Assignment 3

1. The Nash equilibria are (NM,NM) and (JB,JB) and ((2/3,1/3),(1/3,2/3)). The third is in mixed strategies (player 1 plays NM with probability 2/3 while player 2 plays NM with probability 1/3). You can solve for these probabilities by using the indifference condition for each player: at a mixed strategy NE, each player is using a mixed strategy that makes *the other* indifferent to using her two pure strategies (i.e., she is equally happy to play NM, or JB, or mix over the two).[1] The expected payoff to each player under the mixed strategy NE is 2/3. Notice that this is worse than either would get by conceding to the other's wishes. Why is that?

   Note that since we can write pure strategies as mixed strategies, we could also write the three NE as ((1,0),(1,0)), ((0,1),(0,1)), and ((2/3,1/3),(1/3,2/3)).

2.
   a. The NE is ((1/2,1/2),(1/2,1/2)). Expected payoffs are each 50.
   b. The NE is ((0.6,0.4),(0.6,0.4)). Expected payoffs are $u_1 = 56$, $u_2 = 44$.
   c. The NE is ((6/13, 7/13),(7/13,6/13)). Expected payoffs are approximately $u_1 = 52.3$, $u_2 = 47.7$.

   Notice that the (equilibrium) response of player I to the improvement in her F strategy is different in parts b and c. In part b, she increases the likelihood that she uses F (at the NE), while in part c, she decreases the likelihood that she uses F (at the NE). In both cases, her expected payoff at the NE increases.

4. There are three NE: (H,H), (L,L), and ((1/2,1/2),(1/2,1/2)). Under the latter, "sales" occur 75% of the time.

5. The game matrix is 2x2x2 and can be visualized by writing out two 2x2 matrices, one for III playing B and the other for III playing S. There are three payoffs (one for each player) in each cell. E.g., the payoffs for strategy profile (B,B,S) are (-1,-1,3). There are 6 NE in pure strategies, none of which are symmetric. There is one additional NE in mixed strategies which is symmetric.

   At the mixed strategy NE, each player must be indifferent between playing B and S. Thus, for each player i: $u_i(B, \sigma_{-i}) = u_i(S, \sigma_{-i})$. This gives us three sets of equalities that must be satisfied (note that each of the three equations restricts two of the players' probabilities P_i^B of playing B). However, to search for a symmetric NE, we can restrict our attention to the case of each player using the same probability of playing B. Then, using any one of the three equations, you should find that this probability is 0.5. Thus the symmetric NE is ((0.5,0.5),(0.5,0.5),(0.5,0.5)).

6. This is admittedly an unpleasant game, as it assumes that individuals prefer to let others be "good Samaritans." However, given that assumption, the symmetric mixed strategy NE has an interesting property, as seen in part b.

a. Suppose that each of the n players chooses not to call for help with probability p. For this to be a NE, it must be that each player is indifferent between the pure strategies of calling (C) and not calling (N), i.e., $u_i(C, \sigma_{-i}) = u_i(N, \sigma_{-i})$. Hint: To calculate the latter, note that the probability of no one else calling is p^{n-1}.

b. Once you find the NE value of p (which will be a function of v, c, and n), then note that the probability in equilibrium of no one calling is p^n which you will see is *increasing* in the number n of players. I.e., in this game, where each player would make the call if she were alone (since $v > c$) but prefers that someone else do it if she is in a crowd, at the symmetric mixed strategy NE, the likelihood that *no one*

[1] Note that if I am mixing as a BR to your strategy, than I must be indifferent between the pure strategies in the support of my mixed strategy, *given what you are doing*. Thus *your* strategy is making me indifferent.

calls increases as the crowd becomes larger. Specifically, with a single observer, the probability that no one calls is zero, with two observers it is $(c/v)^2$, and in the limit as $n \to \infty$, it $\to c/v$.

