Macroeconomic Gentrification

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Abstract

This paper explores the macroeconomic relationship between product innovation and consumption inequality. A rise in rents accruing to high income consumers can shift both production and R&D spending toward products targeted to that class of consumer. The increase in income inequality will then have a magnified impact on consumption inequality through induced changes in product quality and availability. We label this process *macroeconomic gentrification*. We extend the agent-based macroeconomic model in Georges (2011, 2018) and document three channels that contribute to this process: an income channel, a variety channel, and an innovation channel. We further find that the relationship in the model between the distribution of income and long run economic dynamism can be highly nonlinear.

1 Introduction

There has been substantial recent interest in the causes and consequences of rising income inequality in the U.S. and some other advanced capitalist economies in the past 40 years. There is also evidence, reviewed in the next section, that rising income inequality has gone hand in hand with rising consumption inequality, not simply in consumption levels, but also in the composition of consumption goods and services.

That a variety of consumers with high incomes or rising income expectations, ranging from young professionals to the ultra rich, demand different goods and qualities of goods from consumers with lower incomes and different class locations accords with both academic research and casual observation. Examples include financial services, luxury apartments, fitness trainers, after school programs, private schools and day care, espresso drinks, high end restaurants and boutique shops, delivery services, private entertainment, private tours, exclusive treatment on cruises and in entertainment venues, nannies, art galleries, and limo services.[1] In gentrifying neighborhoods, the entry or quality upgrading of these goods and services displaces goods and services ranging from repair shops, to laundromats, affordable apartments, diners, basic grocery and variety stores, and so forth.[2]

In this paper, we propose that a similar process can

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[1]For one example, the current and planned development in Manhattan around the locations of the new Whitney Museum, High Line park, and Hudson Yards is transforming these formerly low-rent neighborhoods. See the promotional website for the Hudson Yards project http://livehudsonyards.com/ and the Jeremiah's Vanishing New York blog entry on that project http://vanishingnewyork.blogspot.com/2015/02/hudson-yards-effect.html. For another example see "In an Age of Privilege, Not Everyone Is in the Same Boat." *New York Times*, April 23, 2016.

[2]The provision of public services, such as public transportation, housing, schools, parks, and health care is a closely related issue that we will not address here.

operate at the level of the macroeconomy. We consider a model in which rents accruing to high income workers may induce innovation in the high income consumption goods sector, to the benefit of those workers, while also potentially reducing the welfare of lower income workers by reducing their income shares and the variety and quality of goods that cater to them. We label this process *macroeconomic gentrification.*

We build off of the agent based macroeconomic model in Georges (2011, 2018) in which both growth and business cycle dynamics are grounded in product innovation. The hedonic approach to the product space there provides a simple and flexible way to characterize the ongoing emergence of new and improved products through product innovation. That innovation is driven by R&D investments that are also a driver of aggregate demand in the model. The model is capable of producing complex disequilibrium dynamics due to the ongoing churning of firms and product shares and the tension between the near term and medium term effects of R&D investments on individual firms' profits.

In the present paper we develop an extension of that model in which production workers and overhead workers consume different baskets of goods. We document three channels that contribute to macroeconomic gentrification in this model: an income channel, a variety channel, and an innovation channel. An increase in rents accruing to overhead labor shifts relative demand and thus production between the two consumer goods sectors. It also influences R&D spending in the two sectors, yielding a period of increased product innovation in the one sector and innovative stagnation in the other. We further find that the relationship in the model between the distribution of income and long run economic dynamism can be highly nonlinear.

