Econ 460
Game Theory
Assignment 3
Nash Equilibrium in Mixed Strategies (Ch. 11)

1. Consider the Battle of the Sexes game. Use the game matrix from my handout on 2x2 games. Confirm that this game has two Nash equilibria in pure strategies. Then consider mixed strategies. Show that there is a third Nash equilibrium, which is in mixed strategies, by plotting the best response curves for each player in mixed strategies (i.e., in the probabilities that each selects Nikki Minaj). Solve for this Nash equilibrium in mixed strategies (calculate the probability distributions σ_1 and σ_2). What are the expected payoffs for each player in that equilibrium?

2. Consider the following variation on matching pennies applied to a particular volley in tennis. Player one is the offensive player (deciding where to hit the ball) and player two is the defensive player (who wishes to position herself in order to return the ball). We will suppose that both players have to select a strategy simultaneously. Player 1 will decide to hit to her opponent's forehand (F) or backhand (B) and player 2 will take a defensive position against a shot to the forehand (F) or backhand (B). The payoffs in the game matrix below are the likelihoods that each player will win under each strategy profile. If player 2 guesses correctly, and matches player 1's choice, then player 2 wins 80% of the time, whereas if player 2 guesses incorrectly, and fails to match player 1's choice, then player 1 wins 80% of the time.

		Player 2	
		F	B
Player 1	F	20,80	80,20
	B	80,20	20,80

a) Show that ((½,½), (½,½)) is a mixed strategy Nash equilibrium of this game. What are each player's expected payoffs at this equilibrium. Are there any other NE of this game? Illustrate your answer using a best response diagram for the two players.
b) Now suppose that player 1 improves her forehand so that she now has a higher probability of winning when player 2 matches against her forehand. Specifically, suppose that the payoffs to (F,F) become 40,60. Recalculate the mixed strategy NE and expected payoffs at this equilibrium. Does player 1 now play F more or less frequently (i.e., with higher or lower probability)? Explain this result intuitively.
c) Ignore part b. Now suppose that player 1 improves her forehand so that she now has a higher probability of winning when player 2 fails to match against her forehand. Specifically, suppose that the payoffs to (F,B) become 90,10. Recalculate the mixed strategy NE and expected payoffs at this equilibrium. Does player 1 now play F more or less frequently (i.e., with higher or lower probability)? Explain this result intuitively.

3. Confirm (by making the necessary calculations) that ((1/3,1/3,1/3),(1/3,1/3,1/3)) is a NE for the game Rock, Paper, Scissors. Explain intuitively why there are no other NE of this game.

4. Sales: Recall the Bertrand game with brand loyalty from the Problem Set I, #5 part b. Suppose that each firm now chooses between two prices, a high price at which profit per pizza is $12 and a low price at which profit per pizza is $8. Otherwise, the problem is as before. Find all of the NE for this game. If we interpret the mixed strategy NE as an equilibrium in which each firm may or may not run a sale, what is the likelihood of a sale occurring in this market. (Aside: note that in this model the role of the "loyal" consumers could also be played by "uninformed" consumers who do not bother to find out whether there are sales).

5. A Minority Game: Consider the following game in which it is your interest to be in the minority of the population. Suppose that there are three traders of a financial security each of whom must independently chose whether to place a buy order or a sell order this morning. These orders are large, and so will influence the market price. Consequently, for each player, the payoff to Buy is 3 if the other two Sell and -1 if at least one other Buys. Similarly the payoff to Sell is 3 if the other two Buy and -1 if at least one other Sells. I.e., it pays to be in the minority.

   a) Are there any symmetric pure strategy NE in this game?
   b) Is there a symmetric NE in mixed strategies?

6. A Good Samaritan Game: Please do chapter 11, problem 8.

7. Reporting Cheating on an Exam: Suppose that you are one of 2 students who witness another student, Biff, cheating on an exam. Each of you is angry about the cheating, and each believes that he or she will gain utility of 100 if the student is reported, regardless of who reports him. However, there is also a cost to any student who gets personally involved, which each assesses at 80.

   Note that, if you knew you were the only student that witnessed the cheating, you would report Biff, as your benefit would outweigh your cost (you would get net utility of 20). However, you also know that either you or the other student who witnessed the cheating could do so, and you would prefer that the other student bears the cost. You also know that the other student feels the same way.

   Suppose that, after leaving the exam, both you and the other student who witnessed the cheating decide independently whether to report the cheater (R) or not (N). Further, suppose that each of you plays a mixed strategy. What is the probability that neither reports the cheating episode at the Nash equilibrium?

   If the number of witnesses was 10 instead of 2, would the probability that no one reports the incident be higher or lower? Provide intuition for this result.