Economics 460	
Chris Georges	Evolutionary Dynamics in the Ultimatum Game

Consider the ultimatum game in which two players are to divide a dollar. Suppose that we simplify the game so that the proposer can offer either 50 cents or 10 cents, and the responder must accept the "fair" offer of 50 but can reject (R) or accept (A) the "unfair" offer of 10. Then the payoff matrix (strategic form) is:

	Respondent		
Proposer		R	A
	50	50,50	50,50
	10	0,0	90,10

Note that there are two pure strategy NE of this game: (50, R) and (10, A). If you draw the game tree it is easy to see that only one of these, (10,A), is subgame perfect. The other, (50,R), involves the noncredible threat by the respondent to reject an offer of 10, in favor of 0.

Note also that this is an asymmetric game. There are two distinct roles with two distinct sets of strategies and payoffs. Consequently, to study the evolutionary dynamics of the game, we must either (a.) treat this as a two population game, with a population of proposers and a population of respondents, or (b.) symmetrize the game by having a single population in which players don't know which role they will play a priori.

Two Population Evolutionary Ultimatum Game

Let's start with the two population game. This follows Gale, Binmore and Samuelson (GEB, 1995). There are two distinct populations, one of Proposers and one of Respondents. Call x_1 the proportion of Proposers who make "fair" offers of 50 and x_2 the proportion of Respondents who reject (R) unfair offers.

Then for Proposers (Population 1), the fitness of each of the two strategies (or types) is:

$$F_{50}^{1} = x_{2} \cdot 50 + (1 - x_{2}) \cdot 50 = 50$$

$$F_{10}^{1} = x_{2} \cdot 0 + (1 - x_{2}) \cdot 90 = 90 - 90 \cdot x_{2}$$

We thus have $F_{50}^1 > F_{10}^1$ if $x_2 > 4/9$.

I.e, if there are enough Respondents rejecting offers of 10, then offering 50 is successful (and will proliferate in the Proposer population) while if there are few enough Respondents rejecting offers of 10 then offering 10 will be successful (and proliferate in the Proposer population).

For Respondents (Population 2), the fitness of each of the two strategies (or types) is:

$$F_R^2 = x_1 \cdot 50 + (1 - x_1) \cdot 0 = 50 \cdot x_1$$

$$F_A^2 = x_1 \cdot 50 + (1 - x_1) \cdot 10 = 10 + 40 \cdot x_1$$

So $F_A^2 > F_R^2$ if $x_1 < 1$, and $F_A^2 = F_R^2$ if $x_1 = 1$. I.e., it doesn't pay individuals to reject unfair offers, and it only won't cost individuals to have a policy of rejecting unfair offers if no one makes unfair offers.

From this we get the following phase diagram (over):



There is a small basin of attraction for a fair play equilibrium in which $x_1 = 1$ and x_2 is greater than 4/9. In that equilibrium each Receiver benefits from having a large number of other Receivers reject unfair offers, since that behavior keeps unfair offers from proliferating in the Proposer population. However, there is a much larger basin of attraction for the other evolutionary equilibrium which is $x_1 = 0$, $x_2 = 0$ (i.e., all proposers offer 10 and all respondents accept). This evolutionary equilibrium corresponds to the SPNE of the game.

Symmetrized Ultimatum Game

Now if we were to switch to a single population version of the game (by symmetrizing it), we get an even greater role for fair play in evolutionary equilibrium, since now fair minded players can play against their own kind. In this simple version (in which all Receivers must accept 50), the game matrix is 4x4 (a strategy now describes what a player will do if she is the proposer (offer 50 or 10) and what she will do if she is the respondent (A or R).¹ In Skyrms' notation, the four strategies in each players strategy set are Gamesman, S3, Easy Rider, and Fairman. If you write out this game matrix, you will find a number of NE including (Fairman, Fairman) and (Gamesman, Gamesman), as was the case in Skyrms' game. Fairman and Gamesman are both ESS, and there are also a variety of polymorphic evolutionary equilibria, so I expect that, as with Skyrms' game, fair division is fairly likely to emerge in evolutionary equilibrium.

Note that, unlike the two population version, fair play can now have higher fitness than the modularly rational selfish play. E.g., Fairman has higher fitness than Gamesman as long as there are enough Fairmen in the population, since Gamesman does poorly against Fairman.

In either case, commitment to fair play by individuals (i.e., to rejecting unfair offers), though not modularly rational, is good for the population of fair players.

¹ Skyrms (Ch. 2) considers the same game, but allows respondents to chose whether to reject fair offers as well as unfair offers. Consequently, in Skyrms' version, the game matrix is 8x8 and a strategy describes what the player will do if she is a proposer, what she will do if she is a respondent and is offered 50, and what she will do if she is a respondent and is offered 10 (hence 8 possible strategies).