

Application: The Cagan Model of Inflation

The question that we will raise here is whether the general price level is primarily a monetary phenomenon. Particularly, should we expect, on theoretical grounds, that price inflation is tied to the rate of growth of the money supply, or rather that sustained inflation might arise even in the absence of money growth.

Note that the quantity equation, $M \times V = P \times Y$ implies that $\widehat{M} + \widehat{V} = \widehat{P} + \widehat{Y}$, so that if both V and Y are constant, the rate of price inflation (\widehat{P}) must just be equal to the rate of growth of the money supply (\widehat{M}). If, on the other hand, velocity is rising over time ($\widehat{V} > 0$), then the inflation rate will be greater than the money growth rate. In other words, if the public is willing to turn over the same amount of money faster (make larger dollar purchases per year with the same stock of money), then prices can rise independently of the money stock.

We can use a very simple model of inflation due to Cagan to examine this issue.

Cagan's model is intuitively very simple. It models the general price level as being determined by the intersection of the demand and supply of money. This equilibrium condition is given below by equation (3a). That is the equation which we will use to describe the behavior of the price level. However, first I will digress by deriving this condition.

Deriving the Equilibrium Condition (3a)

Cagan supposed that the demand for real money balances MD_t at time t was a negative function of the expected rate of price inflation for the following period $\pi_{t+1}^{e(t)}$. He chose a functional form that would make the analysis relatively simple:

$$(1) \quad MD_t = e^{-\gamma\pi_{t+1}^{e(t)}}$$

If expected inflation is zero, then money demand is one (the public desires to hold a quantity of nominal money balances M equal to the price level). If the public expects inflation to be positive, then they expect that the real value of their money holdings will be eroded, and so will desire to hold less money: MD will be less than one. γ , a positive constant, is the elasticity of money demand with respect to expected inflation.

Expected inflation $\pi_{t+1}^{e(t)}$ is defined as the expected rate of growth of the general price level P from period t to $t + 1$, looking from time t .

$$(2) \quad \pi_{t+1}^e = \frac{P_{t+1}^{e(t)} - P_t}{P_t}$$

We assume that the central bank controls the level of nominal money balances at any time, so that M_t is exogenous to the model. The real money supply is M_t/P_t .

The money market is in equilibrium when money supply is equal to money demand:

$$(3) \quad \frac{M_t}{P_t} = e^{-\gamma\pi_{t+1}^{e(t)}}$$

Mathematically, this equation says that if expected inflation $\pi_{t+1}^{e(t)}$ rises, *ceterus paribus*, money demand falls, driving the price level up. A simple story to go along with this would be that as expected inflation rises, the public reduces their money balances by buying goods, driving the general price level up.

The equilibrium condition (3) is a non-linear equation relating price today to expected price tomorrow. We can transform this into a linear equation (which is easier to work with) by taking logs of both sides.

$$\ln M_t - \ln P_t = -\gamma\pi_{t+1}^{e(t)}$$

and using the approximation that

$$\begin{aligned}\pi_{t+1}^{e(t)} &\approx \ln(1 + \pi_{t+1}^{e(t)}) \\ &= \ln \frac{P_{t+1}^{e(t)}}{P_t} \\ &= \ln P_{t+1}^{e(t)} - \ln P_t\end{aligned}$$

the equilibrium condition is

$$\ln M_t - \ln P_t = -\gamma(\ln P_{t+1}^{e(t)} - \ln P_t)$$

Finally, denoting logs in lower case: e.g., $m_t \equiv \ln M_t$, this can be written

$$m_t - p_t = -\gamma(p_{t+1}^{e(t)} - p_t)$$

where m , p , and p^e are the logs of the nominal money supply and general price level, and the expected log price level respectively.

Let's make one final rearrangement so that p_t stands alone on the right hand side.

$$(3a) \quad p_t = \frac{1}{1+\gamma}m_t + \frac{\gamma}{1+\gamma}p_{t+1}^{e(t)}$$

Equation (3a) is the equilibrium condition that we will use to describe the behavior of prices in this model. It says that price today (p_t) is a weighted average of the current money supply (m_t) and the public's expectation today of what the price level will be tomorrow ($p_{t+1}^{e(t)}$).

Please note that (3a) is just (3) rewritten: i.e., it still just expresses the statement that money demand equals money supply. As before, it says the equilibrium price level today depends on what the public expects the price level to be tomorrow. A rise in the expectation of tomorrow's price is an increase in expected inflation; this causes the public to reduce their money holdings today, which in turn drives up the price level today.

