

In the U.S., the various levels of government receive revenues (mostly from taxes) and make a variety of outlays (payments for materials and services, transfer payments, etc.). Consider for example the federal budget.

The *budget surplus* for a particular year is the revenues taken in by the government in that year minus the outlays made by the government in that year.

$$\text{Budget Surplus} = \text{Revenues} - \text{Outlays}$$

Equivalently, the *budget deficit* = outlays - revenues. Thus, if revenues are 100 and outlays are 120 in a particular year, then the surplus is -20, and equivalently the deficit is 20, in that year.

The *primary surplus* ignores interest payments on the outstanding government debt (which are outlays).

$$\text{Primary Budget Surplus} = \text{Revenues} - \text{Outlays net of Interest Payments}$$

The budget is *balanced* in a given year when revenues are equal to outlays in that year. Then all outlays can be paid for out of the government's revenues in that year. When the government runs a budget deficit, revenues are insufficient to cover all of its outlays, and so it must borrow to cover the shortfall. It does this by issuing new bonds (Treasury Bonds).

The outstanding government *debt* is the accumulated deficits of the past. If the government runs a deficit in one year, and then has a balanced budget in the next, the debt issued in the first year must be rolled over, and so persists. In order to reduce the debt outstanding, the government must run surpluses. The surpluses are then "used" to retire outstanding debt.¹

As a stylized example, suppose that the government has balanced its budget in the past and has no outstanding debt. Now in period one, suppose that the government increases its outlays by 100 billion dollars without increasing taxes or other forms of revenues. Thus it runs a deficit (negative surplus) in period one of \$100 billion. Suppose that in future years it runs a balanced primary budget.² I.e., suppose that in future years its revenues just cover all outlays except interest payments.

Measure the debt in any year as the debt outstanding at the end of that year. Further, suppose that the government borrows at interest rate $i = 5\%$. Further, denote the debt as D and the deficit as d . Then we have:

Year 0: $d_0 = 0$, $D_0 = 0$.

Year 1: $d_1 = 100$, $D_1 = 100$.

Year 2: $d_2 = i \cdot D_1 = 5$, $D_2 = D_1 + d_2 = 105$.

Year 3: $d_3 = i \cdot D_2 = 5.25$, $D_3 = D_2 + d_3 = 110.25$.

Etc.

¹ If the surplus was used in any other way than to retire debt or accumulate financial assets, this use would be classified as an outlay and so this would not actually be a budget surplus.

² You can think of the initial \$100 billion as e.g., a one time investment in education or the military.

So in this case the debt will grow at the rate of interest,³ since we have to issue new bonds to cover the interest payments on the outstanding debt while rolling over all of that outstanding debt each year. Similarly, the interest payments themselves (and thus the deficit) will get larger over time, since the debt is getting larger.

Notice that in this case (balanced primary budget), whether the ratio of debt to GDP grows or shrinks over time depends exclusively on whether the rate of interest that the government pays on the debt is greater or smaller than the rate of growth of GDP. If international investors believe that there is a high risk of default, and consequently demand high interest rates in order to hold the debt, this can cause the ratio of debt to GDP rise over time to unsustainable levels, increasing the likelihood of default, even if the government is currently running a balanced primary budget.

Now, returning to our previous example, suppose that the government ran a balanced budget in each year after year one. Then it would be running primary surpluses to cover the interest payments on the outstanding debt. In that case, the debt would stay at 100 every year, the surplus (deficit) would be zero each year, and the primary surplus would be \$5 billion each year.

If the government wanted to reduce its debt to zero in the future, it would have to run budget surpluses. In each year that it runs a budget surplus, it retires that amount of outstanding debt.

³ I.e., according to the numbers above, $D_t = (1 + i) \cdot D_{t-1}$ in each year $t > 1$.