2. In the extended model (Ch. 15) DAS is: \( \pi_t = E_{t-1}\pi_t + \phi \cdot (Y_t - \bar{Y}) + v_t \). Given \( v_t = 0 \), then for expected inflation to be correct \( (E_{t-1}\pi_t = \pi_t) \), inflation would have to be expected to be the level consistent with \( Y_t = \bar{Y} \).

If the Fed reduces its inflation target, then DAD will shift downward (by exactly the change in the target). Suppose that the change is announced in advanced (and the announcement is credible, i.e., believed by the public). Then under rational expectations, the economy will jump to its new long run equilibrium. I.e. the inflation rate will jump to the new target, and output will stay at \( \bar{Y} \). The public’s expectations will turn out to be correct.

On the contrary, we know from Ch. 15, that under adaptive expectations, both actual and expected inflation will fall more slowly, and so the economy will fall into a recession and then slowly recover, ultimately arriving at the new long run equilibrium. Along the way, expected inflation will be greater than actual inflation in every period, and so the public’s expectations will be systematically incorrect – if they understood how the economy works, they would be able to improve upon these naive adaptive expectations.

4a. Recalling that \( m = \frac{1 + cr}{cr + rr} \), the nominal money supply \( M \) is

\[
M = m \times B \\
= 2 \times 1200 \\
= 2400
\]

and the price level \( P \) is 1. Equating real money supply \( (M/P) \) and money demand, we get \( Y = 2400/0.8 = 3000 \).

Notice that since LM is vertical, equilibrium output \( Y \) is determined solely by LM, but interest rates \( r \) depend also on spending (IS).

4b. The Fed is increasing the monetary base \( B \) by 100. Since the money multiplier is 2, this increases the money supply by 200. Equating the new money supply (2600) with money demand, we get \( Y = 3250 \). Output increases by 250.

The money supply expands by the increase in the base (100) plus a further increase (100) through the process of deposit creation via banks loans. This expansion of the money supply depresses interest rates, stimulating investment spending, and thus causing output to increase.
c. Now we have an increase in the monetary base by 100 as above but also an increase in government purchases by 100. If the K-Cross multiplier is 5, then IS shifts to the right by 500 (i.e., spending rises by 500 if interest rates stay constant). Interest rates will rise, however, unless the money supply rises sufficiently (i.e., LM is shifted out sufficiently) to offset the pressure of increased spending on interest rates. But LM has only shifted to the right by 250, so interest rates do increase, though not by as much as if the increase in the deficit was not monetized (i.e., if the Fed left M constant).

Note then, that the effect on interest rates depends on both IS and LM – they may increase or decrease in the new equilibrium depending on the relative sizes of the two shifts. This is the case even though the Fed is buying exactly the same amount of bonds that the Treasury is selling and so the direct impact of these bond purchases and sales on interest rates is a wash.

5a. According to the life-cycle theory, an individual consumes her/his lifetime income evenly over the course of the 60 years. Lifetime income is the area under the income $Y$ curve in the picture.\footnote{If this is not clear, start by thinking about the individual earning income at rates which are fixed within each year. To do this, build a ‘staircase’ under the $Y$ curve with the width of each ‘step’ being one year. If the annual}
40 \times 10,000 + \frac{1}{2} \times 40 \times 40,000 = 1,200,000. Then, she/he can consume \( \frac{1,200,000}{60} = 20,000 \) in each year, every year, for life.

The individual borrows on future income until age 30, at which time \( C = Y \).

\[
20,000 = 10,000 + 1000 (t - 20) \quad \Rightarrow \quad t = 30
\]

Then she/he saves and accumulates assets for retirement. The loans (negative assets) acquired between ages 20 and 30 are just paid off, and so assets are zero, at age 40. This can be calculated by noting that accumulated debt is the shaded area \( A \). We want to solve for an age \( t \) such that accumulated saving (shaded area \( B \)) just equals this. We can make these calculations, or simply notice that, with a linear income path, it will take the same amount of time to pay off the loans as it did to accumulate them: 10 years. The individual then continues to save and accumulate assets until age 60, and then consumes all of these assets (by selling them and buying consumption goods; i.e., dissaving) in retirement. Assets are zero at ages 20, 40, 80.