Solving the Model For Equilibrium Prices Under Rational Expectations

Since, according to the equilibrium condition (3a), the equilibrium price level today depends on expectations of tomorrow's price level, we must determine how expectations are formed in order to say anything more. Note that these expectations are being made by the economic actors (the public) in the model. Specifically, in order to decide how much money to hold today, each actor needs to form an expectation of tomorrow's price $p_{t+1}^{e(t)}$. Let's assume that these expectations are *rational expectations*: $p_{t+1}^{e(t)} = \mathbf{E}_t p_{t+1}$. Then (3a) is

$$(3b) \quad p_t = \frac{1}{1+\gamma}m_t + \frac{\gamma}{1+\gamma}\mathbf{E}_t p_{t+1}$$

Further, let us suppose that actors in the economy know the model: i.e., they know that the price level obeys (3b). This is an extreme assumption.¹ However, it is often employed in rational expectations model building.

If the average actor in the economy knows that the price level at *any time* t follows (3b), then she can make her forecasts of future prices *based on* (3b). From (3b) she knows that the price level tomorrow will actually be

$$p_{t+1} = \frac{1}{1+\gamma}m_{t+1} + \frac{\gamma}{1+\gamma}\mathbf{E}_{t+1}p_{t+2}$$

¹ If, in reality, economists don't know what the correct model of the economy is, why should the public? In the assumption's defense, it is a simple way of ensuring that expectations are not systematically incorrect in the modeling exercise.

Thus, her rational expectation of this price level must be

$$\mathbf{E}_t p_{t+1} = \frac{1}{1+\gamma} \mathbf{E}_t m_{t+1} + \frac{\gamma}{1+\gamma} \mathbf{E}_t [\mathbf{E}_{t+1} p_{t+2}]$$

which is²

$$(4) \quad \mathbf{E}_t p_{t+1} = \frac{1}{1+\gamma} \mathbf{E}_t m_{t+1} + \frac{\gamma}{1+\gamma} \mathbf{E}_t p_{t+2}$$

Similarly, $\mathbf{E}_t p_{t+2}$ can be based on what is known about the actual behavior of p_{t+2} according to (3b)

$$p_{t+2} = \frac{1}{1+\gamma} m_{t+2} + \frac{\gamma}{1+\gamma} \mathbf{E}_{t+2} p_{t+3}$$

so

$$(5) \quad \mathbf{E}_t p_{t+2} = \frac{1}{1+\gamma} \mathbf{E}_t m_{t+2} + \frac{\gamma}{1+\gamma} \mathbf{E}_t p_{t+3}$$

Now if we continue for $\mathbf{E}_t p_{t+3} \dots$ etc., and substitute (4) into (3b) and then (5) into the resulting equation, and so forth, we get

$$p_t = \frac{1}{1+\gamma} m_t + \frac{1}{1+\gamma} \frac{\gamma}{1+\gamma} \mathbf{E}_t m_{t+1} + \frac{1}{1+\gamma} \left(\frac{\gamma}{1+\gamma}\right)^2 \mathbf{E}_t m_{t+2} + \frac{1}{1+\gamma} \left(\frac{\gamma}{1+\gamma}\right)^3 \mathbf{E}_t m_{t+3} + \dots$$

which can be written

$$(6) \quad p_t = \frac{1}{1+\gamma} \sum_{i=0}^{\infty} \left(\frac{\gamma}{1+\gamma}\right)^i \mathbf{E}_t m_{t+i}$$

with $\mathbf{E}_t m_t$ taken to be m_t .³

This says that p_t is a weighted average of current and future expected nominal money stocks.⁴ More weight is placed on near future than far future money stocks.⁵

Thus, the rational expectations of future price levels resolve into rational expectations of future nominal money stocks. Consequently, so does the current equilibrium price level. So according to this solution, inflation is monetary in that it is based on (rational) expectations of future monetary policy.

Case 1

Suppose that the public believes correctly that the central bank is committed to keeping the nominal stock of money in the economy constant, so that $m_t = \bar{m} \quad \forall t$.

