Game Theory
Assignment 7
Principal Agent Problems

1. **Bonus Pay:** Consider the Efficiency Wage problem on Assignment 5 (problem number 1). In that problem, the worker and the firm made simultaneous decisions about effort and the wages. Notice that the firm was, in that case, unable to tie the current wage to current effort. Cooperation (i.e., both high wages and effort) could only be maintained by a trigger strategy in an indefinitely repeated version of the game: the firm offered a high wage now in order to keep the worker's effort high in the future, and vice versa.

   Now consider a non-repeated version of this game in which the employer cannot observe effort directly, but does observe the output generated by the worker and can condition the worker's pay on that observation. We will assume that the worker's effort and output are positively but not perfectly correlated. We will also assume that the worker can decide not to work for the firm, in which case she gets payoff U_0.

   The timing of the game is as follows: the firm decides whether to offer the worker a particular type of contract; the worker then accepts or rejects the contract (quits – in which case the game is over); if the worker accepts the contract she then the selects her effort level (high or low); and finally the level of productivity is determined (by "nature") given the worker's effort level. If the game reaches this point, the worker's pay is determined according to the contract.

   We will suppose that if the worker selects low effort (E_L), she will generate low revenues (Y_L) for the firm with certainty, whereas if the worker selects high effort (E_H), then she generates high revenues (Y_H) with probability ½ and low revenues (Y_L) with probability ½ . Use the values from Assignment 4: Y_H=10, Y_L=4, E_H=2, E_L=0. Further assume that U_0=3.

   a. If the firm could observe effort directly and perfectly and write a contract in which the worker's wage depends on the level of effort, what wages would the firm pick for each effort level? What would the firm's expected profit be in this case?
   b. Now return to the case at hand (imperfect observation of effort). Consider three contract options for the firm. Contract I has a fixed wage that is paid if the worker accepts the contract (i.e., is paid regardless of effort level). Contract II has a base wage W_0, which is paid if the worker accepts the contract and a bonus B which is paid only if the firm observes high output. The third option is to not hire the worker at all (the firm gets payoff 0 in this case, and the worker gets U_0. Under Contract I, what wage will the firm set? What are expected profits under this contract?
   c. Under Contract II, what is the lowest level at which the firm should set the bonus? If the firm sets this bonus, at what level will it set the base wage? What will its expected profits be if it selects this contract?
   d. Under Contract II, could the firm offer no base wage (i.e., offer all bonus)? What would the firm's expected profit be in this case?
   e. Which contract option will the firm chose?
   f. Qualitative question: above we have assumed that the worker is risk neutral. If the worker was risk averse, how do you think that this risk aversion would this affect the firm's choice of base wage and bonus under Contract II?

g. Ignore part f. Now suppose that the game is as above with the following exception. If the worker selects high effort, she now produces low revenues with probability ¾ and high revenues with probability ¼ . How will this change your answer to part c (calculate the wage and bonus and explain why these numbers are or are not different than your answer to part c).

2. **Efficiency Wages 2:** Now consider a similar game to that above, except now the worker works as a part of a team and the firm can not observe the individual worker's contribution to the output of the team. Rather, the firm now monitors workers directly and can thus observe effort directly but still imperfectly.

   Specifically, we suppose that if the worker select high effort, the firm is able to identify this, but if the worker selects low effort ("shirks"), the firm correctly identifies this with probability pand mistakes it for high effort with probability (1-p).

   Suppose that the firm offers the worker the following contract. If the worker is observed shirking, she will be fired. If the worker is not observed shirking, the firm will pay her a fixed wage W. Assume that a worker who is fired goes elsewhere and gets utility U_0 (as above – i.e., there is no difference to the worker between quitting and being fired).

   What is the smallest wage W that the firm can offer in order to get its workers to select high effort? Explain why, if detection of shirking is imperfect (i.e., if p<1), this wage is strictly greater than $U_0 + E_H$ (i.e., strictly greater than 5 if we were to use the numbers above). I.e., why must the firm must pay the worker a premium over the value of her next best opportunity in order to get her to work hard? Is it profitable for the firm to offer this contract?