A Hedonic Approach to Product Innovation for Agent-Based Macroeconomic Modeling‡

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Abstract

This paper develops a hedonic approach to product innovation suitable for large scale agent-based macroeconomic modeling.

1. Introduction

A central source of ongoing novelty in market economies is product innovation by firms. Product innovation alters the qualities of existing goods and introduces new goods into the product mix. This novelty leads to further adaptation by consumers and firms. In turn, both the innovation and adaptation contribute to the complex market dynamics observed in market economies.

The agent-based or multiagent approach to macroeconomic modeling is well suited to the study of such complex adaptive market dynamics. In the following short paper, we are interested in constructing a product space and representation of consumer utility that will accommodate ongoing product innovation in large multiagent macroeconomic models.

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2. Some Empirical Background: Churning and Heterogeneity

Recent research on product turnover suggests that product innovation is a pervasive force in modern advanced economies. For example, Broda and Weinstein (2010) have documented that flows of both product creation and product destruction are large in the U.S. economy. Using Nielson homescan data, they estimate that about half of consumer products (identified by UPC code) purchased in 2003 (accounting for over 1/3 of expenditures) did not exist in 1999, and close to 80% of these products (accounting for 64% of expenditures) did not exist in 1994. Close to 2/3 of consumer products purchased in 1994 were no longer offered in 2003.

Similar product dynamics are also observed at higher levels of aggregation. Bernard, Redding and Schott (2010) using Census data find that more than one half of U.S. manufacturing firms alter their mix of 5 digit SIC products every five years, with 40 percent altering their product scope by moving in or out of 4 digit SIC industries.

This product innovation and consequent product turnover is part of a more general phenomenon; modern economies are characterized by considerable churning. Firms are born, die, enter new markets and exit existing ones (Dunne, Roberts and Samuelson, 1988), create new products and discontinue old ones, create and destroy jobs (Davis, Faberman and Haltiwanger, 2006), make infrequent large investments in new productive facilities (Doms and Dunne, 1998) and search for new types of financing (Contessi and Francis, 2009). In general, the gross flows of entry, exit, creation and destruction tend to be large relative to the net flows, and variations in these flows contribute to the severity and persistence of business cycle events such as the recent "great recession" (e.g., Elsby, Hobjin, Sabian (2010)).

Product innovation also contributes to the considerable concentration of economic activity observed in modern market economies. For example, the distributions of firm size, firm growth rates, and aggregate growth rates are highly skewed (Axtell (2001), Stanley et. al. (1996), Canning et.al. (1998)) as are product market shares (Brynjolfsson, Yu and Smith (2010).

3. Some Theoretical Background

A standard approach to modeling product innovation in contemporary growth and business cycle models is as growing product variety (e.g., Grossman and Helpman, 1991). In this approach, all products are valued identically by consumers, and competition merely drives the number of varieties to equilibrium. The homogeneity of firms and products in addition to the standard assumption of rational expectations improves the analytical tractability of these models. But variety is only one expression of product innovation. Both the product turnover and skewed distributions of firm sizes and market shares that we observe indicate that some products drive out other products and develop outsized market shares due to superiority in perceived quality. Some recent research (Klette and Kortum (2004), Akcigit and Kerr (2010)) has moved in the direction of adding firm or product heterogeneity into rational expectations models of product innovation.

We are interested here in agent-based computational models of heterogeneous boundedly rational adaptive agents who may operate in perpetual disequilibrium. This approach to modeling in economics and finance has developed rapidly in the past decade in response to advances in computational capabilities and to the desire to relax the severe restrictions imposed on the modeling process by analytical tractability. Important early contributions to the modeling of macroeconomic growth and fluctuations include e.g., Dosi et al. (2005), Delli Gatti et al. (2008) and Deissenberg et al. (2008). The global economic crisis of 2007-2009 and subsequent economic fallout has heightened the interest among economists and policy makers in agent-based macroeconomic modeling and led to calls for further advancement of the approach. See e.g., Colander etal. (2008), LeBaron and Tesfatsion (2008), Farmer and Foley (2009), and Kirman (2010).