² My best guess today about what my expectation will be tomorrow about the following period's price must just be my best guess today about that price: $\mathbf{E}_t[\mathbf{E}_{t+1} p_{t+2}] = \mathbf{E}_t p_{t+2}$. If this sounds confusing, it is. But you might think about the coin toss in the last handout. Before the first coin is tossed, what is your best guess as to what your best guess of Z will be after you discover the value of the first toss? Since you don't yet know what the value of the first toss is, it is just your current best guess of Z, which is \$1.0.

³ Actually, the current value of the nominal money supply enters directly, but we cheat notationally and treat it as though it enters as an expectation but that the expectation is actually correct: i.e., we observe the m_t at t .

⁴ It is easily verified that the weights $\frac{1}{1+\gamma} \left(\frac{\gamma}{1+\gamma}\right)^i$ sum to one.

⁵ $\left(\frac{\gamma}{1+\gamma}\right)^i$ falls as i increases.

Then according to (6), the equilibrium price level obeys

$$\begin{aligned}
 p_t &= \frac{1}{1+\gamma} \sum_{i=0}^{\infty} \left(\frac{\gamma}{1+\gamma}\right)^i \bar{m} \\
 &= \frac{1}{1+\gamma} \bar{m} \sum_{i=0}^{\infty} \left(\frac{\gamma}{1+\gamma}\right)^i \\
 &= \frac{1}{1+\gamma} \bar{m} \frac{1}{1-\frac{\gamma}{1+\gamma}} \\
 &= \frac{1}{1+\gamma} \bar{m} (1+\gamma) \\
 &= \bar{m}
 \end{aligned}$$

The price level will be, at any time t , exactly \bar{m} . Thus, the price level is constant over time, and there is no inflation.

Note that we could see this answer without slogging through the algebra by observing that a weighted average of a constant sequence of \bar{m} s is just \bar{m} .

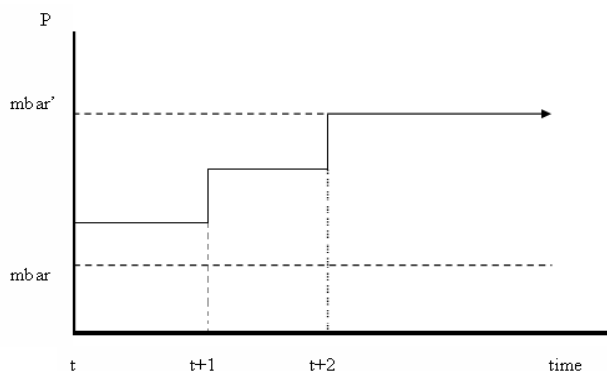
Case 2

Suppose that the public's expectations change. It now discovers that the central bank plans to raise the level of the money stock from \bar{m} to \bar{m}' in two years.

Then according to (6), the equilibrium price level obeys

$$p_t = \frac{1}{1+\gamma} \left[\bar{m} + \frac{\gamma}{1+\gamma} \bar{m} + \left(\frac{\gamma}{1+\gamma}\right)^2 \bar{m}' + \left(\frac{\gamma}{1+\gamma}\right)^3 \bar{m}' + \dots \right]$$

The price level today is a weighted average of current and future expected money stock level, with larger weights placed on money levels in the near future than in the far future. Thus, the price level will jump up this year (time t), but not all the way to \bar{m}' . It will then rise further toward \bar{m}' next year (p_{t+1}), and reach \bar{m}' in the second year (p_{t+2}). It will then stay at \bar{m}' in all future years (as long as the central bank actually sticks to its plan and expectations do not change).



Why do prices rise in anticipation of future increases in money? The public correctly anticipates that in two years the price level will be \bar{m}' . Thus, there will be inflation from $t+1$ to $t+2$. This will drive down money demand in $t+1$, and in turn drive up prices in $t+1$. Again, knowing the model, the public correctly anticipates this inflation from t to $t+1$ and reduces its money demand at t (today). Thus, price rises part way today.

Note that, to calculate the new equilibrium values of p_{t+1} and p_t we can solve backwards from $t+2$ using (3b).

Case 3

Suppose now that the public correctly expects that the central bank will inject new money into the economy at a constant annual rate θ , forever.

Note first that since m and p are in logs, that the actual inflation rate π_{t+1} is $p_{t+1} - p_t$ and the rate of growth of the nominal money stock, which we know to be θ , is $m_{t+1} - m_t$.

