

## Outline of Discussion of Solow Growth Model

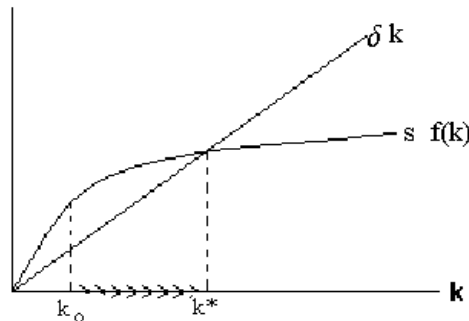
### Simplest Case: Ch. 7.1,7.2

In the simplest case of the Solow growth model, covered in Ch. 7.1 and 7.2 of Mankiw, there is no population growth or technological change. Consequently, the equation of motion for the stock of capital per worker  $k$  is:

$$\Delta k = s \cdot f(k) - \delta \cdot k$$

This simply states that the amount of capital per worker in the economy will grow if investment is greater than depreciation. The equation recognizes that investment is equal to saving in the model.

Plotting the two parts of the right hand side of this equation, we can see graphically the **steady state** equilibrium  $k^*$  and that this steady state is stable: i.e.,  $k$  will gravitate toward  $k^*$  over time from any starting point (such as  $k_0$  in the diagram).



Since output per worker is just a function of capital per worker ( $y = f(k)$ ), it must then be that  $y$  will also gravitate to a steady state level  $y = y^* = f(k^*)$ .

### Implications:

- Note then that for any country starting below its  $k^*$ , growth (of capital per worker  $k$  and thus output per worker  $y$ ) will eventually stop. A country with a current capital stock much lower than its  $k^*$  will grow rapidly for a while, but eventually growth will slow to zero. This is consistent for example with **Krugman's** argument that very rapid growth in Russia and Japan after WWII and China, Singapore, and other Asian economies in the last 20 years can be explained largely by high rates of investment (i.e., high saving rates) and low initial capital stocks.
- An increase in a country's saving rate  $s$  will raise the steady state levels of  $k$  and thus  $y$ , and lead to a temporary burst of growth. Make sure that you can show this graphically.
- Will consumption per worker go up in steady state as a result of the increase of the saving rate above? Not always. See section 7.2 of Mankiw.  $c^* = (1 - s)y^*$ , so while an increase in  $s$  increases steady state output per worker  $y^*$ , it also reduces the fraction of this output that is consumed rather than saved (invested). There is a level of the saving rate  $s$  between zero and one which maximizes  $c^*$ . Call this the *golden rule* saving rate  $s_{GR}$ .

### Adding Population Growth: (Ch. 7.3)

If the population grows, then the amount of capital per worker shrinks over time unless we both replace depreciated capital and buy new capital for the new workers. Thus, our equation of motion for  $k$  is now

$$\Delta k = s \cdot f(k) - (\delta + n) \cdot k$$

where  $n$  is the rate of growth of the labor force.

The new depreciation line  $(\delta + n) \cdot k$  lies above the old line, and so one effect of population growth is to lower the steady state level of capital per worker  $k^*$ . At the new steady state,  $k$  is constant, and so  $y$  is

constant. Since  $k = K/L$  and  $Y = Y/L$  and  $L$  is growing at rate  $n$ , it must be that  $K$  and  $Y$  also grow at rate  $n$  in the new steady state equilibrium. Thus, population growth causes output to grow permanently, but lowers the standard of living  $c$  (by lowering  $y$ ).

So population growth causes sustained growth in output  $Y$ , but output per worker  $y$  gravitates to a lower steady state the higher is  $n$ .

### **Adding Technological Progress: Ch. 8.1**

Then what can cause growth to be sustained in the very long run? The answer according to the Solow model is technological progress. This acts to shift the production function upward over time, and so can generate sustained growth.

Think of the production function as  $Y = (K, L \cdot E)$  and technological progress as increasing the *efficiency of labor*  $E$  over time at rate  $g$ . Note that if there is no technological progress and  $E = 1$ , then this is just our old model.

Now,  $k = K/L$  and  $y = Y/L$  will no longer converge over time to a constant steady state, but will rather grow in very long run equilibrium. However, it can be shown that  $\frac{K}{E \cdot L}$  and  $\frac{Y}{E \cdot L}$  (i.e.,  $K$  and  $Y$  per effective worker) now do converge to steady states in very long run equilibrium.

Since  $\frac{K}{E \cdot L}$  and  $\frac{Y}{E \cdot L}$  will be constant at the steady state, and  $L \cdot E$  grows at constant rate  $n + g$ , it must be that at the steady state (i.e., in very long run equilibrium), the rates of growth of both  $Y$  and  $K$  are  $n + g$  and that the rates of growth of  $\frac{K}{L}$ ,  $\frac{Y}{L}$ , and  $\frac{C}{L}$  are  $g$ . I.e., sustainable growth in the standard of living ( $C/L$ ) is driven exclusively by technological progress.