Here we consider what approach can be taken to product innovation that is appropriate for large scale agent-based macroeconomic modeling in which the number of agents in the model can approximate the number of agents in actual economies. To be useful, the approach should be simple and flexible as well as empirically relevant, supporting the kinds of heterogeneity and churning dynamics that we see empirically.

Our proposed approach revisits Lancaster (1966a,b). Preferences are defined over a set of product characteristics, and products offer various bundles of these characteristics. As Lancaster notes, by comparison with traditional utility theory in which preferences are defined over products, the characteristics approach allows for new products to be introduced seamlessly, as new products simply offer new possibilities for the consumption of an unchanging set of characteristics.

Of course there is a large literature on product innovation, and there are a number of existing models that bear some relation to the one developed here. An important existing approach is given in a series of papers by Chen and Chie (2005,2007). They model both products and preferences as parse trees that describe production processes. Products satisfy preferences to the degree that they match (parts of) consumers' preference trees, and product innovation is modeled via genetic programming. This approach has the appeal that products and preferences are modular, so that products can successively emerge through the emergence and progressive recombination of building blocks. However, preferences are essentially specified in terms of ideal products, which ultimately limits the scope for the emergence of novelty.

In Marengo and Valente (2010), products are designed from a set of components, and preferences depend on the quantities of these components. The preferences of heterogeneous consumers for individual goods depend on the distances of these goods from their ideal products. Windrum and Birechenhall (1998) have an earlier related paper in which consumer heterogeneity can emerge from the coevolution of products and consumer preferences. In both papers, consumer heterogeneity allows distinct market niches to develop as the product mix coevolves with consumer demands.[1]

In Klette and Kortum (2004), Sutton (2007), and Akcigit and Kerr (2010) product quality is modeled as unidimensional, but firms produce multiple products and can grow by either adding new products or improving the qualities of existing products. Thus, there is an evolution of suites of products at the level of the firm.

In the approach proposed below, the representation of products and preferences is practical for large scale agent-based macroeconomic modeling. Further, preferences are not restricted by ideal products, and individual consumers consume bundles of goods, so that the emergence of product market niches need not be driven exclusively by consumer heterogeneity.

4. Hedonic Framework and Innovation

Suppose that there are n products offered in the market and that consumers value m hedonic characteristics of these products.[2] Each consumer has a mapping g from bundles $q \in R_n$ of products into bundles $z \in R_m$ of the characteristics. Thus, for consumer c we have $z_c = g_c(q_c)$. The mapping can be idiosyncratic and is essentially a home production function.[3]

Consumers have preferences defined on the characteristics space. Consumer c's momentary utility from consuming the vector z_c of hedonic characteristics is $u_c(z_c)$. Each consumer searches over time for bundles that will increase her utility.

A product innovation takes the form of the creation of a new or improved product that, from the point of view of an individual consumer, combines a new set of characteristics, enhances an existing set of characteristics, or provides additional units of characteristics when consumed individually or jointly with other products. The new product will be successful if it is perceived as offering utility (in combination with other goods) at lower cost than current alternatives. The product may fail due to high cost, poor search by consumers, or poor timing in terms of the availability or desirability of complimentary goods.

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[1] See also Ciarli, Lorenz, Savona, Valente (2010) in which demand evolves endogenously with the class structure of the population. Hidalgo et. al. (2007) do not model product quality or innovation per se, but provide an empirical network characterization of the product space based on trade patterns. They provide evidence that the sparseness of the product space limits development for countries located in the periphery of that space by restricting opportunities to develop new goods.

[2] Of course, both n and m can be allowed to evolve over time, but we are particularly interested in how much mileage can be gotten out of a model with fixed but potentially large m.

[3] This is essentially the approach taken by Lancaster, and shares some similarities with others such as Becker (1965) and Strotz (1957).

5. Preferences I: Products and Hedonic Characteristics

Realistic mappings from products to hedonic characteristics are complex due to the vast number of products that may be available at any time as well as the importance of complementarities across products, specialization of products in the characteristic space, indivisibilities in the product space, and heterogeneity among consumers. This complexity will impose substantial demands on both model design and computational resources. Therefore, we want to keep the form for the home production function as simple as possible.

