Mixed Strategies in Chicken

Consider the following Chicken game:

	Player 2		
Player 1		**C**hicken	**T**ough
	Chicken	2,2	**1,3**
	Tough	**3,1**	0,0

Cell by cell inspection reveals that there are two pure strategy Nash equilibria of this game: (T, C) and (C, T). We can also consider what would happen if each player decided to let chance select a strategy. Suppose that player 1 uses mixed strategy $\sigma_1 = (p_1^C, 1-p_1^C)$, and player 2 uses mixed strategy $\sigma_2 = (p_2^C, 1-p_2^C)$. Then when it comes time to play the game, player 1 plays C with probability p_1^C and plays T with probability $(1-p_1^C)$ and player 2 plays C with probability p_2^C and plays T with probability $(1-p_2^C)$.

When either or both of the players play a mixed strategy, the outcome of the game is random. We assume that, a priori, when faced with this kind of uncertainty, each player cares about her expected payoff, i.e., the expected value of her payoff under the mixed strategy. Let's spell out the expected payoffs for each player for various combinations of pure and mixed strategies.

$$u_1(C, \sigma_2) = 1 + p_2^C \qquad u_2(C, \sigma_1) = 1 + p_1^C$$
$$u_1(T, \sigma_2) = 3p_2^C \qquad u_2(T, \sigma_1) = 3p_1^C$$
$$u_1(\sigma_1, \sigma_2) = p_1^C + 3p_2^C - 2p_1^C p_2^C \qquad u_2(\sigma_2, \sigma_1) = p_2^C + 3p_1^C - 2p_2^C p_1^C$$

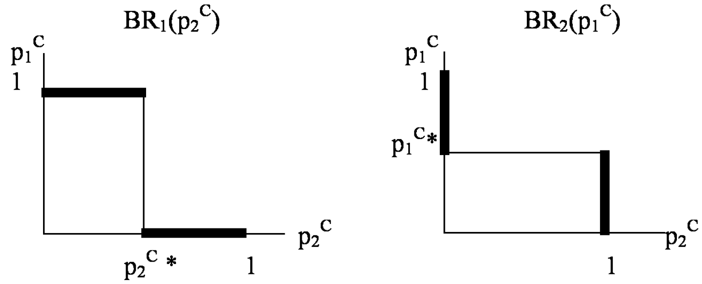
	Player 2			
Player 1		Chicken	Tough	Mixed $(p_2^C, 1-p_2^C)$
	Chicken	4,4	2,6	$2+2p_2^C$, $6-2p_2^C$
	Tough	6,2	0,0	$6p_2^C$, $2p_2^C$
	Mixed $(p_1^C, 1-p_1^C)$	$6-2p_1^C$, $2+2p_1^C$	$2p_1^C$, $6p_1^C$	$2p_1^C + 6p_2^C - 4p_1^C p_2^C$, $2p_2^C + 6p_1^C - 4p_1^C p_2^C$

For example, if Player 2 uses the mixed strategy $(p_2^C, 1-p_2^C)$ and Player 1 plays C, then Player 1 will receive 2 with probability p_2^C and 1 with probability $(1-p_2^C)$. Thus, the expected payoff for player 1 in this case $u_1(C, (p_2^C, 1 - p_2^C))$ is $2 \cdot p_2^C + (1 - p_2^C)$, which reduces to $1 + p_2^C$. Similarly, the expected payoff (not shown in the table above) to Player 2 in this case is $3 - p_2^C$ (you should confirm this).

We are looking for a Nash equilibrium in mixed strategies. Thus, we are looking for values of p_1^C and p_2^C such that $\sigma_1 \in \mathrm{BR}_1(\sigma_2)$ and $\sigma_2 \in \mathrm{BR}_2(\sigma_1)$ (i.e., values of p_1^C and p_2^C such that the mixed strategies that are being played by the two players are best responses to each other). So let's consider the best responses of each player to the other's mixed strategy.

Look first at $\mathrm{BR}_1(\sigma_2)$. If 2 chooses $p_2^C = 1$ (i.e., play pure strategy C), then 1 wants to play T (i.e., 1 wants to set $p_1^C = 0$). If 2 chooses $p_2^C = 0$, 1's BR is to set $p_1^C = 1$. So for some p_2^C in the interval $(0, 1)$ there is a switch in player 1's BR (call this value p_2^{C*}). At p_2^{C*}, player 1 is indifferent between playing C and T.

A similar logic applies to $\mathrm{BR}_2(\sigma_1)$. We can visualized these best responses as follows:



Combining these, we see that the BR functions intersect at (p_1^{C*}, p_2^{C*}). Thus, a Nash equilibrium of the game is the mixed strategy profile $\sigma^* = (\sigma_1^*, \sigma_2^*) = ((p_1^{C*}, 1 - p_1^{C*}), (p_2^{C*}, 1 - p_2^{C*}))$. We see that this game does indeed possess a mixed strategy Nash equilibrium.

Note that the BR curves also intersect at the two pure strategy Nash equilibria of this game (which, written as mixed strategy profiles, are $((1,0),(0,1))$ and $((0,1),(1,0))$). So the BR analysis in mixed strategies above shows us all three Nash equilibria of this game.

Finally, note that the probabilities p_1^{C*} and p_2^{C*} are the probabilities that make each player's opponent indifferent to which pure strategy she uses. We can use this result about indifference to calculate p_1^{C*} and p_2^{C*}. E.g., p_1^{C*} must make player 2's expected payoffs to selecting C and T equal. Thus, p_1^{C*} makes $1 + p_1^C = 3p_1^C$. Solve this for $p_1^C = 1/2$. By symmetry, we will have $p_2^C = 1/2$. Thus $\sigma^* = (\sigma_1^*, \sigma_2^*) = ((1/2, 1/2), (1/2, 1/2))$. Each player uses each of her strategies with probability 1/2 (i.e., "50% of the time").

Thus, under the mixed strategy Nash equilibrium, the two players share the chance of "winning." Unfortunately, they also create the possibility of a crash (which happen with probability 1/4). Thus the expected payoff of each player at the mixed strategy Nash equilibrium is $(1.5, 1.5)$, which is worse than each would get under (C, C). However, as with any Nash equilibrium, it would constitute a credible pre-game agreement, whereas

(C, C) would not, since the latter is not a Nash equilibrium. Finally, note that if the players agreed in advance to throw the game and flipped a coin to determine who will win (who play T while the other plays C), the expected payoffs (prior to the coin toss) would be $(2, 2)$ which is again better than the expected payoffs under the mixed strategy Nash equilibrium.