Chapter 8:
1. Questions For Review numbers 1,4 (p. 238).
2. Country A and country B each have the production function

\[ Y = F(K, L) = K^{1/2}L^{1/2} \]

2a. What is the per-worker production function, \( y = f(k) \)?

2b. Assume that neither country experiences population growth or technological progress and that 5 percent of capital depreciates each year. Assume further that country A saves 10 percent of its output each year and country B saves 20 percent of its output each year. Find the steady-state level of capital per worker for each country. Then find the steady-state levels of income per worker and consumption per worker.

2c. Suppose that both countries start off with a capital stock per worker of 2. What are the initial levels of income per worker and consumption per worker? Why is country B’s consumption per worker initially smaller than country A’s, and why will it grow to be larger than country A’s in the (very) long run? Why do standards of living in both countries converge to steady states rather than continuing to improve in the (very) long run?

Remembering that the change in the capital stock is investment less depreciation, use a calculator or spreadsheet to show how the capital stock per worker and consumption per worker will evolve over time in both countries for the next two years.

2d. The slope of the per capita production function is the MPK. Note that, in this example, this slope is \( \frac{1}{2}k^{\frac{1}{2}} \). Given this information, are the saving rates in countries A and B above or below the Golden Rule level? By altering its saving rate, could each country improve its standard of living in both the short and long runs?

2e. Make a table showing the steady state values of output per worker \( y^* \) and consumption per worker \( c^* \) for saving rates of 0%, 25%, 50%, 75%, and 100%. At which of these saving rates is steady state output per worker the highest? At which of these saving rates is steady state consumption per worker the highest?

2f. (based on chapter 9) Now suppose that in each country population grows at a rate of 1% per year, and technological progress causes the efficiency of labor to grow at a rate of 2% per year. What are the growth rates of output \( Y \) and output per worker \( \frac{Y}{L} \) in each country in the long run?

2g. (based on chapter 9) Consider problem 2f. again. Solve for the steady state level of output per effective worker \( (y^*) \) in terms of \( s, n, \delta, \text{ and } g \).\(^1\) Use this to calculate the ratio of the standards of living \( \frac{C_L}{C_L'} \) in the two countries in the long run.

3. Problems and Applications number 6 (p. 240).

Chapter 9:
4. Questions For Review numbers 1,2,4,6 (p. 267).
5. Problems and Applications number 6 parts a,b (p. 268).
6. Suppose that all assumptions of the Solow model hold except that the production function does not exhibit a diminishing marginal product of capital, and that there is no population growth or technological progress.
   a. Specifically, suppose that the per-worker production function is

\[ y = k \]

\(^1\) Note that with technological progress, \( y \) is now defined as \( \frac{Y}{L-E} \).
Show that, in this case, a higher saving rate leads to a permanently higher growth rate for the economy. Explain this result intuitively. Why does this result not hold in the Solow model?

b. Now suppose (again contrary to the Solow model) that aggregate production functions typically have low but \textit{increasing} returns to capital (low but increasing MPK) at low levels of national capital stock, and that diminishing returns (decreasing MPK) then kick in at high levels of capital stock. Use the saving and depreciation diagram to show that in this case there can be multiple steady state equilibria, so that an economy that starts out with a relatively small capital stock may not catch up with more developed economies even if all countries’ rates of saving and population growth are the same.

Appendix to Chapter 9:

7. Real GDP grew about 2.2% in the U.S. in 2017. That year the real capital stock grew approximately 2.0% and the number of hours worked by labor increase by about 1.2%. Suppose that the elasticity of output with respect to capital ($\alpha$) is one third (1/3). Then according to standard growth accounting, what was the contribution to output growth from labor growth, the growth of the capital stock, and total factor productivity growth in 2017?