2 Background

There is substantial ongoing debate about the importance of various causes of recent increases in income inequality, particularly trade, technology, and market power. Much of the recent literature has focussed on skill biased technological change. Technological advances appear to be contributing to a hollowing out of the middle of the distribution of labor income and may be progressively replacing ever higher skilled but routine types of labor (e.g., Autor and Acemoglu (2011), Brynjolfsson and McAfee (2014), Acemoglu and Restrepo (2018)). There is, however, also increasing interest in rent-based explanations (e.g., Piketty et al. (2014), Reich (2015), Furman and Orszag (2018), Keller and Olney (2018), Bota et al. (2019)). Our analysis fits into this latter category. We consider changes in a variety of income related parameters of the model but focus on the case of an exogenous increase in product markups which drives an increase in rents to salaried employees in the model. While there is disagreement over the magnitude of the increase, (Basu, 2019) a number of researchers have documented a secular increase in average markups in the U.S. over the past several decades (e.g., De Loecker, Eeckhout, and Unger. 2018).

There is evidence that rising income inequality in the U.S. has gone hand in hand with rising consumption inequality. For example, Attanasio and Pistaferri (2016) reviews this literature. While much of that literature considers inequality in *levels* of consumption spending, several recent papers also consider the *composition* of that spending. Eisenberg (2014) provides evidence that, because some upgrades in PC processors in the early 2000s replaced rather than supplemented lower performance chips, the benefits of these innovations went disproportionately to relatively affluent consumers. Jaravel (2019) provides evidence from scanner data that the relative demand for consumer goods influences the direction of product innovation. He finds that innovations in consumer products from 2004-2013 focussed on consumer goods catering to the upper end of the income distribution and that the skewed proliferation of product variety also led to relatively lower consumer price inflation for affluent consumers due to a combination of increased product variety and lower marginal cost. Murphy (2016) provides additional evidence that technological change has been biased not only toward skilled labor, but also toward relatively high income consumption goods. He provides a model in which technological change is assumed to be both skill and sector biased, and shows that not only are high income workers made better off, but welfare may fall in absolute terms for low income workers.

Our analysis complements this empirical and theoretical literature. In our model, increases in rents accruing to high income workers may induce shifts of both product quality and product variety toward the high income consumption goods sector, to the benefit of those workers, independently of process innovation. Proper measurement of the increase in consumption inequality requires adjustment for both product quality and product variety.

Our approach to modeling innovation and economic growth follows in the footsteps of a rich literature that documents and models product innovation (Chen and Chie (2005,2007,2013,2014), Valente (2012), Marengo and Valente (2010), Windrum and Birechenhall (1998), Ciarli, Lorenz, Savona, Valente (2010, 2016), Garas and Lapatinas (2017), Hidalgo et. al. (2007), Klette and Kortum (2004), Sutton (2007), Akcigit and Kerr (2018), Acemoglu, Akcigit, Bloom, and Kerr (2018)) and churning in product markets (Broda and Weinstein (2010), Bernard, Redding and Schott (2010), Hottman et al. (2016) Argente, Lee and Moreira (2017)).

The current paper is also in the recent tradition of agent-based macroeconomics. This literature builds macroeconomic models from microfoundations, but in contrast to standard macroeconomic practice, treats the underlying agents as highly heterogeneous, boundedly rational, and adaptive, and does not assume a priori that markets clear. See e.g., Dosi et al. (2005), Delli Gatti et al. (2008), Dawid et al. (2016), Hommes and Iori (2015), Russo et al. (2016), and Fagiolo and Roventini (2017), Dawid and Delli Gati (2018).

3 The Macroeconomic Environment

Here are the fundamental features of the model.

- There are $n_1 + n_2 = n$ firms, each of which produces a single good. Type 1 firms produce goods consumed by wage earning production workers. Type 2 firms produce goods consumed by salaried overhead workers. The total number of firms n is fixed.
- There are m characteristics of goods that all consumers care about. Each good embodies distinct quantities of these characteristics at any given time. Product innovation affects these quantities.
- The probability that a firm experiences a product innovation at any time depends on its recent investments in R&D, which in turn is the outcome of a discrete choice rule.
- Each firm produces with both overhead and variable labor. It forecasts the final demand for its product by extrapolating from recent experience, sets its price as a constant mark-up over marginal cost, and plans to produce enough of its good to meet its expected final demand given this price.
- There are two distinct representative consumers, one representing production workers, the other overhead workers. They inhabit distinct segments of the product market. Each spends all of her labor income each period on consumption goods and searches for better combinations of products to buy within her budget. The former searches only over the n_1 products (firms) that cater to her consumer type, and the latter only over the n_2 products that cater to her consumer type.
- If a firm becomes insolvent, it exits the market and is replaced by a new entrant. The new entrant selects which of the two markets to enter according to a discrete choice rule.

