1. Chapter 2. Problems and Applications 2,7:

1.2. \$6 has been added to GDP in this example. We don't want to double-count the value of the wheat and flour, since they show up in the price of the bread. Thus, we measure GDP by counting *final* goods only (here the bread) or equivalently by adding up the *value added* for all goods (in this case $\$1 + \$2 + \$3 = \$6$).

1.7. We have the following information on purchases and prices in the two years:

Year	P_{hotdogs}	Q_{hotdogs}	$P_{\text{hamburgers}}$	$Q_{\text{hamburgers}}$
2010	\$2	200	\$3	200
2018	\$4	250	\$4	500

1.7.a. With base year 2010, we have:

	Year 2010	Year 2018
Nominal GDP	\$1000	\$3000
Real GDP	\$1000	\$2000

Here is how I calculated these numbers.

$$\text{Nominal GDP}_t = \sum_i (P_t \times Q_t)_i$$

where t indicates year (either 2010 or 2018) and i indicates good (here hotdogs or hamburgers). So

$$\text{Nominal GDP}_{2010} = \$2 \times 200 + \$3 \times 200 = \$1000$$

$$\text{Nominal GDP}_{2018} = \$4 \times 250 + \$4 \times 500 = \$3000$$

Nominal GDP rose (by 200%) between 2010 and 2018. However, this partly reflects the increases in output of both goods and partly reflects the increases in prices of both goods. Real GDP takes out the effect of rising prices by valuing outputs in each year at the fixed set of base year prices.

$$\text{Real GDP}_t = \sum_i (P_0 \times Q_t)_i$$

So with the base year the year 2010:

$$\text{Real GDP}_{2010} = \$1000$$

$$\text{Real GDP}_{2018} = \$2 \times 250 + \$3 \times 500 = \$2000$$

With the base year the year 2010, Real GDP doubled (increased 100%) from 2010 to 2018 (whereas Nominal GDP tripled). Thus, half of the increase in Nominal GDP is attributed to price increases, and half to overall quantity increases.

1.7.b. The price of hotdogs rose by 100%, whereas the price of hamburgers rose by only 33.333.% (i.e., by one third). So the overall inflation rate will be somewhere between these two numbers. The exact rate will depend on which price index we use. Here we are using the GDP Deflator.

The general price level, as measured by the GDP deflator, in any year t is:

$$\text{GDP Deflator}_t = \frac{\text{Nominal GDP}_t}{\text{Real GDP}_t} = \frac{\sum_i (P_t \times Q_t)_i}{\sum_i (P_0 \times Q_t)_i}$$

So with the base year the year 2010:

$$\text{GDP Deflator}_{2010} = 1$$

$$\text{GDP Deflator}_{2018} = \frac{3000}{2000} = 1.5$$

$$\%\Delta\text{GDP Deflator} = \left(\frac{\text{GDP Deflator}_{2018} - \text{GDP Deflator}_{2010}}{\text{GDP Deflator}_{2010}}\right) \cdot 100 = 50\%$$

This says that on average, prices rose 50% (in total) over the 8 year period.[1] More literally, it says that the basket of all the goods that were produced in 2018 cost 50% more to buy in 2018 than it would have cost at base year (2010) prices.

Aside: This is one measure of inflation for this period. If we changed the base year to 2018, or we used a different price index method, such as the CPI (Laspeyres) method, we would get somewhat different numbers (though usually not dramatically different). E.g., if we calculated the CPI with the data in this problem (you don't need to do this), we would get $\text{CPI}_{2018} = 1.6$, which says that prices on average have risen 60%, rather than 50%, since 2010.

Why is this? Any price index compares the cost of some basket of goods evaluated at base year and current prices. When we change the base year or the index method (e.g., to CPI), we change that basket of goods, and thus the costs in the two years.

Equivalently, any price index can be thought of as averaging over the various prices in the economy using some particular set of weights (those weights depend on the importance of the goods in the reference basket of goods). Changing the base year, or switching to the CPI method, changes the weights used to calculate the average price, and so changes measured inflation.

