1. Chapter 2. Problems and Applications 2.6:

1.2. $6 has been added to GDP in this example. We don’t want to double-count the value of the wheat and flour, since they show up in the price of the bread. Thus, we measure GDP by counting final goods only (here the bread) or equivalently by adding up the value added for all goods (in this case $1 + $2 + $3 = $6).

1.7. We have the following information on purchases and prices in the two years:

<table>
<thead>
<tr>
<th>Year</th>
<th>$P_{\text{hotdogs}}$</th>
<th>$Q_{\text{hotdogs}}$</th>
<th>$P_{\text{hamburgers}}$</th>
<th>$Q_{\text{hamburgers}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>$2$</td>
<td>200</td>
<td>$3$</td>
<td>200</td>
</tr>
<tr>
<td>2015</td>
<td>$4$</td>
<td>250</td>
<td>$4$</td>
<td>500</td>
</tr>
</tbody>
</table>

1.7.a. With base year 2010, we have:

<table>
<thead>
<tr>
<th>Year</th>
<th>Nominal GDP</th>
<th>Real GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>$1000$</td>
<td>$1000$</td>
</tr>
<tr>
<td>2015</td>
<td>$3000$</td>
<td>$2000$</td>
</tr>
</tbody>
</table>

Here is how I calculated these numbers.

Nominal GDP$_t$ = \( \sum_i (P_t \times Q_t)_i \)

where \( t \) indicates year (either 2010 or 2015) and \( i \) indicates good (here hotdogs or hamburgers). So

\[
\text{Nominal GDP}_{2010} = 2 \times 200 + 3 \times 200 = 1000
\]

\[
\text{Nominal GDP}_{2015} = 4 \times 250 + 4 \times 500 = 3000
\]

Nominal GDP rose (by 200%) between 2010 and 2015. However, this partly reflects the increases in output of both goods and partly reflects the increases in prices of both goods. Real GDP takes out the effect of rising prices by valuing outputs in each year at the fixed set of base year prices.

Real GDP$_t$ = \( \sum_i (P_0 \times Q_t)_i \)

So with the base year the year 2010:

\[
\text{Real GDP}_{2010} = 1000
\]

\[
\text{Real GDP}_{2015} = 2 \times 250 + 3 \times 500 = 2000
\]

With the base year the year 2010, Real GDP doubled (increased 100%) from 2010 to 2010 (whereas Nominal GDP tripled). Thus, half of the increase in Nominal GDP is attributed to price increases, and half to overall quantity increases.

Aside: It is worth noting, however, that the traditional way of measuring Real GDP that we are using here is sensitive to the choice of base year. If you were to re-do the calculations for Real GDP using 2015 as the base year, you would find that Real GDP increases from $1600 in 2010 to $3000 in 2015, an increase of 87.5%, which is smaller than the 100% increase that we got above using 2010 as the base year. Neither measure is intrinsically better, and in general there is no perfect measure. The U.S. Commerce Department
now moderates this base year problem by “chaining” (updating the base year each year), and chained Real GDP is the measure that gets reported in the news, etc.

1.7.b. As for the price indexes,

<table>
<thead>
<tr>
<th>Year 2010</th>
<th>Year 2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP Deflator</td>
<td>1</td>
</tr>
<tr>
<td>CPI</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\text{GDP Deflator}_t = \frac{\text{Nominal GDP}_t}{\text{Real GDP}_t} = \frac{\sum_i (P_t \times Q_t)_i}{\sum_i (P_0 \times Q_0)_i}
\]

So with the base year the year 2010:

\[
\text{GDP Deflator}_{2010} = 1
\]

\[
\text{GDP Deflator}_{2015} = \frac{3000}{2000} = 1.5
\]

This says that on average, prices rose 50% (in total) over the 5 year period.\(^1\)

Again, this measure of inflation is somewhat arbitrary. Any price index averages over the various prices in the economy using some particular set of weights. Changing the base year, or switching to the CPI (Laspeyres) method, changes the weights used to calculate the average price, and so changes measured inflation. Consider the CPI:

\[
\text{CPI}_t = \frac{\sum_i (P_t \times Q_0)_i}{\sum_i (P_0 \times Q_0)_i}
\]

So with the base year the year 2010:

\[
\text{CPI}_{2010} = 1
\]

\[
\text{CPI}_{2015} = \frac{4 \times 200 + 4 \times 200}{1000} = 1.6
\]

Thus, according to the CPI, on average, prices rose 60% (in total)\(^2\) over the course of the 5 year period, whereas, according to the GDP Deflator, prices only rose 50% over that period.

The CPI gives a higher measure of inflation in this example because it weighs price increases on each good by the good’s share in spending in 2010 (the base year), rather than in 2015 (the current year) as does the GDP Deflator. Notice that the price of hotdogs rose by 100%, whereas the price of hamburgers rose by only 33.333...% (i.e., by one third). Consumers appear to have reacted by increasing their consumption of hamburgers by more (a larger percentage) than their consumption of hotdogs. Therefore, the GDP Deflator places more weight on hamburgers (the low inflation good) than does the CPI in calculating average price increases.