We assume that each consumer associates with each good a set of *base characteristics* and a set of *complementarities* with other goods.

Base characteristics: For a given consumer c, we assume that each good i provides a base set of individual characteristic magnitudes $\text{z-base}_{c,i} \in R_m$ per unit of the good.

Complementarities: Consumer c associates with each good i a set of other goods k with which it shares complementarities. Each of these goods pairs (i,k) provides an additional set of characteristic magnitudes $\text{z-comp}_{c,i,k} \in R_m$, defined relative to some composite unit $q_{c,i,k} = \theta(q_{c,i}, q_{c,k})$ of the two goods. This vector is associated with good i, and it is convenient to assume that the complementarities are independent across goods (i.e., that the vectors $\text{z-comp}_{c,i,k}$ and $\text{z-comp}_{c,k,i}$ are independent). The aggregator $\theta()$ is assumed to be a quasi-concave function.

For example, for an individual consumer, a unit of spinach may offer certain quantities of subjectively valued characteristics like nutrition, flavor, and crunchiness. Further, the flavor characteristic might also be enhanced for this consumer by consuming the spinach in combination with a salad dressing, so that the total quantity of flavor achieved by eating these in combination is greater than the sum of the flavor quantities from consuming each separately. We can divide the extra quantity arbitrarily between the z-comp vectors of spinach and dressing.

Similarly, in isolation, an Apple iPad may provide a consumer some modest degree of entertainment, but this entertainment value is dramatically enhanced by consuming it along with a personal computer, an internet access subscription, electricity, and so on.

It is worth noting that an important feature of consumption that we will ignore here is its time consuming nature. However, this aspect of consumption provides a further heuristic motivation for the inclusion of complementarities in the model. In reality, since consumption is a time consuming activity, and time is scarce, combining consumption modes can be utility enhancing. A commute to work in a car is enhanced by air conditioning and satellite radio. In turn, listening to the radio in the car can free up time later in the day that had been devoted to similar entertainment. This latter effect can be proxied by a complementarity between the car and the radio in our model in which consumption is not constrained by time.

Aggregation: We assume that both base characteristic magnitudes and complementary characteristic magnitudes are additive at the level of the individual good. I.e., for consumer c, good i and hedonic characteristic j, we have

$$z_{c,i,j} = \text{z-base}_{c,i,j} \cdot q_{c,i} + \sum_k \text{z-comp}_{c,i,j,k} \cdot q_{c,i,k}. \tag{1}$$

These characteristic magnitudes are then aggregated over products by a CES aggregator:

$$z_{c,j} = \left[\sum_{i=1}^{n} z_{c,i,j}^{\rho_{1,c}} \right]^{1/\rho_{1,c}} \tag{2}$$

with $\rho_{1,c} < 1$. Equations (1) and (2) define the mapping $g_c(q_c)$ introduced in section 4. The CES form of (2) introduces some taste for variety across products. [4]

6. Preferences II: Characteristics and Utility

We will assume that the utility function u_c for consumer c over hedonic characteristics is also CES, so that

$$u_c = \left[\sum_{j=1}^{m} (z_{c,j} + \bar{z}_{c,j})^{\rho_{2,c}} \right]^{1/\rho_{2,c}} \tag{3}$$

Thus, utility takes a nested CES form. Consumers value variety in both hedonic characteristics and in products.

Notice that the CES aggregator in this section includes a shifter $\bar{z}_{c,j}$ for each characteristic. These shifters can vary across characteristics as well as across consumers. The variation of these shifters across characteristics breaks the homotheticity of preferences over characteristics and can be interpreted as describing a hierarchy of wants for the consumer.[5]

7. Implementation in a Simple Macro Model

We introduce this approach to product innovation into a very simple macroeconomic environment for the purpose of illustration.

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[4] Note that if $\rho_1 = 1$, the number of viable products in the economy would be strongly limited by the number of hedonic elements, as in Lancaster

[5] For a given characteristic, a lower value of the shifter relative to other characteristics increases the relative desirability of consuming products that supply the characteristic, and the consumption of certain characteristics may therefore only become positive at relatively high incomes. See e.g., Jackson (1983).

