Economics 430
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Testing The Solow Model

Here are some empirical tests of the Solow Growth Model that appear in Mankiw, Romer and Weil (1992)[1]

1. Explaining levels of output per worker:

1a.

Assume that all countries are close to their steady state growth paths, so that transitional dynamics are unimportant. Recall that, according to the Solow Growth Model, when labor augmenting technological progress occurs at rate g, (i.e., the effectiveness of labor A_t at time t equals $A_0 \cdot e^{gt}$), steady state output per worker grows at rate g. Specifically, for a given country,

$$(\frac{Y}{L})_t = \left(\frac{s}{\delta + n + g}\right)^{\frac{\alpha}{1-\alpha}} \cdot A_0 \cdot e^{g \cdot t}$$

Note that A_0, g, and α are not directly observable. Taking logs of both sides,

$$\ln(\frac{Y}{L})_t = \ln(A_0) + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(\delta + n + g) + g \cdot t$$

Now split the unobservable term $\ln A_0$ into a part a that we assume is the same across countries and a part ε that varies from country to country and has zero mean across countries, and assume that g, α, and δ are the same in each country. Then for country i in year t we have

$$\ln(\frac{Y}{L})_{i,t} \approx (a + g \cdot t) + \frac{\alpha}{1-\alpha}\ln(s_i) - \frac{\alpha}{1-\alpha}\ln(\delta + n_i + g) + \varepsilon_i$$

This equation describes the variation in incomes per worker across countries for any given year t as arising from differences in saving rates s and population growth rates n, assuming that α, δ, and g are constant across countries. Thus, MRW use this equation as a cross sectional regression equation, fixing the year at 1985, and using data on s, n, and Y/L for about 100 countries. The fit is not terrible ($R^2 \approx .6$), and the coefficients have the predicted signs (s raises Y/L and n lowers Y/L). However, the estimates imply $\alpha \approx .6$, which is higher than the .3 predicted by the Solow model. See Table I.

1b.

Adding human capital as a separate form of capital into the Solow model leads to a regression equation similar to the one above, but with the gross saving rate s divided into two terms: s_k, the investment rate in physical capital, and s_h, the investment rate in human capital. The regression results imply a combined capital (physical plus human) elasticity ($\alpha + \beta$) of about .6, and the overall fit of the model is improved ($R^2 \approx .8$). See Table II.

[1] "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics*, **107**, 1992, p. 407-37.

2. Explaining Growth Rates:

The discussion above assumed that all countries were close to their steady state growth paths (at least in 1985). If that were the case, then the rate of growth of output per worker (y) should be very close to g for each country, according to the Solow model. However, if some countries are below their steady state paths they should be moving toward them and thus be growing at a faster rate than g. In that case, each country's growth rate at a given time depends on how far it is from its steady state.

Suppose, first that all countries have the same fundamentals and thus the same steady state growth path. Then according to the Solow model, they should be converging over time to the same growth path. The ones that start relatively far below this long run growth path (and thus have relatively low initial levels of output per worker y_0) would have faster transitional growth than those who start closer to the steady state growth path.

We can test for this kind of absolute convergence by testing whether $b < 0$ in the following regression:

$$\hat{y}_i = a + b y_{0i} + \epsilon_i$$

where the left hand side variable is the average growth rate of output per worker over some time period (starting in year 0) for country i, and y_{0i} is the level of output per worker in year 0 for country i.

This regression recognizes that different countries start in different places, but ignores the fact that their ultimate steady states may be different. Indeed, the Solow model predicts that the level of the steady state path depends on s and n, so there will be *conditional convergence* in standards of living – standards of living will converge for countries with similar fundamentals – not absolute convergence.[2]

Of two countries that start at the same initial level of output per worker, the one with the greater s and lower n is moving toward a higher steady state standard of living growth path and so should grow faster along the way. In general, the growth rate of a country at any time will depend on how far it is from its steady state, and this depends on both its current level of output per worker and the level of its steady state growth path.

MRW add s_{ki}, s_{hi} and n_i as explanatory variables in the regression equation above, and find that after adding these variables, the estimated sign of b changes from positive to negative, and that the fit improves considerably. See Tables III-V.