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\(^1\) Note that, due to compounding, this implies an average annual inflation rate of about 8.5% per year over the 5 year period.

\(^2\) An average annual rate of about 9.9% per year.
Specifically, the shares of hotdogs and hamburgers in base year spending were .4 and .6 respectively (i.e., of the $1000 spent in the year 2010, forty percent was spent on hotdogs and sixty percent was spent on hamburgers). So overall inflation measured by the CPI is $0.4 \times 100\% + 0.6 \cdot 33.33\% = 60\%$. The shares of hotdogs and hamburgers in real GDP in 2015 were 0.25 and 0.75 respectively (i.e., 25% and 75% of real GDP respectively). So overall inflation measured by the GDP Deflator is $0.25 \times 100\% + 0.75 \cdot 33.33\% = 50\%$.\(^3\)

Both indexes provide information about how rapidly the cost of living is increasing. However, the information is somewhat different. The GDP Deflator (Paasche index) shows how fast prices have risen for what you are purchasing today (2015). The CPI (Laspeyres index) shows how fast prices have risen for what you purchased in the base year (2010). However, since you didn’t purchase the same things in the two years, neither is clearly better than the other. The Laspeyres method ignores the fact that if something gets very expensive, we will probably buy less of it and more of other things (i.e., we substitute away from the goods with the greatest inflation rate). On the other hand, the Paasche method underestimates the increase in the cost of our original (base year) standard of living.

Aside: While this may all seem arcane (!!), it has big implications for practical matters like what index to use to make cost of living adjustments (COLAs) to Social Security payments (the U.S. currently “indexes” social security payments to the CPI), or what index to use to determine if or by how much the purchasing power of average wages has gone up over time, or what measure of inflation the FED should target.

Aside: One practical way do deal with the differences in methods is to just average over them. As with real GDP above, the Commerce Department now focusses on a chain weighted version of the GDP deflator. And the FED targets 2% inflation according to a chain weighted PCE price index rather than the CPI. These chain weighted indexes both update the underlying base year annually and also average across Paasche and Laspeyres methods (yielding a ‘Fischer’ type index).

Aside: A separate measurement issue is how well the BLS and Commerce Department are doing adjusting their price indices (and real GDP measure) for improvements in product quality (the introduction of new and improved goods) over time. The quick answer is that they both make adjustments for this, but by no means exhaustively.

1.7.c. As noted in the handout on price indexes and in the problem above, any price index can be written as a weighted average of \((P_t/P_0)_i\) for each good \(i\):

\[
\text{Index Value}_t = \sum_i w_{i,t} \times \left( \frac{P_t}{P_0} \right)_i
\]

where \(w_{i,t}\) is the weight given to good \(i\) in the average in year \(t\).\(^4\) Consequently, if the rate of inflation on each good \(i\) is the same, then all price indexes will measure this as the general inflation rate.

In this example (part c), prices on hotdogs and hamburgers both increased by 100 percent. Consequently both the GDP Deflator and the CPI will indicate that the general price level rose 100 percent over the decade.

Notice that \((P_t/P_0)_i = 2.0\) for both goods, and so the value of the GDP Deflator and CPI will both be 2 for the year 2015.

<table>
<thead>
<tr>
<th></th>
<th>2010</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP Deflator</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>CPI</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

2. We can compare the real purchasing power of the minimum wage in the two years (1970 and 2017) by using the CPI.

\(^3\) Similarly, the the level of the CPI in 2015 is \((0.4 \times 2.0 + 0.6 \times 1.333\ldots) = 1.6\), and the level of the GDP Deflator in 2015 is \((0.25 \times 2.0 + 0.75 \times 1.333\ldots) = 1.5\).

\(^4\) For the GDP Deflator, the weights reflect the importance of each good in current year \((t)\) purchases (current year real GDP), while the weights for the CPI reflect the importance of each good in base year purchases.
### Table 1: Comparison of Nominal and CPI for 1970 and 2017

<table>
<thead>
<tr>
<th>Year</th>
<th>Nominal Wage</th>
<th>CPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>$1.60</td>
<td>0.388</td>
</tr>
<tr>
<td>2017</td>
<td>$7.25</td>
<td>2.480</td>
</tr>
</tbody>
</table>

For the purchasing power of the nominal minimum wage to be the same in 2017 as it was in 1970, it would have to be $1.60 \times (2.480/0.388) = $10.23 per hour. I.e., the nominal wage would have to keep up with the general price level which is 6.392 times higher than it was in 1970.5

An equivalent way of doing this is to calculate the nominal minimum wage in 2017 that would make the “real” minimum wage the same in both years:

\[ \text{Real Wage}_t = \frac{\text{Nominal Wage}_t}{\text{CPI}_t} \]

The real minimum wage in 1970 is $4.12 (this is sometimes called the 1970 minimum wage “in 1983 dollars”). A nominal minimum wage of $10.23 would give the same real minimum wage in 2017.