7a. Brief Description of the Model

- There are n firms, each of which produces one type of good at any time.
- There are m characteristics of goods that consumers care about.
- There is a single representative consumer.
- At any time the base and complementary set of hedonic characteristic magnitudes (z-base$_i$ and z-comp$_{i,k}$) associated by the consumer with good i are coded as m dimensional vectors of integers. These characteristics vectors are randomly initialized at the beginning of the simulation.
- A product innovation is a set of random (integer) increments (positive or negative) to one or more elements of z-base$_i$ or z-comp$_{i,k}$. Product innovation is strictly by mutation.
- Product innovations can be positive or negative. I.e., firms can mistakenly make changes to their products that consumers do not like. However, there is a floor of zero on characteristic values. Further, innovations operate through preferential attachment; for a firm that experiences a product innovation, there is a greater likelihood of mutation of non-zero hedonic elements.[6]
- The probability that a firm experiences a product innovation at any time depends on its recent investments in R&D.
- The R&D investment choice is binary. If a firm engages in R&D in a given period, it incurs additional overhead labor costs R in that period.
- In making it's R&D investment decision at any time, the firm compares the recent profit and R&D experiences of other firms and acts according to a discrete choice rule.
- Each firm i produces its good with labor subject to a momentarily fixed labor productivity $A_{i,t}$. It also incurs a fixed overhead labor cost H.
- Process innovation, if it occurs, takes the form of random shocks to labor productivity.
- Each firm forecasts the nominal final demand for its product by extrapolating from recent experience. It sets its price as a constant markup over marginal cost and plans to produce enough of its good to meet its expected final demand given this price.
- The representative consumer spends all of her labor income each period on consumption goods and searches for better combinations of products to buy within her budget by experimenting with variations on the present consumption mix, adopting only those variations that are expected to increase utility (relative to inaction).
- The aggregator for complements $\theta(q_{c,i}, q_{c,k})$ is floor$(\min(q_{c,i} \cdot \frac{1}{\lambda}, q_{c,k} \cdot \frac{1}{\lambda})) \cdot \lambda$. I.e., complementarities are defined per common fractional unit consumed. Note that

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[6] This weak form of preferential attachment supports specialization in the hedonic quality space.

this introduces a fractional integer constraint on the consumer's optimization and search problem.[7]

7b. Features of the Model

If we eliminate all heterogeneity in the model, there is a steady state equilibrium which is stable under the simple dynamics described above. Aggreagate output is a multiple of the overhead labor costs H and R and is independent of product quality. See Appendix I for details. Quality adjusted output and consumer utility, on the other hand, will depend on product quality.

With firm heterogeneity and product innovation, the hedonic characteristics of firms' products will evolve. This will in turn influence product market shares as well as aggregate activity and living standards.

The stochastic evolution of product quality with zero reflective lower bounds on individual characteristic magnitudes will tend to generate skewness in the distribution of individual product qualities and thus in the distribution of market shares. The same is true for process innovation if that is active, through the distribution of labor productivities.

Working against the skewness of distribution of product shares are the variety-loving aspect of consumer preferences and the number of product characteristics and complementarities. The greater the number of characteristics, the greater the opportunities for individual firms to find niches in the characteristics space. A small number of characteristics relative to the number of products creates a "winner take all" environment in which firms compete head to head in a small number of dimensions, whereas a large number of characteristics may create opportunity for the development of a (so called) "long tail" of niche products.[8]

Ultimately, the evolution of firm sizes and product shares will depend on the interaction between the evolution of individual product characteristics, complementarities, and productivity as well as the entry and exit processes. In this paper the complementarity network is random, but we could easily add structure to make some firms or sectors more central. The model can also be easily extended to include a supply network of intermediate goods inputs. In that case, the topology of the supply network will influence the ultimate distribution of firm sizes as well as the nature of aggregate volatility.

Turning to aggregate activity, with heterogeneity, there are several chanels

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[7] This produces non-convexities which can create multiple local optima. The searching consumer may get temporarily stuck at suboptimal local maxima.

[8] A recent literature argues that the growth of internet retail has allowed niche products that better suit existing consumer preferences to become profitable, eroding the market shares of more broadly popular "superstar" products (Brynjolfsson, Hu and Smith (2010), Brynjolfsson, Hu, and Simester (2011))

through which product innovation may be a source of output fluctuations.