From (6) the actual price level in t and $t + 1$ follow

$$p_t = \frac{1}{1 + \gamma} \sum_{i=0}^{\infty} \left(\frac{\gamma}{1 + \gamma}\right)^i \mathbf{E}_t m_{t+i}$$

$$p_{t+1} = \frac{1}{1 + \gamma} \sum_{i=0}^{\infty} \left(\frac{\gamma}{1 + \gamma}\right)^i \mathbf{E}_{t+1} m_{t+1+i}$$

so that the inflation rate is

$$\begin{aligned} p_{t+1} - p_t &= \frac{1}{1 + \gamma} \sum_{i=0}^{\infty} \left(\frac{\gamma}{1 + \gamma}\right)^i (\mathbf{E}_{t+1} m_{t+1+i} - \mathbf{E}_t m_{t+i}) \\ &= \frac{1}{1 + \gamma} \sum_{i=0}^{\infty} \left(\frac{\gamma}{1 + \gamma}\right)^i \theta \\ &= \theta \end{aligned}$$

I.e., the rate of inflation is just the rate of growth of the money supply.

Case 4

Suppose finally that the public does not know what future nominal money stocks will be precisely, but that they do know the stochastic rule that the nominal money stock follows. Specifically, suppose that the public correctly believes that the log of the nominal money supply follows the process

$$m_t = \bar{m} + \epsilon_t$$

where ϵ_t for any year t is a random variable with zero mean. This says that at time t there is a surprise shock ϵ_t to the money supply, which pushes m_t away from \bar{m} . The effect of the shock lasts only one period, after which the money supply in the next period is \bar{m} plus the value of *that* period's shock ϵ_{t+1} (on average this value is zero). Intuitively, this might be the case if the central bank's goal is to keep the nominal money supply fixed at \bar{m} , but there are shocks to the money supply which the central bank can only neutralize with a one period lag.

By (6), the price level is a weighted average of current money and future expected money. From above, actual money in a future period $t + i$ is $m_{t+i} = \bar{m} + \epsilon_{t+i}$. The rational expectation at time t of this level for $i > 0$ is

$$\begin{aligned} \mathbf{E}_t m_{t+i} &= \mathbf{E}_t [\bar{m} + \epsilon_{t+i}] \\ &= \mathbf{E}_t \bar{m} + \mathbf{E}_t \epsilon_{t+i} \\ &= \bar{m} + 0 \\ &= \bar{m} \end{aligned}$$

Since the shocks are zero on average, our best guess about future money stocks is that they will be \bar{m} . For $i = 0$, the expectation is the actual value m_t (recall our convention when we defined (6)). Thus, all of the expected money terms in (6) are just \bar{m} except for the first term ($i = 0$) which is $\bar{m} + \epsilon_t$. Thus, the price level at t will be a weighted average of \bar{m} and $\bar{m} + \epsilon_t$.

Pull the ϵ_t out of the sum, which leaves us with

$$\begin{aligned} p_t &= \frac{1}{1 + \gamma} \epsilon_t + \frac{1}{1 + \gamma} \sum_{i=0}^{\infty} \left(\frac{\gamma}{1 + \gamma}\right)^i \bar{m} \\ &= \frac{1}{1 + \gamma} \epsilon_t + \bar{m} \end{aligned}$$

Thus the equilibrium price level follows the stochastic process

$$p_t = \bar{m} + \frac{1}{1+\gamma}\epsilon_t$$

The shocks to the current money supply, which are temporary, cause temporary shocks to the current price level. The magnitude of the effect is tempered by the belief on the part of the public that the shocks to the money supply are only temporary. Consequently, $\frac{dp_t}{d\epsilon_t} = \frac{1}{1+\gamma} < 1$. Intuitively, the price level today does not increase by the full size of the shock because the public expects the increase in the price level today to be followed by a return of the price level to \bar{m} in the next period, i.e., for $\pi_{t+1} < 0$.

If, for example, $\gamma = 3$, then a positive temporary shock to the money supply of \$1 will temporarily raise the current price level above \bar{m} by 25 cents.

Notice that if the central bank holds its target \bar{m} constant, then it will observe on the basis of past data, that $\frac{dp_t}{dm_t}$ appears to be $\frac{1}{1+\gamma} < 1$. However, it would be a mistake for the central bank to interpret this as predicting that if it were to raise \bar{m} by \$1, the long run impact on the price level would be less than \$1.⁶ What it has actually observed in the data is $\frac{dp}{d\epsilon}$, whereas $\frac{dp}{d\bar{m}} = 1$. As soon as the public catches on that \bar{m} is higher, its expectations will change. We will be back in case 2, and the central bank will be surprised to discover that $\frac{dp}{d\bar{m}} = 1$ for *this* change in m .