The regression equation for Table V, for example, is essentially:

$$\hat{y}_i = a + b \ln y_{0i} + c \ln s_{ki} + d \ln(\delta + n + g)_i + e \ln s_{hi} + \epsilon_i$$

[2] Solow does predict that there will be absolute convergence in growth rates – all countries' growth rates (of y) will eventually converge to g.

TABLE I
ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
ln(I/GDP)	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n+g+\delta)$	−1.97	−2.01	−0.76
	(0.56)	(0.53)	(0.84)
$\overline{R}^2$	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87	7.10	8.62
	(0.12)	(0.15)	(0.53)
$\ln(\text{I/GDP}) - \ln(n+g+\delta)$	1.48	1.43	0.56
	(0.12)	(0.14)	(0.36)
$\overline{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
p-value	0.38	0.26	0.79
Implied α	0.60	0.59	0.36
	(0.02)	(0.02)	(0.15)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g+\delta)$ is assumed to be 0.05.

Three aspects of the results support the Solow model. First, the coefficients on saving and population growth have the predicted signs and, for two of the three samples, are highly significant. Second, the restriction that the coefficients on $\ln(s)$ and $\ln(n+g+\delta)$ are equal in magnitude and opposite in sign is not rejected in any of the samples. Third, and perhaps most important, differences in saving and population growth account for a large fraction of the cross-country variation in income per capita. In the regression for the intermediate sample, for example, the adjusted R^2 is 0.59. In contrast to the common claim that the Solow model "explains" cross-country variation in labor productivity largely by appealing to variations in technologies, the two readily observable

about 0.03 or 0.04. In addition, growth in income per capita has averaged 1.7 percent per year in the United States and 2.2 percent per year in our intermediate sample; this suggests that g is about 0.02.

and I/GDP is 0.59 for the intermediate sample, and the correlation between SCHOOL and the population growth rate is −0.38. Thus, including human-capital accumulation could alter substantially the estimated impact of physical-capital accumulation and population growth on income per capita.

C. Results

Table II presents regressions of the log of income per capita on the log of the investment rate, the log of $n + g + \delta$, and the log of the percentage of the population in secondary school. The human-capital measure enters significantly in all three samples. It also

TABLE II
ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\bar{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\bar{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.32
Test of restriction:			
p-value	0.41	0.89	0.97
Implied α	0.31	0.29	0.14
	(0.04)	(0.05)	(0.15)
Implied β	0.28	0.30	0.37
	(0.03)	(0.04)	(0.12)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

TABLE III
TESTS FOR UNCONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	−0.266	0.587	3.69
	(0.380)	(0.433)	(0.68)
ln(Y60)	0.0943	−0.00423	−0.341
	(0.0496)	(0.05484)	(0.079)
$\overline{R}^2$	0.03	−0.01	0.46
s.e.e.	0.44	0.41	0.18
Implied λ	−0.00360	0.00017	0.0167
	(0.00219)	(0.00218)	(0.0023)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960.

essentially zero. There is no tendency for poor countries to grow faster on average than rich countries.

Table III does show, however, that there is a significant tendency toward convergence in the OECD sample. The coefficient on the initial level of income per capita is significantly negative, and the adjusted R^2 of the regression is 0.46. This result confirms the findings of Dowrick and Nguyen [1989], among others.

Table IV adds our measures of the rates of investment and population growth to the right-hand side of the regression. In all three samples the coefficient on the initial level of income is now significantly negative; that is, there is strong evidence of convergence. Moreover, the inclusion of investment and population growth rates improves substantially the fit of the regression. Table V adds our measure of human capital to the right-hand side of the regression in Table IV. This new variable further lowers the coefficient on the initial level of income, and it again improves the fit of the regression.

Figure I presents a graphical demonstration of the effect of adding measures of population growth and accumulation of human and physical capital to the usual "convergence picture," first presented by Romer [1987]. The top panel presents a scatterplot for our intermediate sample of the average annual growth rate of income per capita from 1960 to 1985 against the log of income per capita in 1960. Clearly, there is no evidence that countries that start off poor tend to grow faster. The second panel of the figure