4 Consumer Preferences

The representative consumers' preferences are defined on the characteristics space, and we assume that the two types of consumers have identical utility mappings in this space. Specifically, for either type of consumer, the momentary utility from consuming the vector $z \in R_m$ of hedonic characteristics is $u(z)$.

In addition to this utility function, each consumer has the same home production function $g(q)$ that maps bundles $q \in R_n$ of products into perceived bundles $z \in R_m$ of the characteristics.[3]

Here we will simplify the model in Georges (2011, 2018) by removing direct product complementarities. Product complementarities are clearly important in practice both for consumer utility and for network effects that may influence the macroeconomic propagation of microeconomic disturbances. However, we will set these aside for the moment in order to focus on other aspects of the model.

Thus, we assume that the representative consumer simply associates with each good i a set of integer characteristic magnitudes $z_i \in R_m$ per unit of the good.[4]

The home production $g(q)$ is a CES aggregator that, for each characteristic j, aggregates these characteristic magnitudes over products i:

$$z_j = \left[\sum_{i=1}^{n} (q_i \cdot z_{i,j})^{\rho_1} \right]^{1/\rho_1} \tag{1}$$

with $\rho_1 < 1$. The CES form of (1) introduces some taste for variety across products.[5]

The utility function u for the representative consumer over hedonic characteristics is also CES, so that

$$u = \left[\sum_{j=1}^{m} z_j^{\rho_2} \right]^{1/\rho_2} \tag{2}$$

Thus, utility takes a nested CES form. Consumers value variety in both hedonic characteristics and in products.[6]

5 Product Innovation

A product innovation takes the form of the creation of a new or improved product that, from the point of view of the consumer, combines a new set of characteristics, or enhances an existing set of characteristics. The new product will be successful if it is perceived as offering utility (in combination with other goods) at lower cost than current alternatives. The product may fail due to high cost, poor search by consumers, or poor timing in terms of the availability or desirability of other goods.

A product innovation is then a set of random (integer) increments (positive or negative) to one or more elements of z_i. Product innovation for continuing firms is strictly by mutation. Product innovations can be positive or negative. I.e., firms can mistakenly make changes to their products that consumers do not like. However, there is a floor of zero on characteristic values. Further, innovations operate through preferential attachment; for a firm that experiences a product innovation, there is a greater likelihood of mutation of non-zero hedonic elements.[7]

The probability that a firm experiences product innovation in any given period t is increasing in its recent R&D activity.

[3] As noted in Georges (2011, 2018), this is essentially the approach taken by Lancaster, and shares some similarities with others such as Becker (1965) and Strotz (1957). The primary deviation of our approach from that of Lancaster is the construction of our home production function $g(q)$.

[4] These are the *base characteristic* magnitudes z-base$_i$ in Georges(2011, 2018). Intuitively, by ignoring product complementarities, we are assuming that products like smart phones or spinach have characteristics (entertainment, nutrition, crunchiness, and so forth) that are invariant with respect to what other goods are consumed. I.e., electricity, programming content, salad dressing, and so forth do not independently alter the utility from consuming these goods.

[5] Note that if $\rho_1 = 1$, the number of viable products in the economy would be strongly limited by the number of hedonic elements, as in Lancaster, who employs a linear activity analysis to link goods and characteristics.