To have the minimum wage keep up with both inflation and productivity growth, it would have had to grow 6.392 times to keep up with inflation (i.e., to preserve the same real purchasing power) and then another 2.536 = 1.0247 times to keep up with productivity growth. This minimum wage would be $25.94.

4. Chapter 3: 8,9,10,11,13. These problems refer to the basic model of saving and investment in the long run. Output is held fixed, so policy changes affect the composition of output, but not its level.

4.8. An increase in taxes (of $100 billion) will reduce disposable income and consequently cause consumers to reduce their consumption expenditures. If the MPC is .6, consumers reduce their consumption by 60 billion and their saving by 40 billion. National saving \( S = Y - C - G \), the income left over after consumption and government spending, increases by 60 billion, driving the interest rate down by just enough to drive investment spending up by the equivalent 60 billion. The composition of output has changed, with consumption falling and investment rising by equal amounts.

Note the chain of causality here. The tax increase raises national saving \( S \), creating a surplus of loanable funds which causes the interest rate \( r \) to fall. This reduction in the interest rate in turn causes investment spending \( I \) to increase. So the chain of causality is \( \uparrow S \rightarrow \downarrow r \rightarrow \uparrow I \).

4.8.a. Public saving is the government budget surplus \( T - G \) (which has been negative in the US for most of the past 40 years). Since tax revenues have risen by 100 billion and government spending has remained unchanged, public saving has risen by 100 billion.

4.8.b. Private saving is disposable income minus consumption \( (Y - T - C) \). This has decreased by 40 billion as indicated above.

4.8.c. National saving has risen by 60 billion. We can see this easily by looking at our definition: \( S = Y - C - G \). Consumption has fallen by 60 billion, while \( Y \) and \( G \) are unchanged. We could also note that national saving is private plus public saving, and use the answers to parts a and b.

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5 The Federal minimum wage in the U.S. is currently (January 2018) $7.25. The NY State minimum wage was just increased to $10.40 (on Jan 1, 2018) and is scheduled to increase to $11.10 in January 2019 and $12.50 by 2021.
4.8.d. *Investment* must increase by the same amount as national saving, since by national income accounting: \( S = I + NX \), and in this problem \( NX = 0 \).

4.9. National saving falls, and so interest rates rise, causing investment to fall. In equilibrium we have a change in the composition of GDP, with consumption rising and investment falling by equal amounts. Consumption spending is said to *crowd out* investment spending (whereas in 8, \( I \) was *crowded in*).

4.10.a. Plugging in the values for \( Y \) and \( T \), we can see that \( C = 5000 \). Consequently, Private Saving \( = Y − T − C = 1000 \), Public Saving \( = T − G = −500 \), and National Saving \( S = Y − C − G = 500 \).

4.10.b. In equilibrium, we have

\[
Y = C + I + G + NX \\
= 5000 + (1200 − 100r) + 2500 + 0
\]

so the equilibrium interest rate is 7%.

4.10.c. A decrease in \( G \) by 500 increases both Public and National Saving by 500 and leaves Private Saving unchanged.

4.10.d. The increase in National Saving by 500 will cause interest rates to fall, subsequently causing \( I \) to increase by exactly 500 in equilibrium. Repeating the steps in part b we can calculate that the new equilibrium interest rate is \( r = 2 \).

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6 Note that you could have equivalently solved for this equilibrium interest rate by setting \( S = I + NX \).
4.11. Use the example in 8. Now consumption falls by 60 billion as in 8, but government spending rises by 100 billion. National saving, the income left over after $C$ and $G$, falls by 40 billion. Interest rates rise and investment spending is crowded out by 40 billion. Notice, however, that higher MPCs lead to smaller decreases in national saving. The government is taxing away and spending income from consumers, some of which the consumers were saving. With a higher MPC, less of that income had been saved, so more of the 100 billion of income that the government is taxing away is just being transferred from $C$ to $G$.

![Diagram of 4.11](image1)

4.13. Consider 8 again. National saving increases by 60 billion due to an increase in taxes. This drives down the interest rate. However, if consumers respond to the fall in the interest rate by consuming more of their income and saving less of it (e.g., racking up credit card purchases, buying new cars, etc.), then national saving will fall back down somewhat. The interest rate will fall by less in equilibrium than in 8 and the equilibrium increase in saving and investment will be smaller than 100 billion.

![Diagram of 4.13](image2)

The general effect of interest sensitive saving in the model is to mitigate the crowding out/in mechanism. For example, increases in $G$ and $C$ will lead to less crowding out of investment, and autonomous increases in investment will actually cause investment to rise in equilibrium (i.e., not crowd itself out completely).