Econ 460
Game Theory
Assignment 2

Nash Equilibrium: (Ch. 9,10), Continuous Strategy Spaces, Games With Many Players (Collective Action Games)

Games With Continuous Strategy Spaces:

1. Spatial location game: Suppose that two hot dog stand owners have been permitted to set up their stands on a quarter mile strip of beach. Beach goers tend to distribute evenly on the beach and tend to frequent the stand nearest to them if there is more than one stand. Each of the owners must commit in advance to a location along the beach without knowing where the other will choose to locate.
   a. Assume that the strip of beach is linear, and so the ends are not connected. What is the Nash equilibrium for this location game (where on the beach does each locate in the Nash equilibrium)? Could this game be used as a metaphor for the selection of platforms by two candidates in a presidential election campaign?
   b. Would the equilibrium of the game be different if the two ends of the beach were connected - e.g, if the beach surrounded a small lake? Would it be different if there were more than two stand owners?
   c. Listen to the Planet Money Episode #128 podcast "Friend or Foe." Compare the cases discussed there with the spatial location game above.

2. Cournot Oligopoly: Consider a market in which there are a small number of firms selling a homogeneous product. Each firm wishes to make as much profit as it can. The market demand is $P=540-Q$, where Q is the joint supply of the firms in the market (q_1+q_2). Each firm faces marginal cost of 90 for each unit produced and no fixed costs of production. Firms simultaneously select individual quantities q_i to supply in the market, and the market price is then determined by the demand formula above (competition among buyers drives the price up to that price). The game ends after this one round of play.

   a. Duopoly: Suppose first that there are only two firms in the market. Notice that, each firm knows that the price in the market will be $P = 540 - q_1 - q_2$. Write down the payoff functions for each firm ($u_1(q_1,q_2)$ and $u_2(q_1,q_2)$). Calculate the best response functions for each firm ($BR_1(q_2)$ and $BR_2(q_1)$). Use these to solve for the Nash equilibrium of this game. Calculate the profit received by each firm at this equilibrium.
   b. Three Firms: Now suppose that there are three firms in the market. Suppose that firm three believed that the first two firms will supply $q_1=q_2=75$. What is the best response of firm three to these quantities? How much profit does firm 3 expect each firm to make in this case?
   c. What is the Nash equilibrium for this (one shot) game with three firms? How much profit does each firm make at the Nash equilibrium? Why are the quantities in part b not a Nash equilibrium?
   d. Cartel: Suppose that the three firms form a cartel and agree to set market supply to maximize their joint profit and to split this market supply evenly amongst themselves. What quantity will each firm supply? How much profit will each make? Will each firm want to violate the agreement?

e. Talk is Cheap: Given that the game is played only once, and the agreement is verbal, do you expect each firm to respect the agreement?

3. Bertrand Duopoly 2: Consider the following Bertrand pricing game with brand loyalty and continuous pricing. Two pizza shops (we'll call them Roma's and Tony's again) are the only shops in a geographical market. Each firm faces MC=3 and no fixed costs of production. Each selects a price (simultaneously) and knows that it's demand depends on both its own and it's rival's price as follows: $q_i = 12 + 0.5\, P_j - P_i$. Find the best response functions of each firm ($BR_1(P_2)$ and $BR_2(P_1)$). Use these functions to determine the Nash equilibrium of this game.

4. R&D Competition: Consider two firms in a high-tech market. Each must decide (independently and simultaneously) how much to spend on R&D this year. We will assume that the profit per unit of R&D spending for each firm is decreasing in the total R&D spending of the two firms. Specifically, the total profit of any firm i, is:

$$\boldsymbol{\pi_i} = (12 - x_i - x_j) \cdot x_i$$

where x_i is the R&D spending of firm i, and x_j is the R&D spending of the other firm.

If there was only one firm in the market, it would choose 6 units of R&D spending and its profit would be 36. With two firms, will total R&D spending in the market be greater than or less than 6? To answer this question, please calculate the R&D spending x_1 and x_2 of the two firms at the Nash Equilibrium of this game (note: the equilibrium is symmetric). What is the total profit (the sum of the two firms' profits) in the market at this equilibrium?

Collective Action Games:

5. Worker Cooperatives: Consider a worker owned company with 100 worker-owners who we will call "members." Each member can select high or low effort. We will call these effort levels "working" (W) or "free riding" (FR). Profits are shared equally by the members, regardless of their individual contributions. When an individual works, she generates extra profits of $1000 for the group (and thus $10 for each member, including herself), but incurs disutility that she values at $200 for herself. Free riding yields no extra profit for the group and generates no disutility to the free rider.

   a. What is the Nash equilibrium for this game (how many members choose W)? (It may be useful to graph the payoff for a representative individual to each of the two strategies as a function of the number n of other individuals who choose W). Is this equilibrium socially efficient? (Here it may be useful to graph the sum of all individual's payoffs as a function of the number of others n who select W). Explain. How might the group try to improve the outcome of the game?
   b. Now suppose that each member enjoys participating in the cooperative, and that her enjoyment of participation depends on the number of members who are working hard. Specifically, when a member selects W, rather than incurring a flat disutility of working hard of $200, she incurs utility of $(– 200 + 3*n), where n is the number of other members who select W. Thus, if enough other members are working hard, she finds working hard enjoyable. What are the (pure strategy) Nash equilibria of this game?

6. Sport Utility Vehicles: Suppose that there are a large number of drivers in a community. Each can choose to drive a compact car (C) or a sport utility vehicle (SUV). Suppose that drivers care only about the safety of the vehicle that they drive and its cost. SUVs cost more (to buy and operate) than compact cars, but compact cars become less safe as the number of SUVs on the road increases.

   Specifically, suppose that the perceived safety of driving an SUV is valued by drivers at $4000, regardless of what type of car other drivers are driving, and the additional cost to driving an SUV is $1000. Suppose further that the perceived safety of driving a compact car is $4000 – 3000*n, where $n \in (0,1)$ is the proportion of drivers on the road who are driving SUVs.

   a. What are the Nash equilibria of this game? Which is socially efficient?
   b. Would your answer to part *a* change if drivers intrinsically liked SUVs more than compact cars?

7. A Game With Social Preferences: Consider two neighbors. Each must decide (independently and simultaneously) how much to consume this year. Each player has income I, and what is not consumed is saved. We will assume that the utility for each neighbor depends positively on both his own saving and consumption, but also on the difference between his consumption and that of his neighbor's. Specifically, assume that the utility of each player is as follows:

   $U_1(C_1,C_2) = (I-C_1)*[0.5*C_1 + 0.5*(C_1-C_2)]$
   $U_2(C_1,C_2) = (I-C_2)*[0.5*C_2 + 0.5*(C_2-C_1)]$

   Which reduces to:
   $U_1(C_1,C_2) = (I-C_1)*(C1 - 0.5*C_2)$
   $U_2(C_1,C_2) = (I-C_2)*(C2 - 0.5*C1)$

   Note that utility for each player here is the product of his saving and an average of his own consumption and his relative consumption. Since saving is income minus consumption, each player i selects a level of his own C_i. An equilibrium is a profile of consumption levels (C_1,C_2) (these equilibrium values will depend on I).

   a. Find the NE for this game. (Note: the equilibrium is symmetric.)
   b. The joint welfare (utility) of the two neighbors would be highest if they each saved half of their incomes (you can take my word for this). Are they doing this at the NE above? Explain intuitively why or why not.