This observation is an example of the *Lucas Critique* of econometric policy evaluation: a change in policy, will change not only the policy variables but also the public's expectations (at least once it is discovered by the public); consequently, central banks need to account for this when they use historical data (econometrics) to predict the effect of a policy change. In the example above, simply regressing p on m would be misleading.

Rational Bubbles

The results so far seem to indicate that equilibrium price levels are tied closely to the nominal money supply. Specifically, by (6), we see that prices are strictly tied to *expected future* nominal supplies of money. To the degree to which expectations are accurate, prices follow actual money supply closely, as indicated in the examples above. By (6), as long as expectations are not systematically incorrect, it would appear, for example, that we can not get *persistent* inflation occurring spontaneously in the absence of money growth.

There is a caveat, however. (6) is indeed *a* solution to (3b). We can call it the *fundamental solution*. However, it is *not* the only solution. It turns out that there are other solutions to (3b), which we will call *bubble solutions*. These are rational expectations solutions to the model under which inflation arises spontaneously.

To see that this is at least plausible, suppose that expected inflation rises, even though money growth is expected to remain at zero (as in case 1 above). Then money demand will fall, driving the price level up. In other words, the expectations of inflation are at least partly self fulfilling. The expectations would actually be *rational* (i.e., consistent with rational expectations) *if* they are *fully* self fulfilling (at least on average).

Rational Bubble Solutions in the Cagan Model (optional)

To see how rational bubbles might arise (at least in theory), consider the simplest (and most extreme) case of rational expectations. Suppose that economic agents' expectations are actually correct: i.e., that they have *perfect foresight*.

Then (3b) is

$$p_t = \frac{1}{1+\gamma}m_t + \frac{\gamma}{1+\gamma}p_{t+1}$$

⁶ E.g., it would be misleading to think that a permanent change in the money supply will be expansionary once the public understands that it has happened. Once the public has caught on, the increase in AD from the increase in m will be completely offset by an increase in the price level, leaving the level of real spending unchanged.

Now rearrange so that p_{t+1} is on the left hand side.

$$p_{t+1} = -\frac{1}{\gamma}m_t + \frac{1+\gamma}{\gamma}p_t$$

Further suppose that the central bank holds m constant at \bar{m} over time (as in case 1 above).

$$(7) \quad p_{t+1} = -\frac{1}{\gamma}\bar{m} + \frac{1+\gamma}{\gamma}p_t$$

(7) is a simple (constant coefficients) linear difference equation in the price level. We can first solve for a *steady state* equilibrium (p^*) for which the price is constant ($p_{t+1} = p_t$). If the price level starts out at p^* , it will stay there forever. For (7), this solution is

$$p^* = \bar{m}$$

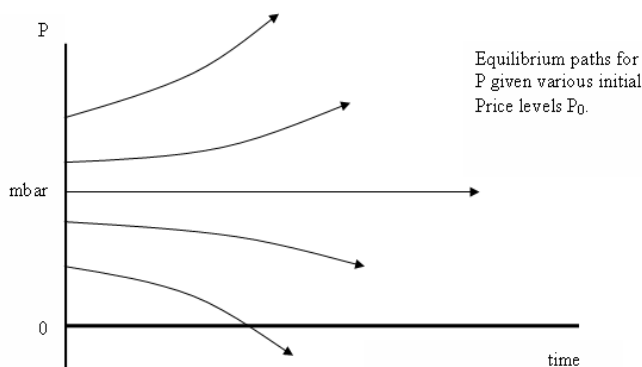
This is the fundamental solution that we found above using (6). However, what happens to the price level if it starts away from p^* ?

Suppose that we know the value p_0 of p at time 0. We can then use (7) to calculate p_1 . We can then use this value of p_1 in (7) to find p_2 . Iterating out t periods in this manner yields a price for period t of

$$(8) \quad p_t = p^* + \left(\frac{1+\gamma}{\gamma}\right)^t(p_0 - p^*)$$

I show how to get this solution in Appendix I at the end of this handout.⁷ Notice that $\left(\frac{1+\gamma}{\gamma}\right) > 1$. Consequently $\left(\frac{1+\gamma}{\gamma}\right)^t$ becomes large over time. Thus, if price starts away from p^* , it will grow further away from p^* over time.