TABLE IV
TESTS FOR CONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	1.93	2.23	2.19
	(0.83)	(0.86)	(1.17)
ln(Y60)	−0.141	−0.228	−0.351
	(0.052)	(0.057)	(0.066)
ln(I/GDP)	0.647	0.644	0.392
	(0.087)	(0.104)	(0.176)
$\ln(n + g + \delta)$	−0.299	−0.464	−0.753
	(0.304)	(0.307)	(0.341)
$\overline{R}^2$	0.38	0.35	0.62
s.e.e.	0.35	0.33	0.15
Implied λ	0.00606	0.0104	0.0173
	(0.00182)	(0.0019)	(0.0019)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

TABLE V
TESTS FOR CONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	3.04	3.69	2.81
	(0.83)	(0.91)	(1.19)
ln(Y60)	−0.289	−0.366	−0.398
	(0.062)	(0.067)	(0.070)
ln(I/GDP)	0.524	0.538	0.335
	(0.087)	(0.102)	(0.174)
$\ln(n + g + \delta)$	−0.505	−0.551	−0.844
	(0.288)	(0.288)	(0.334)
ln(SCHOOL)	0.233	0.271	0.223
	(0.060)	(0.081)	(0.144)
$\overline{R}^2$	0.46	0.43	0.65
s.e.e.	0.33	0.30	0.15
Implied λ	0.0137	0.0182	0.0203
	(0.0019)	(0.0020)	(0.0020)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

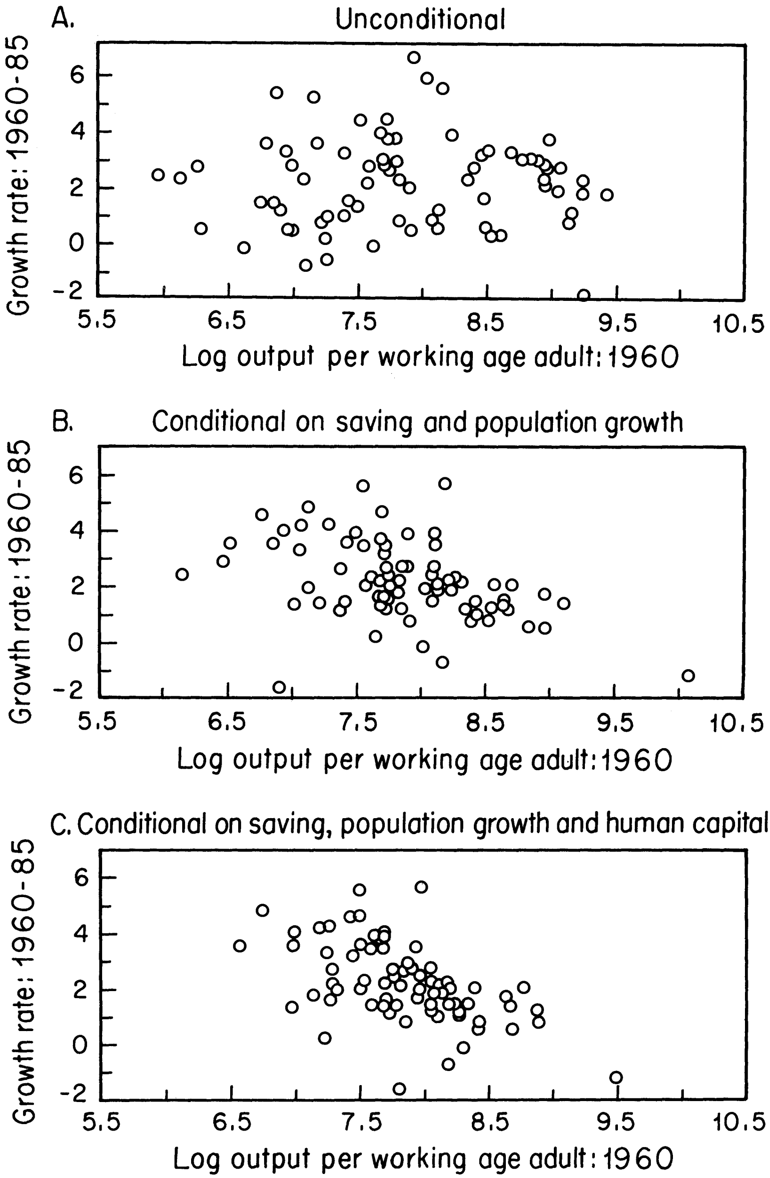

FIGURE I
Unconditional versus Conditional Convergence