Assignment 1

1b. The rationalizable strategies are the pure strategies that survive iterated dominance. In this game, all of the players pure strategies are *rationalizable*, as none of them are (strictly) dominated. I.e., the set of rationalizable strategies for BK is $\{C, L\}$ and the set of rationalizable strategies for Arthur is $\{F, T\}$. Thus, any outcome is consistent with rational play.

This may seem odd, since looking at the extensive form of the game it looks as though the Black Knight should not challenge, as he should expect Arthur to fight if challenged. However, looking at the normal form of the game, C is not dominated by L, and so C is still rationalizable. Intuitively, if Arthur thinks that BK will play L, then both F and T are best responses by Arthur to this (rational) belief (as Arthur does not expect to be challenged and so doesn't expect to actually have to turn back if he chooses strategy T). Thus, BK can rationally believe that Arthur will choose T, and thus chose C himself as a BR to that belief. These beliefs and actions may not seem particularly smart, but they can't be ruled out as "irrational" if we define rational play as playing "rationalizable" strategies (i.e., as *not* playing dominated or iteratively dominated strategies).

1c. The unique pure strategy NE is (L,F). Thus, whereas any outcome of this game is rationalizable, only one is a NE. Further, in this case, the NE makes sense as a solution to the game: Arthur fights if challenged, and BK doesn't challenge.

1d. If we make the BK a better fighter, and change the payoffs to strategy profile (C,F) from (-5,5) to (5,-10), then we have a new game. In this game, L is dominated by C. Removing L from the game, F is then dominated by T in the reduced game. Consequently, this game can be solved by iterated dominance, the solution being (C,T). This is also the unique NE of the game.

2a. The strategy profile being played is $\sigma = ((1, 0), (0.5, 0.5))$. Then we have expected payoffs $u_1(\sigma) = 0$ and $u_2(\sigma) = 0$.

2b. The strategy profile being played is $\sigma = ((0.8, 0.2), (0.8, 0.2))$. Then we have

$$u_1(\sigma) = 0.64 \cdot 1 + 0.16 \cdot (-1) + 0.16 \cdot (-1) + 0.04 \cdot 1 = 0.36$$
$$u_2(\sigma) = 0.64 \cdot (-1) + 0.16 \cdot 1 + 0.16 \cdot 1 + 0.04 \cdot (-1) = -0.36$$

Since both players play H with a greater probability than T, the probability of matching is higher than in part *a* (the probability of a match is now $0.64+0.04 = 0.68$, whereas in part *a* it was $0.25+0.25 = 0.5$). This makes player I's expected payoff greater than that of player II.

3. The only games that are solvable by iterated dominance in this set of games are the Prisoners' Dilemma and Pigs. The Nash equilibria in pure strategies are e.g.,

- Prisoners' Dilemma: (D,D)
- Assurance: (E,E) and (DE,DE)
- Chicken: (C,T) and (T,C)
- Rock, Paper, Scissors: no Nash equilibria in pure strategies

5c. This game can be solve by iterated dominance. This solution (E,N) – Verizon enters, Time does not advertise – is also the unique NE of the game.

5f-h. The strategy space for Verizon is $S_{Verizon} = \{E, N\}$, where E stands for enter and N stands for not enter. The strategy space for Time is $S_{Time} = \{AA, AN, NA, NN\}$, where A stands for advertise, and N stands for not advertise. I.e., each of Time's strategies is a rule that specifies *both* what Time should do if Verizon enters, and what Time should do if Verizon doesn't enter. Thus, the normal form representation of the sequential move game is given by a 2 by 4 matrix.

The game can not be solved by iterated dominance. All pure strategies other than Time's strategy AA are rationalizable. There are three NE in pure strategies (and one additional NE in mixed strategies).

6. This is an assurance type of coordination game. The two pure strategy NE of the games are (Leave, Leave) and (Withdraw, Withdraw). In the first, there is no bank run, in the second there is a bank run. Beliefs that there will be a bank run are self fulfilling in the following sense: if each player believes that the other will withdraw, then each will withdraw. (There is also a third mixed strategy NE in which each player randomizes.)

7. Notice that in part a. there are two Prisoners' Dilemma games (one in H and M, the other in M and L) nested in the game. In both parts a. and b. there are unique NE. In both cases the NE can be found by cell-by-cell inspection or by iterated dominance. With respect to iterated dominance, in each of the two games, one pure strategy is dominated by a mixed strategy (with the other two pure strategies in its support), and once the dominated strategy is eliminated, we can iteratively remove additional strategies that are dominated in the reduced games.

   In part a., the unique NE is (L,L). In part b., the unique NE is (M,M). Brand loyalty removes the strong incentive to try to undercut your competitor's price in order to steal market share.

   The game in part c. can not be solved by iterated dominance. It has two NE in pure strategies (L,L) and (HLPG,HLPG).[1] Lowest price guarantees eliminate the incentive to try to undercut your competitor's price in order to steal market share.