7. This is a variation on the good Samaritan game in #6. With two players, you can write down the 2x2 normal form game. You should find that each observer reports with probability 0.2. The probability that neither reports is thus 0.64. That probability rises as the number of observers increases as in #6.

Assignment 4

1-2. See class notes and readings.

3. Player 1. Note that this game is similar to a patent race. If two firms in a patent race had perfect information, they would be able to accurately predict who would win the race. A difference however is that patent races are costly to "play" (firms incur large R&D fixed costs in each round). Consequently, the losing firm should drop out at the start of the game.

   The game is also similar to the dollar auction game that we played the first day of class. Suppose that one player gets to go first, and it is common knowledge that both players have exactly \$1.72 at their disposal. Then if bids can be made in any increment, the first player should bid exactly 72 cents for the dollar. Why?

4. Solve by backward induction. Firm I moves first firm II moves second. Thus firm II will be reacting to firm I's output selection q_1. Note that this is equivalent to player II playing a best response to q_1 in the Cournot game. I.e., $q_2 = \frac{1}{2}(450 - q_1)$.

   Now go to the first stage. Firm I should maximize its profit given its expectation about firm II's response. Firm I sees its demand as

$$\begin{aligned} P &= 540 - q_1 - q_2 \\ &= 540 - q_1 - \frac{1}{2}(450 - q_1) \\ &= 315 - \frac{1}{2}q_1 \end{aligned}$$

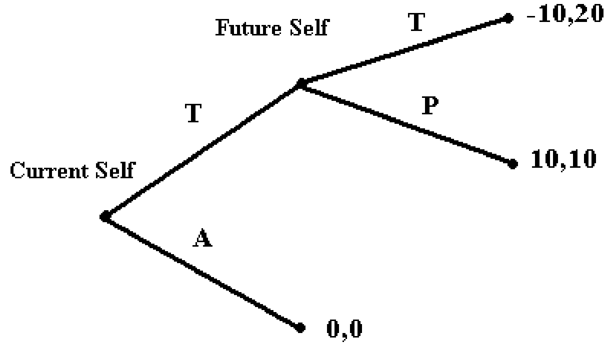
   Thus firm I sees its profit function as

$$\begin{aligned} \pi_1 &= TR - TC \\ &= (315 - \frac{1}{2}q_1) \cdot q_1 - 90 \cdot q_1 \\ &= 225 \cdot q_1 - \frac{1}{2} \cdot q_1^2 \end{aligned}$$

   Setting $\frac{d\pi_1}{dq_1} = 0$, we find $q_1 = 225$.[2] This implies $q_2 = 112.5$. Thus, market quantity is $Q = 337.5$ and market price is $P = 202.5$. Notice that the first mover has an advantage in the Stackelberg game. Also note that the aggregate outcome of the two firm Stackelberg game matches the aggregate outcome for the three player Cournot game, so the joint profit is lower than in the two firm Cournot game.

[2] Equivalently, we could note that firm 1 sees its MR function as $MR = 315 - q_1$, and so setting $MR = MC$ we find $q_1 = 225$.

5a. Without the option to be lashed to the mast, Odysseus will opt to sail away from the Sirens. The game tree is:



6. Draw the tree (Lisa chooses whether or not to invest; if she does invest, then Bart gets to chose whether to have high or low effort).

   a. Note that if Lisa invests, Bart's payoff will be $\$60,000 - \$10,000 \cdot E$ where $E = 0$ if Bart puts in low effort and $E = 1$ if Bart puts in high effort. Lisa's payoff will be $\$50,000 + \$70,000 \cdot E - \$60,000 - \$40,000$.

   b. Note that if Lisa invests, Bart's payoff is now $\$45,000 + (\$15,000 - \$10,000) \cdot E$, and Lisa's is $(\$50,000 + \$70,000 \cdot E) - (\$45,000 + \$15,000 \cdot E) - \$40,000$.