1.7.c. As noted in the handout on price indexes (and in the aside above), any price index can be written as a weighted average of $(P_t/P_0)_i$ for each good i:

$$\text{Index Value}_t = \sum_i w_{i,t} \times \left(\frac{P_t}{P_0}\right)_i$$

where $w_{i,t}$ is the weight given to good i in the average in year t.[2] Consequently, if the rate of inflation on each good i is the same, then all price indexes will measure this as the general inflation rate.

In this example (part **c**), prices on hotdogs and hamburgers both increased by 100 percent. Consequently both the GDP Deflator and the CPI will indicate that the general price level rose 100 percent over the decade.

Notice that $(P_t/P_0)_i = 2.0$ for both goods, and so the value of the GDP Deflator (and CPI, or any other price index over these two goods) will be 2.0 for the year 2018.

[1] Note that, due to compounding, this implies an average *annual* inflation rate of about 8.5% per year over the 8 year period.

[2] For the GDP Deflator (Paasche index), the weights reflect the importance of each good in current year (t) real purchases (current year real GDP).

1.7.d. Yes. The traditional way of measuring Real GDP that we used in part 7a. and the traditional GDP Deflator that we calculated in part 7b are both sensitive to the choice of base year.

If you were to re-do the calculations for Real GDP using 2018 as the base year, you would find that Real GDP increases from \$1600 in 2010 to \$3000 in 2018, an increase of 87.5%, which is smaller than the 100% increase that we got above using 2010 as the base year. Neither measure is intrinsically better, and in general there is no perfect measure.

The U.S. Commerce Department now moderates this base year problem by essentially averaging these two methods (i.e., splitting the difference) year by year and "chaining" the results. Chained Real GDP is the measure that typically gets reported in the news, etc.

Similarly, if we recalculate the GDP Deflator for each year with base year 2018, we will get GDP Deflator = 0.625 in 2010 and 1.0 in 2018.[3] The percent change between the two years is $((1.0 - 0.625)/0.625) \cdot 100 = 60\%$. So, just as with changing the index method, changing the base year gives you a different measured inflation rate for the traditional GDP Deflator. As with Real GDP, the Commerce Department now moderates this base year problem by measuring and reporting a "chained" implicit GDP Deflator. This is again essentially an average of the measures with different base years, or equivalently, an average of Paasch and Laspeyres measures, and is called a Fischer type index.

2. We can compare the real purchasing power of the minimum wage in the two years (1970 and 2017) by using the CPI.

	1970	2017
Nominal Wage	\$1.60	
CPI	0.388	2.480

For the purchasing power of the nominal minimum wage to be the same in 2019 as it was in 1970, it would have to be $\$1.60 \times (2.585/0.388) = \$10.66$ per hour. I.e., the nominal wage would have to keep up with the general price level, which is 6.662 times higher than it was in 1970.[4]

An equivalent way of doing this is to calculate the nominal minimum wage in 2019 that would make the "real" minimum wage the same in both years:

$$\text{Real Wage}_t = \frac{\text{Nominal Wage}_t}{CPI_t}$$

The real minimum wage in 1970 is \$4.12 (this is sometimes called the 1970 minimum wage "in 1983 dollars"). A nominal minimum wage of \$10.66 would give the same real minimum wage in 2019.

Aside: Since 1970, average labor productivity has increased about 2.6 times (it has grown by roughly 2% per year). So for the minimum wage to have kept up with both inflation **and** productivity growth, it would have had to increased 6.662 times to keep up with inflation (i.e., to preserve the same real purchasing power) and another 2.6 times to keep up with productivity growth. This minimum wage would be \$27.72.

3. As we noted above, changing the base year or the price index method will change the measured price level and inflation rate. While this may all seem arcane (!!), it has big implications for practical matters like what index to use to make cost of living adjustments (COLAs) to Social Security payments (the U.S. currently "indexes" social security payments to the CPI), or what index to use to determine if or by how much the purchasing power of average wages has gone up over time, or what measure of inflation the FED should target.

Chain weighted measures of the price level both update the base year each year and average over the Laspeyres and Paasche methods (yielding a Fisher type index). The FED targets 2% inflation according to

[3] Notice that price indexes are always 1.0 in the base year.

