Game Theory
Assignment 8
Incomplete Information and Bayesian Games

1. **A Rat Race:** Consider two workers in a firm. Suppose that there are *three types of job* at the firm and that each worker can select which job she wants to do. In job type 1, the worker works 40 hours per week at low effort. In job type 2, the worker works 40 hours per week at high effort. In job type 3, the worker works 50 hours per week at high effort.

   There are also *two types of worker*. Type A workers (go getters) have higher productivity at each job and incur less disutility from exerting high effort than do type B workers (slackers). Suppose that the firm can't tell the two types of worker apart when they do the same job (even though type A is more productive) and pays the workers within each job the average productivity of the group working at that job. The productivities and disutilities of each type at each job are given in the table below.

   Now, while the firm does not know each worker's type, suppose that the workers each know what type the other worker is. Suppose further that the two workers choose job types simultaneously. Then, given type of contract that it uses, we can ignore the firm and think of this as a game of complete information between the two workers.

		Workers			
		Productivity of Type A	Productivity of Type B	Disutility of Type A	Disutility of Type B
Jobs	1	22	20	15	15
	2	40	30	22	26
	3	50	40	35	50

   a. If both workers were of type A, which job would each select in Nash equilibrium? What if both workers were of type B? Are these outcomes efficient (i.e., do they maximize the joint payoffs to the two workers in each case)?
   b. Now suppose that one worker is of type A and the other of type B. Explain why there is no NE in pure strategies in this case. For example, why is (Job 2, Job 1) not a NE? What about (Job 3, Job 1)?
   c. Calculate the mixed strategy NE of the game in part b (to simplify your calculations, note that each worker has a strongly dominated strategy, so that each will mix over only two of the jobs). Provide an intuitive explanation of this equilibrium and also explain why it is inefficient. I.e., explain the difference between this mixed strategy equilibrium and the first best equilibrium that would obtain if the firm could correctly identify each worker's type and pay her according to her actual productivity.
   d. What would the NE be in the game in part b if Job 3 is not available (so each worker's strategy set is {Job 1, Job 2}?

Bayesian Nash Equilibrium: (Ch. 24,26,27)

2. **Incomplete Information in a Game of Chicken:** Two players are playing a game of chicken. Payoffs are as in the standard chicken game except for the payoffs to (T,T). Player II is unsure whether these payoffs are (2,0) or (-2,0). She correctly believes that there is probability p that Player I is *strong* and doesn't mind crashing so that $u_1(T,T)=2$, and probability $1-p$ that Player I is *weak* and finds crashing very unpleasant so that $u_1(T,T)= -2$. These beliefs are common knowledge (i.e., player I knows that player II holds these beliefs). Find all possible Bayesian Nash equilibria (in pure strategies) of this game, assuming that $p\in(0,1)$. How large would p have to be for player II to never play T in a pure strategy Bayesian Nash equilibrium?

3. **Cournot Competition with Private Information:** Consider again our Cournot duopoly problem. Market demand is given by P=540-Q as before. Further both firms know that Firm I has MC=90, but now Firm I does not know whether Firm II has MC=80 or MC=100 and must pick it's quantity to supply without knowing this. Both firms know that Firm I believes that the likelihood of Firm II being of either cost type is equal, *a priori*. What quantities are supplied at the Bayesian Nash Equilibrium of this game (solve for the quantities supplied by Firm I and each possible type of Firm II)? Which type (low or high cost) firm does Firm I prefer to actually face at this equilibrium? Why? If Firm I knew which firm it was going to face when it chose it's quantity, would it act differently?

4. **An Auction:** Suppose that you want to sell a painting that is of no value to you. There are two potential buyers. You know that each can be of either type A or type B. A type A buyer values the painting at $4 million, while a type B buyer values the painting at $3 million. You believe that for each of the two potential buyers, the probability that she is of type A is ½ . Suppose further that, while each buyer knows her own type, she also holds the prior p = ½ about the other buyer's type. For each potential buyer, if she buys the painting, her payoff is her value for the painting minus the price she pays.

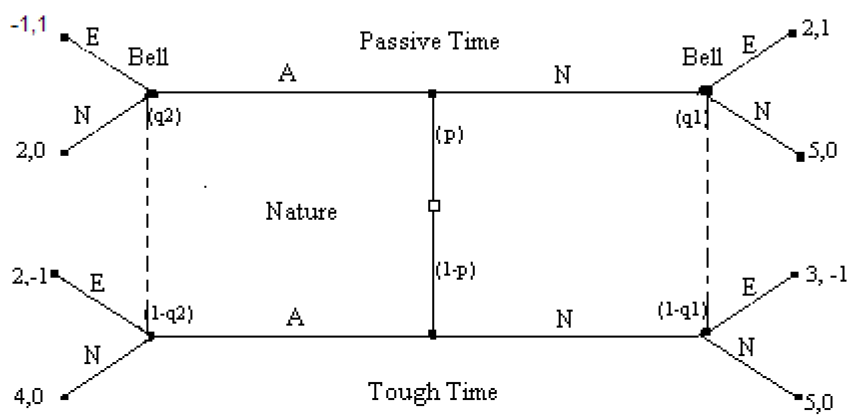
   a. Suppose that you run a second price sealed bid auction with the two potential buyers. Explain why you expect each buyer to bid her true valuation in this auction. What is your expected price from this auction?
   b. Explain why you should expect the same result if you were to run an English Auction (ascending price sequential oral bidding).
   c. Now suppose that you run a first price sealed bid auction. Explain why, at the Bayesian Nash equilibrium of this game, a type B buyer will bid her valuation ($3 million) whereas a type A buyer will play a mixed strategy of placing a bid (b) between $3 and $3.5 million with probability distribution function (b-3)/(4-b). Hint: for the mixed strategy to be played in equilibrium, the type A buyer must be indifferent between any bid in this range.

Perfect Bayesian Equilibrium: (Ch. 28,29)

5. Entry II: Consider again the game between Bell and Time from Assignment 1.

a. Simplify the game so that if Bell doesn't enter, Time has no move, and the payoff is (0,5). Now suppose that Bell can be of two types. If Bell is *strong*, the payoffs are as in Assignment 1. If Bell is *weak*, the payoff to Time when Bell enters and Time advertises is 3 rather than 0. I.e., when Bell is weak, Time improves its profit by advertising against the weak entrant. Suppose that Bell knows its own type, but Time does not know Bells type, and that the common prior is that Bell is strong with probability ½ . Draw the game tree for this game of incomplete information (with nature moving first). Show that the pooling equilibrium (EE,N,q = ½) is a PBE of this game. Why is there not a separating equilibrium (EN,N,q =1) in this game?

b. Ignore part a. Suppose now that Time can be of two types, *passive* or *tough*. Bell does not know Time's type but has common prior p = 2/3 that Time is passive. If Time is passive the payoffs are as in Assignment 1. If Time is tough the payoff to Time advertising when Bell enters is 3 rather than 0. Now let's simplify the game tree by recognizing that Time will advertise only if Bell enters and Time is tough. However, let's also complicate things a bit by supposing that Time can make a costly signal to Bell before Bell enters. The signal is to act aggressively (A) against another firm or not (N). Acting aggressively as a signal is easier for Time if its type is tough (cost = 1) than if its type is passive (cost =3). The game tree for this game is thus:



Is there a separating PBE for this game?