the 2x2 games handout as well as the game Rock, Paper, Scissors.

    a. Which of these games are solvable by iterated dominance?
    b. Find all Nash equilibria (in pure strategies) of these games.
    c. Which type of game is the simultaneous move version of the Nintendo/Sony competition game that we discussed in the first class.

4. Imperfect Information: Please do Chapter 2 Exercise 6 (p. 22). Please also write down a normal form representation for this game.

5. A Market Entry Game: A phone company (Bell) is considering offering high speed internet service (ISP) in a market in which currently a cable company (Time Warner) is the only such provider. In the game, Bell can *enter* or *not enter* and Time can choose to *advertise* or *not advertise* to protect its market share.

   If Bell doesn't enter, Time keeps the whole market. We assume that in this situation it makes profit of 5 if it doesn't advertise and profit of 3 if it does advertise (i.e., we are assuming that advertising is costly and doesn't generate much benefit if there is no competition). Further, if Bell doesn't enter, Bell's profit is 0.

   If Bell enters and Time doesn't counter by advertising, then they compete by cutting prices, so profits are 2 for Time and 1 for Bell. If Bell enters and Time advertises, Time keeps many of its customers from switching to Bell, but only at substantial advertising cost, so profits are 0 for Time and –1 for Bell.

   Assume that this payoff structure is common knowledge – i.e., each firm knows both its own and the other firms exact payoffs for each possible outcome of the game (this then is a game of "complete information").

   Suppose first that this is a simultaneous move game – i.e., each makes its decision not knowing what decision the other firm is making (this then is a game of complete but "imperfect" information).

   a. Write down the extensive form representation of this game (i.e., draw the game tree).
   b. What is the set of available strategies (strategy space) for each player? Write down the strategic form representation of this game (i.e., draw the game matrix).
   c. What are the rationalizable strategies for each player in this game? Is the game solvable by iterated removal of strictly dominated strategies (iterated dominance)?
   d. What are the Nash equilibria (in pure strategies) of this game?

   Now suppose instead that this game is played sequentially. Assume that Bell moves first and that Time sees Bell's move before acting (i.e., this is now a game of "perfect" information).

   e. Write down the extensive form representation of this game (i.e., draw the game tree).
   f. What is set of available strategies (strategy space) for each player? Write down the strategic form representation of this game (i.e., draw the game matrix).
   g. What are the rationalizable strategies for each player in this game? Is the game solvable by iterated dominance?
   h. What are the Nash equilibria (in pure strategies) of this game?

For later:

i. What is the backward induction solution of this sequential move game? Consequently, which of the Nash equilibria of that game are subgame perfect? How convincing are those that are not as solutions to this game?
j. In what sense would Time be better of if it had less choice in this game?
k. Is there a first or second mover advantage in this game? To see this, suppose that Time moved first rather than Bell. Does each firm have a preference between moving first and moving second?

6. Bank Runs: Suppose that a bank has only two customers. Each customer has deposited $100 in the bank. The bank has invested the combined deposits ($200) in a long term investment. The bank can liquidate the investment now for $120, but if it waits another period, it can liquidate the investment for $250. Each customer must choose (simultaneously) whether to leave her deposit in the bank. If both leave their money in the bank, then they will each receive $125 next period. If both withdraw now, then they get back only the $60 each that the bank can recover now. If one withdraws and the other stays in, then again the bank must liquidate the investment early, but the withdrawer gets $100 and the other gets only the remaining $20. Suppose that the customers do not care when they get their money, but rather only care about the amount of money that they will receive? What outcomes in this game are rationalizable? What are the Nash equilibria of this game? What kind of game is this (chicken, prisoners' dilemma, etc.)?

7. Bertrand Game I: Suppose that there are only two pizza shops in a particular geographic market – let's call them Roma's and Tony's. Suppose that they each only consider three possible prices for a pie: a high price (H), a medium price (M) and a low price (L). The profit per pie sold at each price is known to be $12, $10, and $6 for each firm (regardless of the volume of sales). Further, there is a perfectly inelastic demand in the market as a whole: customers will buy 10,000 pies per week regardless of the price. However, if the prices at the two shops are different, all of the demand goes to the lower priced shop, and if the prices at the two shops are the same, the shops will split this market demand evenly.

    a. Write down the game matrix for this game (note that the game is symmetric, so you can figure out the payoffs below the main diagonal and mirror these above the diagonal). Find all Nash equilibria (in pure strategies) of the game. Please provide intuition for each equilibrium.
    b. Brand Loyalty: Now let's change the game a bit. Suppose now that there is some brand loyalty, so that the player with the higher price loses some but not all of the customers in the market to the lower priced firm. to the if we price above our rival's price). Assume now that each shop has a loyal customer base of 3,000 pies per week. The remaining 4,000 pie sales goes to the lower priced firm or are split evenly by the firms if the firms set the same price. Recalculate the payoffs for this modified game (note that the

payoffs on the main diagonal of the matrix are the same). Find all Nash equilibria (in pure strategies) of this game. Compare this answer to the answer to part a.

c. Lowest Price Guarantees: Return to part a. Now suppose that we add a fourth possible strategy to each firm's strategy set. This strategy is to price *high* but offer a *lowest price guarantee* (HLPG). I.e., if the other firm prices below you, customers can still buy from you at the lower price. Write down the game matrix with for the new game (noted that you are just adding a row and a column, the payoffs in the original cells don't change). Find all Nash equilibria (in pure strategies) of the game, and compare this solution to the solution in part a.
d. Are there any dominated strategies in the games in parts a, b or c? Can any of these games be solved by iterated dominance?

8. A three player game: Please do Ch. 2 Exercise 2 (p. 21). What are the strategy spaces of the three players? Could the normal form of this three player game be represented using matrices?