5b. If there has been no population growth for some time, there are the same number of individuals (households) at each age in the life-cycle. Then, since over the life-cycle of each household there is an equal amount of saving and dissaving, there must be as many households currently saving (middle aged) as dissaving (young and old). Thus aggregate personal saving in the economy is zero. If there are currently no other sources of saving in the economy, then aggregate investment spending \( (I) \) must also be zero.

To see how this works in the National Income Accounts, recall that \( I + NX = S \), and that national saving \( S = Y - C - G \) can be decomposed into private saving \( S_p = Y - T - C \) and government saving \( S_g = T - G \); so that \( S = S_p + S_g \). Private saving, in turn, consists of personal saving (done by households) and corporate saving (retained earnings). With the trade and budget deficits equal to zero, we have \( I = S_p \). With corporate and personal saving equal to zero, we then have that \( I = 0 \).

5c. At age 40 there are 20 more years of work life and 40 more years of life. If the individual receives a one time (unanticipated) increase in disposable income of 1000, the life-cycle theory says that she/he will spread it evenly over all future consumption – i.e., will increase consumption in this and every remaining year of life by the increase in remaining lifetime resources divided by the number of years remaining in life. Then \( \Delta C = \frac{1000}{40} = 25 \) in each year. So she/he saves 975 today and spends 25 today. The current \( MPC = \frac{\Delta C}{\Delta Y} = \frac{25}{1000} = .025 \).

If the change in \( Y \) were permanent, then the individual would get an increase in lifetime income of \( 1000 \times 20 = 20,000 \) and would spread it over 40 years of consumption. \( \Delta C = \frac{20,000}{40} = 500 \). \( MPC = \frac{\Delta C}{\Delta Y} = \frac{500}{1000} = 0.5 \)

Suppose that the individual didn’t know whether the \( \Delta Y \) was permanent or transitory, but guessed that it was permanent with probability \( \frac{1}{2} \) and transitory with probability \( \frac{1}{2} \). Then she/he might expect her lifetime income to increase by \( \frac{1}{2} \times 20 \times 1000 + \frac{1}{2} \times 1000 = \$ 10,500 \). Then she/he would plan to increase \( C \) by \( \$ 262.50 \) each year for the remainder of her/his life. Then \( MPC = \frac{\Delta C}{\Delta Y} = \frac{262.50}{1000} = 0.2625 \), i.e., a bit more than 0.25.

rate at which the individual earns income at any point in time was given by the height of this step function, then it is clear that each year’s income would be given by the area under that year’s respective ‘step,’ and that total lifetime income would be given by the sum of these (rectangular) areas. Since in the problem the rate that we earn income is adjusted continuously rather than annually, total lifetime income is actually the total area under our \( Y \) function, not just the area under the steps.
5d. People are taxed (lose disposable income of) 2,000 per year while they are working, but get it all back in retirement. Thus, lifetime disposable income is unchanged, and so they should still want to keep consumption at 20,000 every year. Thus, working families reduce their saving by 2,000 in every year until retirement. For instance, a family at age 30 must borrow exactly 2,000 to maintain its desired consumption stream. The current MPC and MPS out of this change in disposable income are: \( MPC = \frac{\Delta C}{\Delta Y} = -\frac{0}{2000} = 0 \). \( MPS = \frac{\Delta S}{\Delta Y} = -\frac{2000}{2000} = 1 \). Since this is true for every working household, it is true at the aggregate level.

Each household is taxed 2,000 and lowers saving by 2,000. Thus the rise in public saving \( (S_g) \), from the increased tax revenues, is offset by the fall in private saving, leaving national saving unchanged.

If some households are liquidity constrained, they can’t borrow, and so can’t reduce their saving. For these households private saving is unchanged (or falls by less than 2,000 if they are currently doing some saving), while tax revenues still rise by 2,000. National saving then rises.

6. False. If preferences are time inconsistent, then discounting is applied inconsistently over time. For example, if a person discounts the future at a constant rate (i.e., has a constant degree of impatience), then there is no time inconsistency. However, if a person always has extra impatience in the present, then he will feel that waiting (delaying gratification) now is more unpleasant than it will be in the future. However, he will feel the same way tomorrow (hence the inconsistency). If he recognizes the time inconsistency, then he can solve the problem rationally (i.e., accurately maximize his utility), whereas if he does not recognize the inconsistency, he may fall into the trap of ongoing procrastination.