[4] The Federal minimum wage in the U.S. is currently (January 2020) \$7.25. The NY State minimum wage was just increased to \$11.80 (on Jan 1, 2020) and is scheduled to increase to \$12.50 in January 2021.

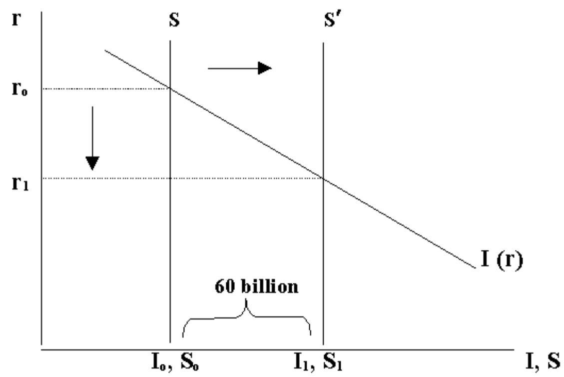
a chain weighted PCE price index rather than the traditional CPI. It also tends to focus more on the "core" index, which excludes food and energy. Why do you think that this is the case?

Aside: A separate measurement issue is how well the BLS and Commerce Department are doing adjusting their price indices (and real GDP measure) for improvements in product quality (the introduction of new and improved goods) over time. The quick answer is that they both make adjustments for this, but by no means exhaustively.

4. Chapter 3: 8,9,10,11,13. These problems refer to the basic model of saving and investment in the long run. Output is held fixed, so policy changes affect the composition of output, but not its level.

4.8. An increase in taxes (of $100 billion) will reduce disposable income and consequently cause consumers to reduce their consumption expenditures. If the MPC is .6, consumers reduce their consumption by 60 billion and their saving by 40 billion. National saving $S = Y - C - G$, the income left over after consumption and government spending, increases by 60 billion, driving the interest rate down by just enough to drive investment spending up by the equivalent 60 billion. The composition of output has changed, with consumption falling and investment rising by equal amounts.

Note the chain of causality here. The tax increase raises national saving S, creating a surplus of loanable funds which causes the interest rate r to fall. This reduction in the interest rate in turn causes investment spending I to increase. So the chain of causality is $\uparrow S \rightarrow \downarrow r \rightarrow \uparrow I$.



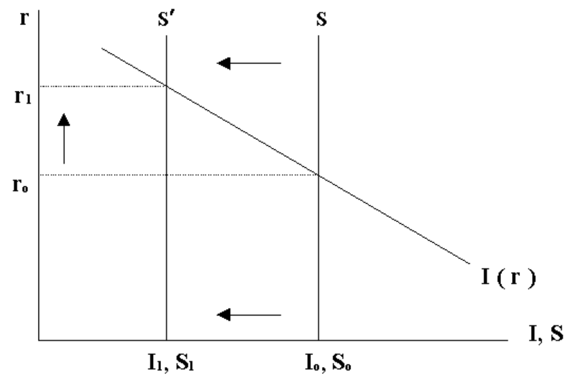
4.8.a. *Public saving* is the government budget surplus $T - G$ (which has been negative in the US for most of the past 40 years). Since tax revenues have risen by 100 billion and government spending has remained unchanged, public saving has risen by 100 billion.

4.8.b. *Private saving* is disposable income minus consumption $(Y - T - C)$. This has decreased by 40 billion as indicated above.

4.8.c. *National saving* has risen by 60 billion. We can see this easily by looking at our definition: $S = Y - C - G$. Consumption has fallen by 60 billion, while Y and G are unchanged. We could also note that national saving is private plus public saving, and use the answers to parts **a** and **b**.

4.8.d. *Investment* must increase by the same amount as national saving, since by national income accounting: $S = I + NX$, and in this problem $NX = 0$.

4.9. National saving falls, and so interest rates rise, causing investment to fall. In equilibrium we have a change in the composition of GDP, with consumption rising and investment falling by equal amounts. Consumption spending is said to *crowd out* investment spending (whereas in **8**, I was *crowded in*).



4.10.a. Plugging in the values for Y and T, we can see that $C = 5000$. Consequently, Private Saving $= Y - T - C = 1000$, Public Saving $= T - G = -500$, and National Saving $S = Y - C - G = 500$.