Consequently, for any initial price level, there is a corresponding rational expectations equilibrium. All but one of them (the steady state equilibrium, which is the fundamental solution) is a bubble solution to the model: price diverges spontaneously over time from the level of the money stock, driven by speculative expectations rather than by money growth.



In these rational expectations equilibria, the price *level* is growing at an increasing rate,⁸ so that the expected inflation rate is rising over time at a rate which drives money demand down at just a sufficient rate over time to cause the price level to actually rise at the expected rate. Thus the inflationary expectations are self fulfilling.

⁷ In Appendix I, I show that if the coefficient on p_t in (7) is greater than one, which it is in this case, then price paths starting away from the steady state equilibrium will diverge further over time from the steady state. Thus, the steady state equilibrium is *unstable*. On the other hand, if the coefficient lay between 0 and 1, then the price would converge to p^* over time, regardless of where it starts out. Thus, the steady state equilibrium in that case would be *stable*. An example of a difference equation similar to (7) (but nonlinear) for which the steady state is stable is the Solow growth model equation.

⁸ Recall that a constant growth rate would show on the diagram as a *linear* path for p (the log of the price level).

Objections

Among possible objections to these bubble solutions are the following. First, how will a large number of independent economic actors coordinate on a particular initial value of the price level other than $p = m$. Of course the same criticism can be levied at the fundamental solution $p = m$. Second, why should anyone expect that the rate of inflation can rise indefinitely in the absence of money growth? Once a bubble starts, above, it must continue on forever. If the public does not believe that the inflation rate can grow forever, then the only solution to (7) left is the fundamental solution.

However, an alternative to believing that a bubble *will* last forever is to expect any bubble to burst eventually, but not to know with certainty when it will end. The public might, for example, collectively coordinate the timing of bubbles with the outcomes of otherwise irrelevant events. Such equilibria are called *sunspot equilibria*. Again, sceptics would argue that these rational bubbles are unreasonable in that they require the public to believe that the bubble *could* continue for a long time (and to act on this belief).

Some sceptics of *rational* bubbles therefore prefer the fundamental rational expectations solution, whereas others argue that bubbles *are* indeed relevant, but that they are grounded in irrational or boundedly rational behavior (e.g., some people blindly follow the trend, or believe that the bubble will burst but that they will get out first leaving others to hold the bag).

For more on rational bubbles, see Appendix 2.

Summary: The Causes of Inflation

The Cagan model was designed to explain extreme inflationary episodes such as the hyperinflations in Germany in the 1920s and Argentina in the 1980s and the deflation in the U.S. during the great depression of the 1930s. The model could also potentially explain more moderate changes in inflation such as the run-up of inflation in the U.S. in the 1970s.

According to the fundamental solution of the Cagan model under rational expectations, inflation is a monetary phenomenon, driven entirely by the current money supply and expectations about future money supplies. Hyperinflations are caused by the central bank printing money at an increasing rate (or at least being expected to by rational observers) and deflations are similarly caused by anticipated contractions of the money supply.

If we accept the plausibility of bubble solutions to the model, it is also possible for inflation to be driven in the Calvo model by self fulfilling expectations of inflation, purely independently of the money supply and expectations about the money supply. However, many macroeconomists are sceptical that hyperinflation or disinflation could be sustained for extended periods of time purely on the basis of self fulfilling expectations.

Bounded Rationality and Learning

While rational expectations is a very strong modeling assumption, it is popular in part because it is a straightforward way to close a model that has expectations in it. It is also considered to be an important modeling benchmark (how does the model behave if agents in it have rational expectations).

There may also be a back-door rationale for using rational expectations in macroeconomic modeling. Suppose that in reality, many economic agents have fairly naive expectations, but learn over time from their mistakes. Then this learning may cause expectations to converge to rational expectations over time.

To see this, let's consider the Calvo model. Consider case 1 above in which the money supply is \bar{m} in each period. Suppose that a typical person in the economy forms the naive expectation that the price level in the future will be constant at some level \bar{P} . Then, according to (3b), the actual price level today will be a weighted average of \bar{m} and \bar{P} , and the price will stay at this level as long as the public retains its naive expectation.