   This is a variation on what is called a "hold up" problem. The investment has the potential to benefit both Bart and Lisa. However, Lisa knows that once she has made the investment, Bart has no incentive to make it profitable for Lisa (he can "hold it up"[3]). This will cause Lisa to not make the investment (part *a*) unless she can find a mechanism that creates an incentive for Bart to make the project profitable after he is hired (part *b*).

7.

   i. Note that the strategy spaces for the two players are: $S_B = \{E, D\}$ and $S_T = \{AA, AD, DA, DD\}$. The subgame perfect NE is (E,DD). At this equilibrium, Bell enters and Time does not advertise. The two other pure strategy NE ((E,DA) and (D,AD)) are not subgame perfect. E.g., the latter involves a non-credible threat by Time to advertise if Bell enters. That outcome would be great for Time (its payoff would be 5), but (by the logic of backward induction) Bell should not believe Time's threat and so should go ahead and enter.

   k. There is a first mover advantage. Each firm would do better (for itself) if it moved first. E.g., Time would get a payoff of 3 instead of 2 if it moved first. Bell would get 0 in that case.

      Note that not all games have first mover advantages. E.g., matching pennies and Rock, Paper, Scissors games have second mover advantages.

8. See class notes and the answer to Assignment 3 #2 above.

9.

   a. (O,C)

   b. (C,OO) is subgame perfect. This is equivalent to the outcome in part **a**.

   c. Would either country like the WTO to require all countries to be open?

[3] In the classic hold up problem Bart uses Lisa's sunk cost as leverage to bargain for a better contract.

10. Yes, of course I should! The NE are (E,HL) and (N,LL). The SPNE is (E,HL). Note the non-credible claim made by the student in the non-SP NE that she won't study even if I do give an exam (implying that I shouldn't bother).

11. Use backward induction. If play reached the 4th round, player 2 would offer \$0 and get \$10. Knowing this, and working backward through the game, we discover that player 1 should offer \$40 in the first round, thus receiving \$60 and ending the game there and then.

    Note that in sequential bargaining games with perfect information (such as this one and the Rubinstein (1982) bargaining model that we covered in class), the solution has player two accepting an offer by player one in the first round. Since both players can look forward in the game and solve it by backward induction, there is no room for disagreement that causes the negotiations to break down - i.e., strikes, lockouts, and long negotiation periods are not explained by these models. To explain breadkdowns, we would need to modify these models, e.g., by relaxing the assumption of perfect information.

12-13. See class notes and Watson, p. 251.

15. **Conditionally Repeated Prisoners' Dilemma:** Yes. The NE of the proper subgame (following (C,C) in the first round) is (D,D). Working backward, the payoffs to (C,C) in the first round are then (3,3). Thus the first round reduces to the standard prisoners' dilemma, but with (3,3) rather than (2,2) as the payoff for (C,C). There are two NE of this reduced game: (C,C) and (D,D). Thus there are two SPNE of the full game: (CD,CD) and (DD,DD). In the first, each play the strategy (CD) of cooperating in the first round, and defecting in the second. Cooperating in the first round allows the game to move to the second round.

16. **Repetition of a Stage Game with Multiple NE:** Yes. You should confirm that there are SPNE in which each player cooperates (C) in the first round and then plays "partial cooperate" (P) in the second round only if the other player cooperated in the first round, and otherwise plays defect (D) in the second round.

    Since (P,P) and (D,D) are both NE of the stage game, they are both NE of all of the proper subgames corresponding to the second round in the game tree. So the promise/threat to play P in the second round if the other player cooperates in the first round but D in the second round if she defects in the first round is credible. What remains to show is that this promise/threat can induce each player to play C in the first round.

    The easiest such SPNE to confirm is (CPDDDDDDDD,CPDDDDDDDD). Here, each plays P in the second round only if both played C in the first round. You can also confirm that (CPDDPDDPDD,CPPPDDDDDD) is a SPNE. Here each plays P in the second round as long as the other played C in the first.