First, product innovation produces ongoing changes in the pattern of demand across goods. It takes time for production to adjust to the changing pattern resulting in both direct and indirect effects on output. Further, as demand patterns change, demand may shift between goods with different production technologies.

A second mechanism is that investment has a direct impact on aggregate demand. Since R%D investment decisions are conditioned by social learning, there can be cyclical herding effects in this model.

Third, if the product share distribution is not too skewed, then the idiosyncratic component of the output fluctuations of individual products will tend to wash out in the aggregate as the number of products grows large. However, if product innovation is either correlated across sets of products or results in highly skewed distributions of firm sizes and product shares, then idiosyncratic fluctuations may not wash out in the aggregate, even as the number of firms and products grows large.[9]

We have excluded other sources of business cycle fluctuations and growth from the model to focus on the role of product innovation.

7C. Some Simulation Results

Runs from a representative agent baseline version of the model behave as expected. If all firms have the same parameters and face the same constant productivities and hedonic qualities, then output converges to the analytical equilibrium discussed in Appendix I.

For the heterogeneous firms model, ongoing innovation tends to generate stochastic growth in consumer utility and ongoing fluctuations in total output. As the number of firms increases, the output fluctuations become increasingly dominated by variations in R%D investment spending through the herding effect discussed above.

Figure 1 is produced from a representative run with 1000 firms. In this case the number of product characteristics is 50, $\rho_1 = \rho_2 = 0.8$, and mutation is multiplicative.[10] Process innovation is turned off, so all innovation is product innovation.

[9] Intuitively, if some firms or sectors remain large and/or central to the economy, even under highly disaggregated measurement, then idiosyncratic shocks will have aggregate effects. For formalizations, see for example Gabaix (2011), Carvahlo and Gabaix (2010), Delli Gatti et. al. (2008), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2010) and Horvath (2000).

[10] The number of firm in this simulation is small (1000), but can easily be scaled up to several million. Similarly the number of hedonic characteristics (50) can be increased easily (though, in both cases, of course at some computational cost).

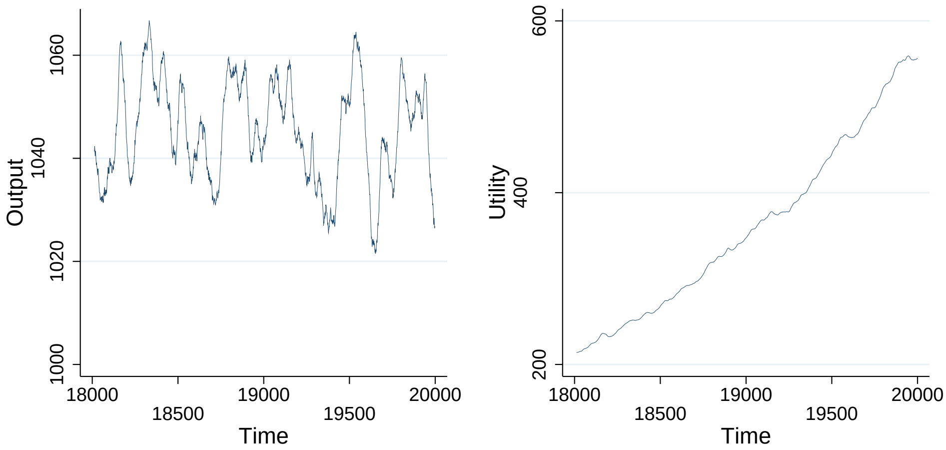

Figure 1: Representative run. Output and utility, for rounds 18,000 – 20,000. Output is measured weekly as average output for the last quarter.

The time period is considered to be a week so we are showing approximately 40 years of simulated data well after transitory growth dynamics have died out. Aggregate output exhibits irregular fluctuations and cycles; peak to trough swings in GDP are on the order of 1 to 3 percent. Utility also fluctuates with both output and the evolution of the quality of products produced and consumed, but also grows due to long term net improvements in product quality. These net improvements are driven both directly and indirectly by innovation. Existing firms' product qualities may rise or fall in response to innovations, but consumers shift toward higher quality products, less successful firms are driven from the market, and new firms enter.