[6] While we allow for substantial heterogeneity among products and firms, our representation of consumers as belonging to only two types and exhibiting fixed rather than evolving preferences is clearly highly stylized. While, richer representations of consumers and their behavior have been developed elsewhere (e.g., Windrum and Birchenhal (1998), Aversi etal. (1999), Valente (2012), Garas and Lapatinas (2017)), we can get a fair bit of mileage out of our reduced heterogeneity formulation while mitigating the computational bottleneck that arises in consumer search when the numbers of firms and characteristics is large (see below).

[7] This weak form of preferential attachment supports specialization in the hedonic quality space.

6 R&D

The R&D investment choice is binary – in a given period a firm either does or does not engage in a fixed amount of R&D. If a firm engages in R&D in a given period, must hire additional overhead labor r at salary rate s and so incurs additional overhead labor costs $R = r \cdot s$ in that period.

In making its R&D investment decision at any time, the firm follows a discrete choice rule. It compares the recent profits of two reference groups, and decides probabilistically on this basis whether to switch its investment state. The two reference groups are either firms with relatively high and low recent R&D investment or firms in the two markets.

For example, if the reference groups are firms with relatively high and low recent *R&D* investment, the firm observes the average recent profits π_H and π_L of other firms with relatively high and low recent R&D activity.[8] If the firm is in the lower profit group, it then switches its R&D behavior with a probability related to the profitability differential between the two groups. Specifically, it switches its behavior with probability $2\Phi - 1$, where

$$\Phi = \frac{e^{\gamma\pi_1}}{e^{\gamma\pi_1} + e^{\gamma\pi_2}} \tag{3}$$

$\gamma > 0$ measures the intensity of choice and π_1 and π_2 are measures of the average recent profits of the high and low profit R%D groups.[9]

A similar rule is followed if the firm's current reference groups are firms in market 1 and firms in market 2. Firms select between the two pairs of reference groups probabilistically, and there is additionally some purely random variation in R&D choice.

7 Production and Employment

Each firm i produces its good with labor subject to a fixed labor productivity A_i and hires enough production labor to meet its production goals period by period. Each firm also must employ a fixed number h of overhead workers at salary rate s and so incurs fixed overhead labor cost $H = h \cdot s$ in each period as well as the additional overhead R&D labor cost $R = r \cdot s$ in any period in which it is engaged in R&D. In this paper, our focus is on product innovation rather than process innovation. Consequently, we suppress process innovation and hold A_i, h, and r constant over time.

8 Consumer Search

Each of the representative consumers spends her entire income each period and selects the shares of her income to spend on each good. Each period, she experiments with random variations on her current set of shares. Specifically, she considers randomly shifting consumption shares between some number of goods in her own market (market 1 or market 2), over some number of trials, and selects among those trials the set of shares that yields the highest utility with her current income. While each consumer engages in limited undirected search and is not able to globally maximize utility each period, she can always stick to her current share mix, and so never selects new share mixes that reduce utility below the utility afforded by the current one. I.e., the experimentation is a thought exercise not an act of physical trial and error.

9 Entry and Exit

When a firm does not have enough working capital to finance production, it shuts down and is replaced. At this time, the new firm may have the opportunity to switch markets. In making this decision it follows a discrete choice rule based on the relative recent profitability of firms in the two markets. The new firm adopts (imitates) the product characteristics of a randomly selected existing firm, and either retains the exiting firm's current share of consumer demand or has some market share reallocated to it from other firms, depending on which market it enters. The im-

[8] Each firm's recent profits and recent R&D activity are (respectively) measured as exponentially weighted moving averages of its past profits and R&D activity.

[9] I.e., if $\pi_H > \pi_L$, then $\pi_1 = \pi_H$ and $\pi_2 = \pi_L$, and firms with relatively low recent R&D activity switch R&D on with a probability that is greater the larger is the difference between π_1 and π_2.

itated firm may or may not be in the entering firm's own market, depending on a bias parameter, allowing some diffusion of product innovation across markets over time. The new firm is also seeded with startup capital.