Notice then that if $\bar{P} > \bar{m}$, then the *actual* price level will be less than the public's *expected* level ($p_t \in (\bar{m}, \bar{P})$). Similarly, if $\bar{P} < \bar{m}$, then the actual price level will be greater than the public's expected level

($p_t \in (\bar{P}, \bar{m})$). So if, over time, the public reacts to its mistakes, it will push its expectation \bar{P} toward \bar{m} over time. But this is exactly the rational expectations equilibrium. I.e., if the public expects the price level to be \bar{m} each period, the price level will be \bar{m} each period, and so this expectation will turn out to be correct.

So we see that in this very simple case, learning should eventually drive expectations to be rational. Indeed, in this example, learning selects the *fundamental* rational expectations equilibrium (not bubble bubble solutions to the model). So at least in this very simple example, learning causes inflation to become a purely monetary phenomenon (eventually) and does not lead to arbitrary self fulfilling inflationary expectations.

Before we get too excited about this result however, note that for expectations to respond rationally in a changing environment (such as case 2), the learning process would have to be substantially more sophisticated. Some economists have argued that we should expect this to be the case, while others remain sceptical.

One More Word on Inflation

Note that we can think of the Cagan model as grounding inflation in the demand side of the economy. If the public expects prices to rise in the future, then they reduce their money demand today, driving up the price level today. As noted above, we can think of them deciding to buy goods rather than hold money in anticipation that the price of goods will rise (though the Cagan model does not model this spending mechanism explicitly). Thus, Calvo's model can be thought of (at least roughly) as corresponding to an AS-AD model in which expected inflation shifts the AD curve.

We could alternatively consider the influence of expected inflation on the supply side of the economy. For example, the standard Phillips curve price adjustment model of AS models inflation as depending on expected inflation and the expected output gap. Note that, in that model, an increase in expected inflation (on the part of firms) causes inflation to increase today (as firms raise their own prices in response to the expectation that other prices will rise). The resulting increases in the general price level reduce real spending. Thus, the effect on real spending in the economy from this AS channel (expected inflation shifting AS) is the opposite of the effect through the AD channel. An increase in expected inflation will cause both higher inflation and lower real activity, i.e., stagflation. Consequently, if the central bank wanted to avoid an economic slowdown or recession, it would have to print more money, validating the inflationary expectations.

Appendix I (optional)

Digression on Solutions to Difference Equations

Consider a first order linear difference equation in a variable x with constant coefficients a and b :

$$(9) \quad x_{t+1} = a + b x_t$$

Here are some useful results concerning the behavior of x under (9). The steady state value of x is $x^* = \frac{a}{1-b}$. If $0 < b < 1$, then x converges monotonically to x^* from any starting value. If $b > 1$, then x diverges from x^* monotonically at rate $b - 1$, if it starts away from x^* .⁹ To solve for the path of x , either plug in a known initial value of x on the right side of (9), and solve forward iteratively, or use the general solution which can be calculated as follows. Calculate the value of x^* . Subtracting x^* from both sides of (9) gives

$$(x_{t+1} - x^*) = b(x_t - x^*)$$

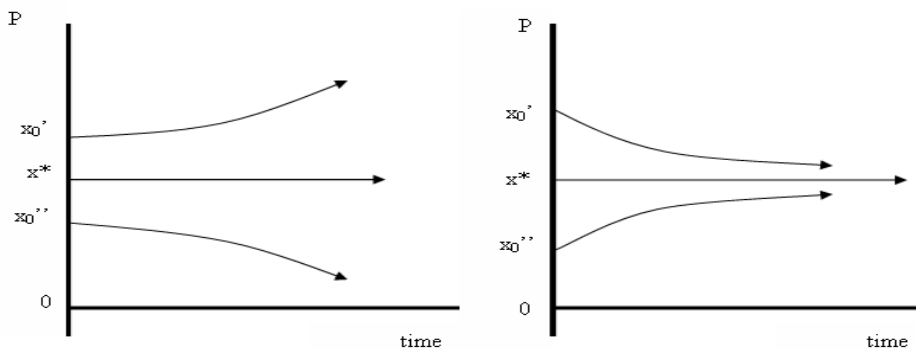
Iterating this forward from some time zero implies that

$$(x_t - x^*) = b^t(x_0 - x^*)$$

or

$$x_t = x^* + b^t(x_0 - x^*)$$

This formula gives the time t value of x for *any* initial value x_0 and any value of b . Again, we can see that if $|b| < 1$, x converges to x^* over time, since b^t gets smaller over time, and if $b > 1$, the gap between x and x^* grows at constant growth rate $(b - 1)$.