Now consider the distribution of firm sizes as measured by final demand shares. We start the simulations with the representative consumer setting equal shares across firms and characteristic magnitudes initialized to a random sequence of zeros and ones. As each simulation proceeds, the consumer searches for better bundles of goods.

If there is no product or process innovation, then the optimum bundle for the consumer is static and demand is redistributed across firms over time, leading to a skewed distribution of demand shares. The degree of skewness can be tuned by the CES utility elasticities. Lower values of ρ_1 and ρ_2 indicate a greater taste for variety in characteristics and products, limiting the impact of product quality on market shares. The shape of the distribution is also affected by the ratio $\frac{m}{n}$ of product characteristics to goods, as well as the distribution of complementarities and the entry and exit process. For example, if $\frac{m}{n}$ is small, then there is less room for firms to have independent niches in the product space, and so the market tends to become more concentrated.

Including ongoing product innovation leads to ongoing changes in relative qualities of goods and so ongoing churning in demand shares. The share distribution follows a similar pattern to that above in early rounds. Starting from a degenerate distribution with all the mass on $1/n$, it first spreads out and then becomes skewed as shares are progressively reallocated among products.

However, as innovation proceeds, the degree of skewness may relax and then stabilize. The ultimate limiting distribution depends on the stochastic process for product innovation in addition to the factors above (ρ_1, ρ_2, $\frac{m}{n}$, the distribution of complementarities, and the entry and exit process) and other parameters of the model, such as the parameters of the discrete choice rule for R&D investment.

The dependence of the skewness of the distribution of market shares on the ratio $\frac{m}{n}$ is illustrated in Figure 2 below. Here the number of firms n is 1000 and we vary the number of characteristics m. Mutation is additive (i.e., increments are of fixed size). We see that, if there is only one product characteristic, then we have a winner take all environment tempered moderately by consumers' taste for variety. After 500,000 rounds, close to half of the firms have zero shares, and most of the remaining firms have shares well below $1/n$. As we increase the number of characteristics to 5 and then 50, we see the effect of increased opportunities for specialization and niching on the distribution of shares.[11]

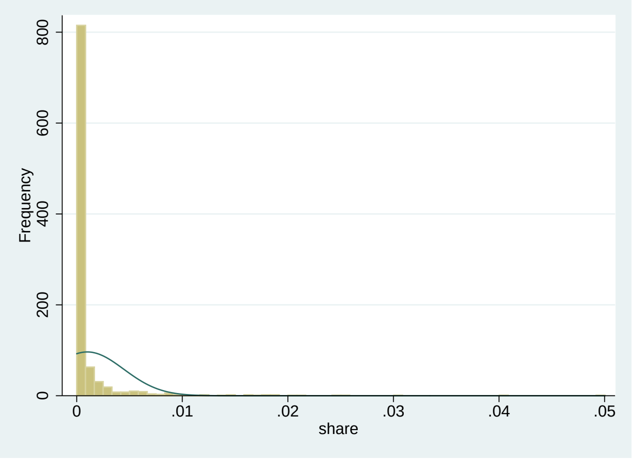

Figure 2a. Distribution of product shares. n=1000, m=1, round 500,000.

[11] The same pattern is seen if mutation is multiplicative (i.e., increments are proportional), but in each case (i.e., for each level of m) the degree of skewness tends to be greater under multiplicative mutation.

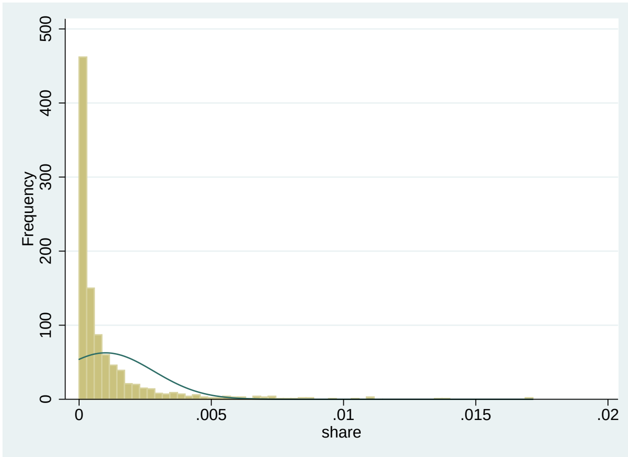

Figure 2b. Distribution of product shares. n=1000, m=5, round 500,000.