10 Timing

The timing of events within each period t is as follows:

- R&D: firms chose their current R&D investment levels (0 or 1).
- Innovation: firms experience product innovation with probabilities related to their recent R&D investments.
- Production: firms forecast sales, hire production labor, and produce to meet forecasted demand.
- Incomes: all firms pay wages to their production workers and salaries to their overhead workers.
- Consumer Search: the representative consumers search and update their consumption baskets.
- Sales: the consumers spend all of their labor incomes (above) on their consumption baskets.
- Entry and Exit: firms with insufficient working capital are replaced. The new entrants may migrate across the two markets.

11 Representative Agent Benchmark

Before moving to the full heterogeneous agent model, it is worth considering a representative agent equilibrium benchmark.

Consider the case in which all firms are identical with the exception of which of the two markets it inhabits (i.e., which consumer type it caters to). There are n_1 firms catering to the consumption of production workers (market 2) and n_2 firms catering to the consumption of salaried (overhead) workers (market 2). Each firm of type i starts with a $1/n_i$ share of the total market. Suppose further that all firms engage in R&D in every period and experience identical innovations over time.

In this case, if n_1 and n_2 are fixed, then the equal individual market shares will persist, since there is no reason for consumers of either type to switch between firms in its own market with identical product qualities and identical prices. Further, there is a unique equilibrium for the real production and sales of consumer goods at which demand and supply are in balance. At this equilibrium, aggregate real activity depends on the markup η, the per firm overhead labor requirements h and r, the salary rate s (for overhead workers), the wage rate W (for production workers), labor productivity A (for production workers), and the numbers of firms n_1 and n_2 in the two markets. Specifically, at this equilibrium, total production $Y = Y_1 + Y_2 = (\frac{1}{\eta-1}) \cdot \frac{A}{W} \cdot (h+r) \cdot s \cdot (n_1+n_2)$. See below for details. Note that this total equilibrium production is independent of both product quality and the relative proportion n_1/n_2 of the two types of firms. Further, this equilibrium is a steady state of the agent dynamics in the model and is locally stable under those dynamics. If, for example, firms all start with production less than steady state production, then since the markup $\eta > 1$, demand will be greater than production for each firm, and production will converge over time to the steady state equilibrium.

We can see this as follows. Since firms within each of the two markets are identical, they produce identical quantities q_1 or q_2 of their goods. The total income (E_1) spent in market 1 is the total income of production workers, and in market 2 (E_2) is the total income of overhead workers:

$$\begin{aligned} E_1 &= \frac{W}{A} \cdot (n_1 \cdot q_1 + n_2 \cdot q_2) \\ E_2 &= (h \cdot s + r \cdot s) \cdot (n_1 + n_2) \end{aligned} \tag{4}$$

Each firm, regardless of market, charges an identical price p for its good, which is a fixed markup η on marginal cost

$$\begin{aligned} p &= \eta \cdot \mathrm{MC} \\ \mathrm{MC} &= \tfrac{W}{A} \end{aligned} \tag{5}$$

and so produces and sells

$$q_i = \frac{E_i}{n_i \cdot p} \tag{6}$$

units of its good.

These relationships yield the following steady state equilibrium value for (per firm) production for each of the two firm types.

$$\begin{aligned} q_1^* &= \frac{(\frac{n_1+n_2}{n_1}) \cdot (h+r) \cdot s \cdot \frac{A}{W}}{(\eta - 1) \cdot \eta} \\ q_2^* &= \frac{(\frac{n_1+n_2}{n_2}) \cdot (h+r) \cdot s \cdot \frac{A}{W}}{\eta} \end{aligned} \tag{7}$$

Thus the total output in markets 1 and 2 in steady state equilibrium are:

$$\begin{aligned} Y_1^* &= \frac{(n_1+n_2) \cdot (h+r) \cdot s \cdot \frac{A}{W}}{(\eta - 1) \cdot \eta} \\ Y_2^* &= \frac{(n_1+n_2) \cdot (h+r) \cdot s \cdot \frac{A}{W}}{\eta} \end{aligned} \tag{8}$$

and total steady state output is:

$$Y^* = n_1 \cdot q_1^* + n_2 \cdot q_2^* = \frac{(n_1+n_2) \cdot (h+r) \cdot s \cdot \frac{A}{W}}{(\eta - 1)} \tag{9}$$

