We want to develop a very general method for modeling investment spending decisions. Suppose that a firm or individual is considering making a particular real investment. As with any optimal choice problem we suppose that the decision maker assesses the costs and benefits of this investment.[1] However, investment projects involve benefits and costs which accrue over time. Thus, the decision maker must weigh future benefits and/or costs against current benefits and/or costs.

**Present Value:**

This time consideration makes the problem a bit tricky. How should we model the decision maker's attitude toward time? Intuitively, we would expect that the decision maker will prefer having a dollar today to getting a dollar tomorrow. Similarly, she will prefer paying a dollar tomorrow to paying a dollar today. However, how strong is this preference? I.e., how much does she discount money received or paid out in the future? We often assume that the rate at which she discounts the future is the rate of interest at which she can borrow and lend.

This assumption can be justified as follows. The rate of interest determines how the individual or firm can transfer money across time. Suppose that I know that I will receive \$100 one year from now. Then suppose that I want to borrow against that income so that I can have the money today. How much can I borrow today and use the \$100 payment one year from now to pay off my loan? If I borrow \$Z today, then I will pay $(1+i) \cdot Z$ in one year when I pay off the loan (principle plus interest). Thus, if I want this payment to be \$100, then $Z$ must be $\frac{100}{1+i}$. E.g., if the interest rate is 5% ($i = 0.05$) then $Z \approx 95.24$. This is the amount that I can borrow today and pay off the loan exactly with the \$100 that I will receive next year. Thus, we can move the future income (\$100) to the present, but only by giving some of it up as an interest payment. Similarly, suppose that I have \$95.24 today. I can lend this forward at the interest rate $i$ and receive \$100 back next year (principle plus interest).

We call the number \$95.24 the *present value* of \$100 received 1 year from now. It is the amount of money that can be borrowed today (at interest rate $i$) against this future income, or equivalently, the amount of money that would have to be put in a bond today (at interest rate $i$) in order to receive \$100 in one year. An individual or firm who can borrow and lend at $i = .05$ should be indifferent between receiving \$95.24 today and receiving \$100 in one year with certainty.

Now, what about \$100 received two years from now? Well, how much can be borrowed against this income today? If we borrow $Y$ today, for one year, we would pay back $(1+i) \cdot Y$ next year. If instead we rolled over the loan at that time for another year adding the interest owed to it, we would then pay back $(1+i)$ times the new loan the following year. I.e., in year two, we would pay $(1+i) \cdot (1+i) \cdot Y$, or $(1+i)^2 \cdot Y$. For this to be equal to \$100, we need $Y$ to be $\frac{100}{(1+i)^2}$. This is the present value of \$100 received two years from now. If $i = 0.05$, this number is approximately \$90.70. Similarly, if I have \$90.70 today, I can lend this forward at the interest rate $i = 0.05$ and receive \$100 back two years from now.

[1] If the investment is all or nothing, then we look at the total benefits and costs. The project should be undertaken if the benefits are greater than the costs. If the individual can select a level of investment, then we want to look at the marginal benefits and costs. The project should be financed up to the point at which the marginal benefit is no longer greater than the marginal cost.

Extrapolating the argument above, the present value of any sum of money $X$ received $t$ years from now is $\frac{X}{(1+i)^t}$. An individual or firm who can borrow and lend at rate $i$ should be indifferent between receiving (or paying) the present value today and receiving (or paying) \$X in year $t$. We will assume, therefore, that our decision maker values sums of money paid or received in the future at their present values.[2]

**Investment Decision:**

Now consider a business that is considering whether to undertake a particular investment project. The project will generate various costs and benefits over time. To see whether the project is worth undertaking, calculate the present value of each cost and benefit incurred in each year (present and future). Then sum up the present values of the costs and sum up the present values of the benefits. If the PV of the benefits is greater than the PV of the costs, the project is worth undertaking. If the PV of the costs is greater than the PV of the benefits, then the project is not worth undertaking.

Why is this? Consider a simple case. The project costs \$100 today and will generate a benefit of \$125 two years from now. There are no other costs and benefits from the project. Suppose that the rate at which the firm can borrow and lend is $i = 0.07$. Then consider the present values of the costs and benefits. The PV of the cost is just the \$100 incurred today. The PV of the benefits is the PV of \$125 received two years from now, which is $\frac{125}{(1.07)^2}$ which is approximately \$109.18. Since the PV of the benefits is greater than the PV of the costs, the project is worth doing according to our present value rule.

We can see this in a variety of ways. First, the \$125 received two years from now is worth \$109.18 to the firm today, since it could borrow that much today against the future income (paying the loan off with the \$125 when it comes due). Thus, the firm could immediately be \$9.18 better off by undertaking the project and borrowing against the future income (assuming that the firm is certain about the future income). Second, the firm could borrow the \$100 to finance the project (rather than using its own finances). It would then pay back $100 \cdot (1.07)^2 =$\$114.49 on the loan in year two, but receive \$125 from the project at that time. Again, the firm would come out ahead.

A third way to see this is to consider the alternative to undertaking the project. Suppose that the firm was to put the \$100 into a bond for two years rather than into the project. Then, it would receive $100 \cdot (1.07)^2 =$\$114.49 from the bond, which is worse than the \$125 that it would have gotten from the project.

**Comments:**

The nice thing about the present value rule is that it is very general and flexible. The benefits and costs of the project can be incurred at any time. Just take present values of each cost and benefit incurred in each period and sum them. If the present value of the benefits is greater than the present value of the costs, then you should undertake the project. If you borrow to fund the project, then you can structure the timing of the loan payments so as to pay less in principle and interest than the benefits that you will receive on the project in each individual year. Similarly, if you use your own funds to finance the project, you can move the benefits around over time (by borrowing and lending) in such a way as to generate greater benefits than costs in each year. Similarly, there is no way to invest the costs of the project in bonds and earn more off the bonds in each period than you would earn on the project.

[2] Note that, in practice, uncertainty about future payments and imperfect access to capital markets introduce additional complications which we are ignoring here.

**Another Example:**

As another example, suppose that a project costs \$200 in the current year and \$200 in the next year. The benefits are \$300 three years from now and \$200 four years from now. Suppose that the interest rate is 5%.

The PV of the cost is $200 + \frac{200}{1.05} \approx 390.48$. The PV of the benefits is $\frac{300}{(1.05)^3} + \frac{200}{(1.05)^4} = 423.69$. The firm should undertake the investment project. It is better off, for example, investing its own funds in the project than in bonds.

Suppose that the firm put the costs into bonds instead of into the project. Then the firm puts the \$200 into bonds today and the second \$200 into additional bonds tomorrow. Suppose that the firm cashes in the first set of bonds in year three. Then it receives about \$231.50 from those bonds at that time. By year three the second set of bonds will have grown to \$220.50. If we cash in \$68.50 of that to bring our year three revenue from the bonds up to the \$300 we would have gotten on the project in that year, and then leave the remaining \$158.00 in bonds for another year, we can cash those bonds in for \$165.90 in year four. This falls short of the \$200 we would have gotten in that year under the project. Thus, we can't match the benefits on the project by making an equivalent investment in bonds.