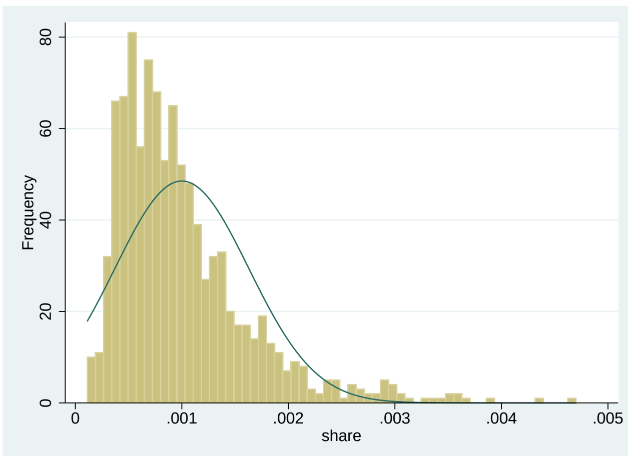

Figure 2c. Distribution of product shares. n=1000, m=50, round 500,000.

8. Conclusion

We have developed a hedonic approach to product innovation suitable for large scale agent-based macroeconomic modeling. As with Lancaster (1966a,b), innovation expands the product space without altering the characteristics space and thus consumers' preference orderings on that space. The approach is parsimonious, flexible and easily scalable. We illustrate by applying the approach to a very simple agent-based macro model with autonomous firms and ongoing consumer search. The simple model generates skewed market share distributions, with the degree of skewness depending on the opportunities for niching in the characteristics space. Business cycle dynamics are dominated by an innovation driven investment cycle.

Appendix I: Representative Agent Benchmark for the Simple Macro Model

Consider the case in which all firms are identical. Suppose further that there are no innovations to product quality or to the production technology, that each firm selects to engage in R&D each period, and that each firm starts out with a 1/n share of the total market. These market shares will persist, since there is no reason for consumers to switch between firms with identical product qualities and identical prices. However, aggregate demand and supply might be out of balance initially.

There is a unique equilibrium for real production and sale of consumer goods Y at which demand and supply are in balance. At this equilibrium, aggregate real activity depends on the markup η, the per firm overhead labor costs H and R, the wage rate W (for production workers), labor productivity A (for production workers), and the number of firms n. Specifically, at this equilibrium $Y = (\frac{1}{\eta-1}) \cdot \frac{A}{W} \cdot (H+R) \cdot n$. See below for details. Further, this equilibrium is a steady state of the agent dynamics in the model and is locally stable under those dynamics. E.g., if firms all start with production less than steady state production, then demand will be greater than production for each firm, and production will converge over time to the steady state equilibrium.

Since all firms are identical, they produce identical quantities q of their goods. Then total labor income is:

$$E = n \cdot [\frac{W}{A} \cdot q + H + R]$$

Each also charges an identical price p for its good, which is a markup on marginal cost

$$p = \eta \cdot \text{MC}$$

$$\text{MC} = \frac{W}{A}$$

and so produces and sells

$$q = \frac{E}{n \cdot p}$$

units of its good.

These relationships yield the following steady state equilibrium value for (per firm) production.

$$q = \frac{(H+R) \cdot \frac{A}{W}}{\eta - 1}$$

We can see that $\partial q / \partial A > 0$, $\partial q / \partial w < 0$, and $\partial q / \partial \eta < 0$. These are all demand driven. With a constant mark-up, an increase in productivity will cause firms to lower their prices, raising aggregate demand. An increase in the wage (of production workers) or the mark-up will cause firms to raise their prices, lowering AD. Further, it is straightforward to show that, if $q > 0$ at this steady state equilibrium and firms follow simple one period adaptive expectations for demand, then the equilibrium is also locally asymptotically stable under the model's dynamics.

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