7. To see whether the project is profitable, calculate

\[
P V_{\text{benefits}} = \frac{.38}{1.1} + \frac{.38}{(1.1)^2} + \frac{.38}{(1.1)^3} \approx .95 < 1 = P V_{\text{costs}}
\]

so don’t take the project. We would be better off investing our money in a bond than in this project, or equivalently would lose money if we borrowed to finance the project.

To see why we needed to do this present value calculation, note that the firm gets a rate of return of 0.38 for 3 years, which is better than that on a bond, but 0 thereafter, which is worse. The sum of the returns is \( 3 \times 0.38 = \$1.14 \), which is greater than the price of the machinery \( (\$1) \), but we shouldn’t value \( \$0.38 \) received three years from now as highly as \( \$0.38 \) today, since to get this we need only put \( \frac{0.38}{(1.1)^3} \approx \$0.29 \) in a bond today. We can receive the exact stream of payments that we get from the project over the three years by putting only \( \$0.95 \) into a bond instead of putting the \( \$1 \) into the project.

8. The answer is that the firm should not purchase the equipment. The interest rate would have to be less than 5% for the firm to find the investment profitable.

To get this answer, we need to work out the present value calculation for the firm. First we can make the following useful observation: If the interest rate \( i \) is constant over time, the PV of a constant perpetual flow of $x per period, that starts next period, is \( \frac{x}{i} \). Let’s derive this:

\[
\frac{x}{i} = \frac{x}{1 + i} + \frac{x}{(1 + i)^2} + \frac{x}{(1 + i)^3} + \ldots
\]

This makes intuitive sense, since the PV of this set of future payments is the amount of money that we would have to invest in bonds today at interest rate \( i \) in order to be able to generate this flow of payments in the future.
\[ PV = \frac{x}{1 + i} + \frac{x}{(1 + i)^2} + \frac{x}{(1 + i)^3} + \ldots \]
\[ = \frac{x}{1 + i} \times (1 + \frac{1}{1 + i} + (\frac{1}{1 + i})^2 + (\frac{1}{1 + i})^3 + \ldots) \]
\[ = \frac{x}{1 + i} \times \frac{1}{1 - \frac{1}{1+i}} \]
\[ = \frac{x}{i} \]

Now back to the problem. Suppose that the firm finances the equipment out of cash on hand. Then the cost of purchasing and running the equipment is the $100,000 outlay today plus the future maintenance costs of $5000 per year. The PV of these maintenance costs is

\[ PV_{\text{maintenance costs}} = \frac{5000}{1.07} + \frac{5000}{(1.07)^2} + \frac{5000}{(1.07)^3} + \ldots \]
\[ = \frac{5000}{0.07} \]
\[ \approx 71,400 \]

Thus, the PV of the costs of buying and operating the equipment is $100,000 + $71,400 = $171,400.

The benefit of operating the equipment is $10,000 per year in increased revenues. The PV of this flow of benefits is

\[ PV_{\text{benefits}} = \frac{10,000}{1.07} + \frac{10,000}{(1.07)^2} + \frac{10,000}{(1.07)^3} + \ldots \]
\[ = \frac{10,000}{0.07} \]
\[ \approx 142,800 \]

Since the PV of the benefits is less than the PV of the costs, you should not purchase the machine.

At what interest rate would this project be profitable? The PV of the benefits is \(\frac{10,000}{i}\) and the PV of the costs is \(100,000 + \frac{5000}{i}\). Thus, for the PV of the benefits to be greater than the PV of the cost we need

\[ \frac{10,000}{i} > 100,000 + \frac{5000}{i} \]

which will be true if

\[ \frac{5,000}{i} > 100,000 \]

and thus if

\[ i < \frac{5,000}{100,000} = .05 \]
If the interest rate on bonds was less than 5%, the investment would be profitable.

Other Investment Models: Mankiw presents a per period capital rental rule for investment rather than the present value rule that we are working with. Since, in the current problem, the flow of benefits and costs over time are constant, the two rules turn out to be equivalent in this case. However, in problem seven above, the returns varied over time (we get benefits in three years only). Since the investment is irreversible and has different returns in different years, we can’t decide period by period whether to make the investment – we have to look at the entire sequence of expected future costs and benefits. Thus, in that case, the PV rule works, whereas Mankiw’s period by period rule would not. The PV rule is the more general rule.