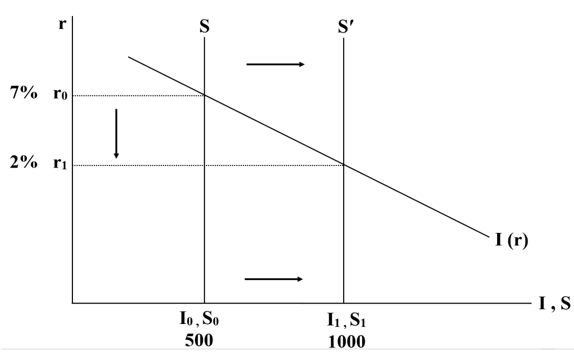
4.10.b. In equilibrium, we have

$$\begin{aligned} Y &= C + I + G + NX \\ &= 5000 + (1200 - 100\,r) + 2500 + 0 \\ r &= 7 \end{aligned}$$

so the equilibrium interest rate is 7%.[5]

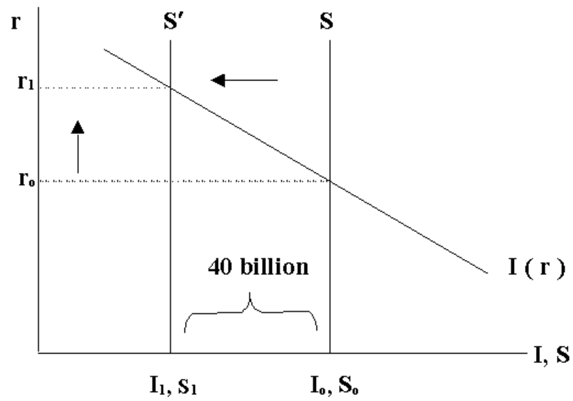
4.10.c. A decrease in G by 500 increases both Public and National Saving by 500 and leaves Private Saving unchanged.

4.10.d. The increase in National Saving by 500 will cause interest rates to fall, subsequently causing I to increase by exactly 500 in equilibrium. Repeating the steps in part **b** we can calculate that the new equilibrium interest rate is $r = 2$.

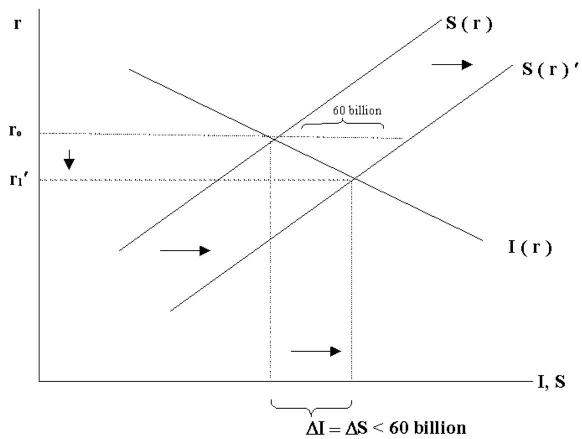


4.11. Use the example in **8**. Now consumption falls by 60 billion as in **8**, but government spending rises by 100 billion. National saving, the income left over after C and G, falls by 40 billion. Interest rates rise and investment spending is crowded out by 40 billion. Notice, however, that higher MPCs lead to smaller decreases in national saving. The government is taxing away and spending income from consumers, some of which the consumers were saving. With a higher MPC, less of that income had been saved, so more of the 100 billion of income that the government is taxing away is just being transferred from C to G.

[5] Note that you could have equivalently solved for this equilibrium interest rate by setting S = I + NX.



4.13. Consider **8** again. National saving increases by 60 billion due to an increase in taxes. This drives down the interest rate. However, if consumers respond to the fall in the interest rate by consuming more of their income and saving less of it (e.g., racking up credit card purchases, buying new cars, etc.), then national saving will fall back down somewhat. The interest rate will fall by less in equilibrium than in **8** and the equilibrium increase in saving and investment will be smaller than 100 billion.



The general effect of interest sensitive saving in the model is to mitigate the crowding out/in mechanism. For example, increases in G and C will lead to less crowding out of investment, and autonomous increases in investment will actually cause investment to rise in equilibrium (i.e., not crowd itself out completely).