Econ 460
Game Theory
Assignment 6
Evolutionary Game Theory (D&S, Mailath, Skyrms)

1. **Bertrand Duopoly**: Consider the pizza pricing game from Assignment 1, problem 7. Consider the variation in part 7b. in which firms have brand loyalty. Suppose now that each firm can price high (H), medium (M), or low (L), or price high with a lowest price guarantee (HLPG). Then the payoff matrix is:

	Firm 2				
Firm 1		H	M	L	HLPG
	H	60,60	36,70	36,42	60,60
	M	70,30	**50,50**	30,42	50,50
	L	42,36	42,30	30,30	30,30
	HLPG	60,60	50,50	30,30	**60,60**

Note that there are two pure strategy Nash equilibria (M,M) and (HLPG,HLPG).

a. Suppose now that there are a large number of such firms and that they are randomly matched in pairs of two over time to play this game. Further, assume that the firms don't try to solve the game rationally, but rather try a strategy and decide over time whether to stick with it or change to another one based on its performance. If the population starts with a fraction x of the population playing HLPG and the remaining fraction (1-x) playing M, what should happen to these fractions over time under evolutionary dynamics? Explain this result intuitively.
b. Among the two Nash equilibrium strategies M and HLPG, which are evolutionary stable strategies (ESS)? (You can ignore H and L and just look at whether a population consisting of all M could be invaded by "mutants" playing HLPG, and vice versa.)

2. **Finitely Repeated Prisoners' Dilemma**: Consider the standard one shot prisoners' dilemma game (use the payoffs from the 2x2 games handout).
a. Verify that D is an ESS of this game.
b. Now suppose that when players are randomly matched, the pairs play 5 rounds of PD (before moving on to new partners). Suppose further, that the players have not thought very carefully about the game and have only come up with three possible strategies for the game: Always Cooperate (AC), Always Defect (AD), and Grim (G). Write down the 3x3 game matrix for this game (assume that players don't discount the future within the five rounds played). What are the Nash equilibria of this game?
c. Suppose that 20% of the population is currently playing G and 80% are playing AD. What will happen to these fractions over time under evolutionary dynamics? Would your answer be different if more of the population was currently playing G?
d. What are the ESS of this (restricted) game?

e. Consider the mixed strategy σ' = (0,3/4,1/4). Verify that (σ',σ') is a NE of this game. Verify formally that σ' is not an ESS of the game.
f. Now suppose that some players thought some more about the game and came up with ("invented", or "discovered") some new strategies (notice that there are many, many possible strategies – compete plans of action – for this repeated PD game). Are there strategies that could be discovered that would always be fitter than G?

3. **Two Population Chicken**: Consider the Chicken game from our handout on evolutionary games. Recall that the only ESS is the mixed strategy (1/2,1/2). Now suppose that we think of the game as a two population game. I.e., there are distinct populations of type I and II players (row and column), and type I players are always matched (randomly) with type II players to play the game. Call x^I the proportion of type I players who play tough (T) and x^{II} the proportion of type II players who play tough (T). Draw a phase diagram illustrating the evolutionary dynamics for this game. Show that from (almost) any initial mix of T and C players (in each population), a convention will emerge in which one population will be aggressive (always playing T) and the other will be compliant (always playing C). Compare the expected payoffs in these evolutionary equilibria to the equilibrium payoffs in the single population version of the evolutionary game.