8. The Owner has strategy space $\{H, N, M\}$ (i.e., hire, not hire, or leave it to the manager to decide). The Worker has strategy space $\{W, S\}$ (where W stands for work hard, and S stands for shirk). The Manager has strategy space $\{H, N\}$ (i.e., hire or not hire). The game matrix is three dimensional (3x2x2). However, we can represent it by writing down two 3x2 matrices (e.g.) with the Owner as the row player and the Worker as the column player. The first matrix contains payoffs (for the three players) for the case that the Manager plays H. The second matrix contains payoffs (for the three players) for the case that the Manager plays N. I.e., we are just dividing up the three-dimensional matrix into two two-dimensional matrices.[2]

Assignment 2:

1a. Given one stand's location, the best response of the other stand is to locate right next to the first stand's location on the side with more beach (i.e., more customers). Thus, the only NE is for both to locate in the middle.

2a. For starters, let's calculate the best response of firm 1 to any quantity set by firm 2. For any quantity q_2 set by firm 2, firm 1 sees its demand as $P = (540 - q_2) - q_1$. Thus, its profit is

$$\begin{aligned}\pi_1 &= TR_1 - TC_1 \\ &= P \cdot q_1 - 90 \cdot q_1 \\ &= (540 - q_2 - q_1) \cdot q_1 - 90 \cdot q_1 \\ &= (450 - q_2) \cdot q_1 - q_1^2.\end{aligned}$$

Firm 1's best response to q_2 will be its profit maximizing quantity given q_2. To find this we can set $\frac{d\pi_1}{dq_1} = 0$, or equivalently recognize that, given the demand function above, firm 1's MR is $MR_1 = (540 - q_2) - 2\,q_1$, and that its profit maximizing quantity must satisfy $MR_1 = MC_1$. Using either method, we get

$$q_1 = \frac{1}{2}\,(450 - q_2)$$

[1] There are three additional NE in mixed strategies, each of which involves at least one player playing (1/5,0,0,4/5).

[2] To look for Nash equilibria, work down columns for player I, across rows for player II, and across matrices for player 3.

Thus, firm 1's best response function is $BR_1(q_2) = \frac{1}{2}(450 - q_2)$. By the same argument, firm 2 has best response function $BR_2(q_1) = \frac{1}{2}(450 - q_1)$.

Solving the two best response functions simultaneously[3] ($q_1 = BR_1(q_2)$ and $q_2 = BR_2(q_1)$), yields

$$q_1 = q_2 = 150.$$

Thus, the quantity supplied in the market is 300 and the market price is \$240. Note that the combined profits of the two firms is \$45,000.

2b. $BR_3(75, 75) = 150$ (again, get this by setting $\frac{d\pi_3}{dq_3} = 0$ or setting $MR_3 = MC_3$). $\pi_i = P \cdot q_i - 90 \cdot q_i$. Since $P = 240$ (to get this plug 75+75+150 into the demand function), the firms profits are (\$11,250,\$11,250,\$22,500). Firm 3 makes twice as much profit as the others because it sells twice the quantity.

2c. In the Nash equilibrium, each firm produces and sells $q = 112.5$, so the price in the market is $P = \$202.5$. The combined profits made by the three firms is \$37,968.75, which is less than in the two firm case.

2d. Each firm splits the monopoly quantity ($Q = 225$), so that $q_1 = q_2 = q_3 = 225/3 = 75$. Thus $P = 315$ and combined profits for the three firms is \$50,625. Recall that the best response $BR_1(75, 75) = 150$, so this behavior is not self enforcing.

3. Here, we want to set $\frac{d\pi_i}{dp_i} = 0$ to get the best response of each firm. You should find that $P = 10$ for each firm at the Nash Equilibrium.

4. You should find that, at the NE, R&D spending by each firm is 4 and that profit for each firm is 16. Thus, competition between the two firms drives up total R&D spending and down total profit in the market.

5a. For any member, $u(W, n) = 10\,n - 190$, whereas $u(FR, n) = 10\,n$, where n is the number of other members who choose to work (W). This is a multi-player prisoners' dilemma game.

5b. Now the game is converted into a multi-player assurance game. If all workers believe that at least 63.33% of the other members will work, work is a best response.

6. This is a multiplayer assurance game. In this case, the best response of any player to the strategies of the other players depends only on the proportion n of the players who are driving SUVs. We are assuming that each player is so small relative to the population that her decision has (for practical purposes) no impact on n. There are three NE. In one, all pick SUVs, in the second other all pick compact cars, and in the third, 1/3 of the players pick SUVs and 2/3 pick compact cars. With any other mix of choices (other than these three equilibrium mixes), some members of the population are not playing BRs to the others' choices.

   In the first equilibrium each player has utility of \$4000, while in the other two equilibria each player has utility of \$3000. Since all players are better off in the first equilibrium, it Pareto dominates the other two. And since total utility (over all people in the population) can't be higher in any other configuration (including non-equilibrium configurations), the first equilibrium is Pareto efficient.

7. You should find that, at the NE, each of the neighbors consumes 2/3 of his or her income.

   Note that both neighbors would be better off if *both* consumed only 1/2 of his or her income, but has no incentive to do so unilaterally. This is sometimes referred to as a "hedonic treadmill" or "keeping up with the Joneses" problem.

[3] A short cut is to look for a symmetric equilibrium in which the two quantities are the same. Setting $q_1 = q_2$ in either of the best response equations will give us the answer $q = 150$.