It should also be that the Tobin’s Q rule would give a similar result as the PV rule. Tobin’s Q is the ratio of the stock market’s valuation of the firm’s capital stock to the replacement cost of that capital stock. If Q is greater than one, then stock holders anticipate that this firm will generate more profits in the future than can be earned on bonds (of similar risk). If Q is less than one, then stock holders view this firm as unprofitable relative to bonds. In the latter case, and if new investments by the firm are perceived by the stock market to be no more profitable than the firm’s past investments, then you should not purchase the equipment, because doing so will drive down the price per share of your company’s stock further. The central difference between the PV rule and the Tobin’s Q rule is that Tobin’s Q uses stock market investors’ expectations of future benefits and costs rather than your’s (i.e., rather than the company managers’ expectations) to evaluate the expected profitability of investment projects.

9.
   a. The capital stock is currently

   \[ K = K^* = \frac{Y}{r + d} \]
   \[ = \frac{2000}{0.05} \]
   \[ = 40,000 \]

   Since 3% of this depreciates each period, gross investment \( (I) \) must be \( 0.03K = 1200 \).

   b. If \( Y \) rises to 2100, then the desired capital stock in the economy rises to 42,000. To achieve this, firms must have net investment \( I_n \) this year of 2000, which requires gross investment of

   \[ I = I_n + dK \]
   \[ = 2000 + 1200 \]
   \[ = 3200 \]

3 If the interest rate on bonds was exactly 5%, we would be indifferent between investing in the project and investing in bonds. This interest rate 5% is sometimes called the “internal rate of return” of the project and is a measure of the marginal net benefit of the project expressed as a rate of return.
However, in the following year, and all subsequent years, gross investment falls back to the new (slightly higher) level of annual depreciation, $0.03 \times 42,000 = 1260$, which is just enough to maintain the capital stock at its desired level. Notice that $I$ is much more sensitive to changes in $Y$ (the accelerator effect) than to the level of $Y$ (look at the graph above: $\Delta Y$ is temporary but produces a large increase in $I$, whereas the level of $Y$ remains high, but has a small impact on $I$).

This analysis has assumed that GDP remains fixed at 2100. As a result of the short run rise in investment spending, we would actually expect GDP to rise further. This would raise desired capital stocks further, and thus boost investment further. Notice that the accelerator effect is destabilizing for $Y$ (makes $Y$ fluctuations larger).

c. The desired capital stock falls to $2100/0.06 = 35,000$. Once firms have adjusted their capital stocks downward, maintaining them will require investment of $0.04 \times 35,000 = 1400$ which is greater than the prior level. Thus, in this example, though the rise in the rate of depreciation reduces the desired (and thus actual) capital stock, it increases the total amount of depreciation per year, causing gross investment spending $I$ to rise in the long run.

Note that depreciation rates have been rising in the U.S. in the past 30 years (equipment has become less durable on average, especially due to the increasing use of computers). Some of the increase in investment spending during that time is simply due to the fact that computers become obsolete very rapidly (have very high depreciation rates) and so businesses are now spending more money replacing them than they did with more durable plant and equipment.

10. The investment tax credit, once implemented, will reduce the effective purchase cost of equipment. If businesses get a tax credit worth a fraction $\tau$ of their purchases of capital goods, then the effective cost of such goods with price $P_K$ becomes $(1 - \tau)P_K$. This should have the effect of stimulating investment spending.

However, if businesses expect the ITC to be implemented next year, they may wait to buy capital goods until the credit is in place, rather than buying now at the higher effective cost. Thus, we may actually see investment spending fall this year, following the campaign announcement, making the recession temporarily deeper.

11a. False. The real value of the debt has fallen due to the inflation by $\pi \times D = $120 billion. If the real deficit is defined as the change in the real debt, then the nominal deficit understates the real deficit (and equivalently the nominal surplus understates the real surplus) by $120$ billion, by not accounting for the fact that inflation has implicitly retired this amount of the real debt. According to this definition, we are running a real budget surplus of $220$ billion.

---

4 Equivalently, the real surplus is the negative of the change in the real debt.