Game Theory
Assignment 5
Indefinitely Repeated Games (Ch. 22, 23)

Indefinitely Repeated Games:

1. **Efficiency Wages 1**: Consider the following game between a worker and her employer. To simplify matters, suppose that the two are stuck with each other: the worker can neither be fired nor quit. The employer can pay a high wage or a low wage (W_H or W_L). The worker can work with a high or low level of effort (E_H or E_L). The level of effort chosen by the worker determines the firm's revenues. High effort generates revenues of Y_H, low effort generates revenues of Y_L for the firm.

   Assume that the worker likes getting paid and dislikes expending effort. Specifically, assume her utility from working is W-E. For example, with high wage and high effort, her utility is W_H-E_H.

   Assume that the employer cares about the firms profits, so her utility from hiring the worker is Y-W. For example, with high wages and high effort, her utility is Y_H-W_H.

   a. Suppose that Y_H=10, Y_L=4, W_H=6 , W_L=2 , E_H=2 , E_L=0. Draw the game matrix for this game assuming that it is played once and that moves are simultaneous. Verify that this game is a prisoners' dilemma type game. What is the Nash equilibrium of the game? Provide an intuitive explanation for this outcome.
   b. Now suppose that the worker and employer play this game once each period forever. Assume further that the interest rate is 25% per period (i.e., r=.25) and that the discount factor used by each of the two players in evaluating payoffs in future periods is $\delta = 1/(1+r)$. Suppose further that the employer plays a *grim trigger* strategy: she pays a high wage in the first period, and then continues to pay a high wage as long as the worker selects high effort. If the worker selects low effort in any period, the employer switches to a low wage for the rest of the game (i.e., for eternity). Show that, in response to the employer's trigger strategy, the worker is better off selecting high effort in every period than selecting low effort in every period.
   c. Now suppose that the employer is considering changing W_H. What is the smallest level of W_H that she can use to elicit high effort from the worker in the infinitely repeated game?
   d. Ignore part c. Is (grim,grim) a subgame perfect Nash equilibrium of this infinitely repeated game?

(over)

2. **Tacit Collusion in the Stock Market:** Suppose that there are 20 dealers who are market makers for a particular stock, and that buyers and sellers of this stock must buy and sell from these dealers. Assume for simplicity that there are no dealer fees, so dealers only make money if there is a spread between the prices at which they buy (their "bid" price) and sell (their "ask" price) and that the cost to dealers of operating is zero. The dealers set ask and bid prices individually. Customers buy from the dealer(s) with the lowest ask price and sell to the dealer(s) with the highest bid price.

   Suppose that the demand for the stock by the public is given by the demand equation

   $P_a=25-0.1\ Q_a$

   and the supply from the public is given by

   $P_b=15+0.1\ Q_b$

   where P_a and P_b are the active ask and bid prices (set by the most attractive dealers) and Q_a and Q_b are the quantities demanded and supplied at those prices per day.

   a. What would the price for this stock be in competitive equilibrium? What are the profits made per day by each dealer in this equilibrium? Do you think that this is a Nash equilibrium price for this game?
   b. Suppose that dealers can only set prices in increments of 1/8 of a dollar (e.g., \$21 3/8). If all dealers were to set their ask prices \$2/8 above the competitive price and their bid prices \$2/8 below the competitive price, what would each dealer's profit be per day?
   c. Now suppose that each dealer adopts a *grim trigger* strategy of setting prices at the levels specified in part b above, as long as all others do the same, but then revert to the competitive price for all future periods once any dealer defects from this strategy. Suppose further that defection involves setting an ask price \$1/8 above the competitive price and a bid price \$1/8 below the competitive price (thereby stealing the entire market if the dealer is the only one to defect). If this game is repeated indefinitely, how large must the effective discount factor be in order for this trigger strategy to induce cooperation?
   d. If dealers colluded overtly in this market, what would be the optimal spread between bid and ask prices?