If in this representative agent case firms follow simple one period adaptive expectations for demand, then firm level output dynamics will converge to the steady state. For type 2 firms, demand is constant at $(H+R) \cdot (n_1+n_2)/n_2$, so supply will converge to q_2^* in a single period. For type 1 firms, output dynamics are then given by:

$$q_{1,t} = \frac{1}{\eta} \cdot q_{1,t-1} + \frac{1}{\eta} \cdot \frac{n_2}{n_1} \cdot q_2^* \tag{10}$$

Thus, given $\eta > 1$, the steady state equilibrium q_1^* for type 1 firms is also asymptotically stable.

Focussing on the steady state equilibrium, we can see that for both firm types, $\partial q^*/\partial(h+r) > 0$, $\partial q^*/\partial s > 0$, $\partial q^*/\partial A > 0$, $\partial q^*/\partial W < 0$, and $\partial q^*/\partial \eta < 0$. These are all demand driven.

- An increase in the salaries s or employment requirements $h+r$ of overhead labor raises the incomes of overhead workers, raising demand in market 2. Since more production labor is required in market 2, the incomes and demands of production workers for goods in market 1 also increases. Thus there is a general increase in demand distributed across the two markets and an increase in output and the employment of production workers in both markets.

- An increase in labor productivity A will cause firms to lower their prices, raising demand by both types of worker and thus equilibrium output in both markets.

- Similarly an increase in the wage rate W of production workers or in the mark-up η will cause firms to raise their prices, lowering AD and equilibrium output.

Clearly Y_1^*, Y_2^* and Y^* will be constant as long as there is no change in the parameters h, r, s, A, W, η and n. If we were to allow them to change, growth in the number of firms n or the productivity of production labor A would cause equilibrium total production Y^* to grow over time, while balanced growth in production wages W and labor productivity A would have no impact on equilibrium production. Note that, in the full heterogeneous agent model, while all of the parameters above are fixed, the fraction of firms adopting R&D investment, rather than being fixed at 1, varies endogenously, contributing to the disequilibrium dynamics of the model.

Note that the steady state levels of both total and market level output Y^*, Y_1^*, and Y_2^* are independent of the relative number of firms n_1/n_2 in the two markets. Thus, varying that ratio, while holding the total number of firms $n = n_1 + n_2$ constant, affects the relative sizes q_1^* and q_2^* of individual firms but leaves total production in each of the two markets unchanged.

Further, and importantly, the relative shares of incomes earned by each of the two classes (wage and salary labor) and spent in the two markets in steady state equilibrium is entirely independent of all parameters in the model with the exception of the markup

η. For example the ratio Y_1^*/Y_2^* is:

$$\frac{Y_1^*}{Y_2^*} = \frac{1}{\eta - 1} \tag{11}$$

and both the share of real income and real consumption going to production workers in steady state is $1/\eta$.

The latter result is not an artifact of the perfectly elastic supply of labor assumed here and is the case for any elasticity of labor supply in either labor market. Equilibrium requires that the supply and demand for labor come into balance in this way via some combination of output and income adjustment, given the fixed markup.[10]

This places a strong restriction on income inequality in the model. For example, if the salary rate or employment of overhead workers increases exogenously ($H + R$ increases), this will cause the economy to expand but will not lead to an increase in the income share of overhead labor nor the relative size or profitability of market 2. However, an increase in the markup η will have those effects in steady state equilibrium, while shrinking output and the employment of production labor in both markets.

