Economics 460
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More on Auctions

First consider auctions in which the bidders have private valuations of the item being auctioned. In our example in class, all players believe that there are two types of bidders. Type A bidders have private valuation \$4 and Type B bidders have private valuation \$3. The probability that a player is of either type is 0.5.

- If the auction is an **ascending price sequential bid** auction (English auction), then the BNE is $(4^A 3^B, 4^A 3^B)$. The expected price for the seller is \$3.25.
- If the auction is a **second price sealed bid** auction (Vickrey auction) then the BNE and expected price is the same as above. These two types of auctions are structurally equivalent.
- However, if the auction is a **first price sealed bid** auction, then rational bidders will no longer bid their private valuations in Bayesian Nash Equilibrium. Rather, the BNE is $(M^A 3^B, M^A 3^B)$, where M is a mixed strategy in which the bidder mixes over all bids in the interval [3.0,3.5] according to the cumulative probability distribution: Prob[bid $< b$] $= (b-3)/(4-b)$. As it turns out, the expected price in this case is still \$3.25, so while the Bayesian Nash Equilibrium strategies are quite different from those in the first two auctions above, the expected price is exactly the same.

Now note that if the seller knew the types of the two bidders and could set the price at the higher of the two players' private valuations, then the expected price would be \$3.75. So while each of the three auction designs above have the good properties that the bidder with the highest valuation wins the object, and the seller gets a price no lower than the second highest valuation in the population of bidders, it is also the case that the seller is not getting the maximum possible prices from the bidders. It is possible to design more complicated auction mechanisms that would extract more of the surplus from the (A type) bidders, but this may require that the seller know something about the distribution of bidder types (notice that in the three auctions above, the seller does not need to know anything about this distribution).[1]

Finally, note that Watson's example in the text has each player's valuation being drawn from a continuous probability distribution. In that case, the BNE strategies in the first price auction are pure strategies rather than mixed strategies (the large set of possible types keeps each player guessing even though each plays a pure strategy). Otherwise the qualitative results are the same as those above (truthful bidding in the first two auctions, bidding below value in the third, and same expected price in all three).

Now consider **common value auctions** and the **Winner's Curse**. Above, we assumed that each bidder knew her true valuation of the object. E.g., a Type B bidder

[1] In some auctions, reservation prices can be used to extract more revenue from high valuation bidders. In the present example, a simple modified second-price mechanism will work: charge the highest bidder the average of the top two bids.

knows that she would get utility of exactly \$3 from the object. In a common value auction, the true value of the object to each player is the same (i.e., does not vary across players as it did above) but each player privately observes a unique *estimate* of that value.

For example, suppose that two players are bidding on a resource (e.g., drilling rights on public land, spectrum use, etc.) that is being auctioned off by a government. Suppose that the true value to each of the players of the resource is \$2, but neither knows this. Rather each sends out engineers to estimate this value. Suppose further that each estimate will be \$1 with probability 0.5 and \$3 with probability 0.5. Thus each player's estimate is unbiased (the expected value of the estimate is \$2, which is the true value), but the estimate will not be correct (it will be either higher or lower than the true value).

Now we know (from above) that if the auction was a first price sealed bid auction, and players knew their true valuations, then they should not bid their valuations, but rather bid below them. However, now that each player has only an estimate of her true valuation, there is a further reason to bid below this estimate: if a player wins (has the highest bid), it is likely that her estimate was greater than the true value. Thus, the bid should be lowered even further to avoid the Winner's Curse. I.e., each bidder should understand that the winner will have the highest estimate and thus, that estimate will likely be higher than the true value of the object. If she bids her estimate and wins, it is likely that she will have paid too much for the object.

To see this in our example, suppose that Player 1 foolishly bids her estimate of the true value. In the games above, bidding your true value in a first price auction would leave you with zero utility. However, in the common value auction, bidding your estimate of the true value will give you a negative expected payoff. The expected price that Player 1 will pay if she wins is greater than the true value, \$2.

Indeed, you should be able to show for Player 1 that:

- Prob[win] $= 1/2$: this is the unconditional probability of winning
- Prob[win|high estimate] $= 3/4$: this is the conditional probability of winning if she draws the high estimate
- Prob[high estimate|win] $= 3/4$: i.e., if she wins, the probability that she overpaid is 0.75
- E[price|win] $= 2.5$: the expected (i.e., average) price, conditional on winning is greater than the true value

As an aside, note also that by Bayes' Rule we have:

- Prob[high estimate|win] = (Prob[high estimate]$*$Prob[win|high estimate])/Prob[win]

The upshot is that, for the common value auction, rational bidders should bid below their estimated value in order to avoid the Winner's Curse. We will not bother with the precise rational strategy in this course, but you should be aware that one does exist.