A very simple example might be useful. Suppose that $a = 0$, so that the difference equation that we want to solve is

$$x_{t+1} = b x_t$$

Then the steady state equilibrium is $x^* = 0$. Suppose, however, that x starts at time 0 at $x_0 = 1$. First let $b = 2$. Then $x_1 = 2$, $x_2 = 4$, $x_3 = 8$, etc. $x - x^*$ grows at rate $b - 1 = 1$ (100% per period). Now let $b = 0.5$. Then $x_1 = 0.5$, $x_2 = 0.25$, $x_3 = 0.125$, etc. x converges on x^* over time.

In (7), since the coefficient on p_t is greater than one, p^* is an unstable steady state equilibrium. If p_0 starts either above or below p^* it will grow further away from p^* over time.

⁹ If $-1 < b < 0$, x also converges to x^* , but oscillates, and for $b < -1$ x undergoes explosive oscillations away from x^* .

Appendix II (optional)

Digression on Rational Bubbles in Asset Markets

Recall that the EMT predicts that the price of a stock at time t should be

$$(10) \quad \begin{aligned} P_t &= \mathbf{E}_t PV_t \\ &= \sum_{i=1}^{\infty} \left(\frac{1}{1+R}\right)^i \mathbf{E}_t d_{t+i} \end{aligned}$$

This fundamental solution, however, is again only one of a large set of possible solutions to the arbitrage condition from which it is derived. The other solutions are rational bubble solutions.

The expected one period holding return to a stock from t to $t+1$ is the expected dividend plus the expected capital gain as a percentage of the purchase price.

$$h_t = \frac{\mathbf{E}_t d_{t+1} + (\mathbf{E}_t P_{t+1} - P_t)}{P_t}$$

If other securities in a similar risk class yield an expected rate of return R , then arbitrage would require that $h_t = R$. Rewriting,

$$(11) \quad P_t = \frac{1}{1+R} \mathbf{E}_t d_{t+1} + \frac{1}{1+R} \mathbf{E}_t P_{t+1}$$

We can solve this, just as we did (3b) above to get (10). However, this is only one solution to (11). To see this, we follow the analysis above by considering (11) under a very simple case. Specifically, simplify things by assuming perfect foresight and setting the actual and expected dividend stream constant over time. Then, rewriting (11) so that P_{t+1} is on the left hand side, we have

$$(12) \quad P_{t+1} = -\bar{d} + (1+R)P_t$$

The steady state equilibrium is $P^* = \frac{\bar{d}}{R}$, the fundamental solution. Since the coefficient on P_t is $(1+R)$ which is greater than one, the steady state solution is unstable. If the price starts at $\frac{\bar{d}}{R}$ it will stay there forever. If it starts above this value, however, the price level will follow a rational speculative bubble. The price level will diverge from P^* over time at rate R .

The general solution to (12) is

$$P_t = P^* + (1+R)^t (P_0 - P^*)$$

Suppose for example that you expect to get a dividend of \$100 per year forever on this stock ($\bar{d} = 100$). Further, you believe that you can get a rate of return of 10% per year ($R = 0.1$) on other comparable securities. Then it makes intuitive sense that you are willing to pay up to \$1000 for the stock. That price gives you a rate of return from the dividend of just 10%, leaving you indifferent between the stock and other securities. This is indeed the fundamental value of the stock at which the EMT predicts that it will be priced.

Under what condition would you pay \$2000 for the stock? If you expected the price to rise by \$100 during the period, then this expected capital gain plus the \$100 dividend would yield the required holding return of 10%. Therefore, $P_t = \$2000$ is a rational expectations equilibrium if $P_{t+1} = \$2100$, and the price continues to grow away from \$1000 at 10% per year in the future.

Similarly, suppose that you thought that the price may rise next period or the bubble might burst. Suppose for example that you believe that with probability 0.2 the price will fall back to \$1000 next period

(the bubble will burst), and with probability 0.8 the price will rise next period (the bubble will continue). Then, it is rational to pay \$2000 for the stock today, if you believe that, *if* the bubble does not burst tomorrow, the price tomorrow will be \$2375. In that case, you believe that there is an 80% chance of getting a \$375 capital gain, and a 20% chance of a \$1000 capital loss, in addition to the dividend of \$100, and your expected rate of return on the stock is exactly 10%.

So 'rational' bubbles are theoretically possible. Again, whether they are plausible or empirically relevant is a matter of some controversy.