

In the Solow Model, growth of output per capita in long run equilibrium is driven by technological progress. But this technological progress is itself not modeled - i.e., is considered to be exogenous. Endogenous growth models attempt to ground the engine of long run growth in choices made by economic actors within the model, such as rates of saving and population growth or investments in capital equipment, education, and R&D.

### **Non-Diminishing Returns to Capital**

The Solow model assumes that the economy exhibits diminishing returns to both labor and capital. A very simple endogenous growth model is simply the Solow model without diminishing returns to capital. I.e., suppose that we have the production function  $Y = AK^\alpha L^{1-\alpha}$ , but that we replace the Solow assumption of  $\alpha < 1$  with the assumption that  $\alpha = 1$ . Then even without any exogenous technological progress (i.e., even if  $A$  is constant over time), we have:

$$\dot{k}_t = s \cdot A k_t - (d + n) \cdot k_t$$

and so

$$\hat{k}_t = sA - (d + n)$$

Note then that in this new model, the growth rate of capital per worker  $k$  (and thus output per worker  $y$ ) is constant, is positive as long as  $sA > (n + \delta)$ , and depends on  $s$  and  $n$ . This is a radical departure from the predictions of the Solow model. Here growth can occur indefinitely through capital deepening even in the absence of technological progress, and the long run growth rate of an economy depends on behavioral parameters such as the saving rate in that economy.

Could there be such non-diminishing returns to capital at the economy wide level? One argument is that individual firms experience diminishing returns to capital, but the economy as a whole does not because individual firms benefit from the capital put in place by other firms. This *externality* might be due for example to the ability of firms to learn from each others' experiences in the production process ("learning by doing") or to the ability to benefit directly from their production (e.g., from the availability of better or lower cost inputs). The empirical evidence for non-diminishing returns has been mixed with a number of studies finding diminishing returns at the aggregate level for the U.S. economy.

### **Endogenous Technological Progress**

It could be there are diminishing returns to capital ( $\alpha < 1$ ), as in the Solow model, but that the path of total factor productivity  $A_t$  is endogenous. For example technological progress ( $\dot{A}_t$ ) may depend on R&D expenditures made by private businesses, universities, and the government. Similarly, in developing countries, private and public investments and government policies may influence the degree to which technology developed in other countries diffuses into these economies.

Then, for example, we might have two equations of motion: one for  $\dot{k}$  (which can be the same as in the Solow model) and one for  $\dot{A}$  (where we have now replaced the exogenous growth rate of  $A$  in the Solow model with an endogenous path of technological progress).

### **Endogenous Constraints on Productivity and Growth**

The Solow model assumes that all saving is invested productively, all capital equipment is used productively, and technological progress takes place exogenously. In practice, geography, institutions, cultural norms, and government policies might determine the level and efficiency of investment. For example a well functioning legal system and well developed domestic financial markets may induce both a high rate of investment and a relatively efficient allocation of capital to different uses. An optimal degree of openness to trade and capital flows may facilitate specialization as well as the diffusion of technologies developed in other countries. Thus either the level or growth rate of total factor productivity  $A$  may be influenced by these factors.