There is nothing in the analysis above that requires profit to be equalized across markets. Since all income in the model accrues to the two types of labor (production and overhead), i.e., to wages and salaries, average profit will be zero in steady state equilibrium. However, if the representative firm in one market has greater output q^* than its counterpart in the other market, then it will spread its overhead costs over more output and so earn positive profit, while the representative firm in the other market earns negative profit.[11]

Once, however, we add entry and exit across markets, then n_1/n_2 can adjust to equalize profits of the representative firms at zero in the representative agent steady state equilibrium. This adjustment will be central to the main results of the paper.

Finally, but importantly, note that the representative agent steady state equilibrium levels of production above are entirely independent of product quality. Improvements in product quality will, however, increase consumer utility, or equivalently, the quality adjusted value of total production at this equilibrium. Specifically, given the nested CES formulation of utility, if the magnitudes of all product characteristics in a given market grow at rate g due to innovation, then the growth rate of consumer utility in that market will converge in the long run to g. If the magnitudes of different characteristics grow at different rates, the growth rate of utility will converge to the rate of growth of the fastest growing characteristic. All else equal, the long run growth path of utility for consumers in that market will be lower in the latter case than in the former case.

The CES formulation also implies that consumers in either market benefit from greater variety of products in their market. Thus, at any moment in time, the utility of each worker type i depends on its own income and the past product innovation in its own market, but also the number of products n_i in its own market.

[10] For example, with the fixed nominal wage w assumed in this paper, an increase in η raises prices, lowering real incomes spent in both markets. Production workers are laid off, while overhead workers are not (since $n_1 + n_2$ is unchanged), skewing demand toward market 2.

[11] On average, across the two representative firms, revenues net of the cost of production labor are just great enough to cover all overhead labor costs. Once we move to the heterogeneous firm case, in the comparable steady state equilibrium, profits are distributed around zero across all firms and around market specific averages that need not be zero within each market. On the one hand, firms will vary as to R&D investment status, with firms who are not engaging in R&D investment saving overhead cost R per period. On the other hand, firms will also vary with respect to demand shares (driven by both aggregate market shares and individual product qualities which are themselves related to past investments in R&D), and firms with relatively high demand shares are able to spread overhead cost over greater sales. Thus, for two firms with the same overhead cost (i.e., the same current R&D investment status), the firm with greater demand for its product will have higher profit, while for two firms with the same product demand shares, the one with the lower overhead cost (lower current R&D investment) will have higher profit. Firms that face chronic losses eventually fail and are replaced.

12 Features of the Heterogeneous Agent Model

As noted above, if we eliminate all heterogeneity in the model, other than the two types of consumer and firm, there is a steady state equilibrium which is stable under the simple dynamics described above. Assuming that all firms engage in R&D investment, steady state aggregate output is a multiple of the combined overhead labor cost $H + R$ and is independent of product quality. Quality adjusted output and consumer utility, on the other hand, will grow over time in equilibrium as product quality improves.

With firm heterogeneity, the evolution of the hedonic characteristics of firms' products will influence product market shares as well as aggregate activity and living standards. The stochastic evolution of product quality with zero reflective lower bounds on individual characteristic magnitudes will tend to generate skewness in the distribution of individual product qualities and thus in the distribution of market shares.[12]

Turning to aggregate activity, with heterogeneity, as discussed in Georges (2018), there is a tension between the short term and medium term effects of R&D at the firm level that feeds back with the social learning to allow complex dynamics to emerge in aggregate R&D spending. That spending in turn drives both aggregate output dynamics as well as innovation and thus growth in consumer utility. Any individual firm's R&D investment creates an aggregate demand externality that initially impacts market 2 and then (relatively rapidly) filters throughout the economy. Any innovation that results stochastically from that investment is initially purely market, and thus class, specific, though may also eventually diffuse across the two markets.

Here, we are particularly interested in the potential for the divergence of consumer utility across classes of worker/consumers. This can be driven by changes in the relative incomes of the two consumer classes, by different rates of product innovation in the two markets, and by changes in relative variety across the two markets. Below we focus on the impact of exogenous increases in markups and in the salary rates of overhead workers on these three drivers of consumption inequality.

According to the analysis in section 11, an increase in the common markup will cause prices to rise and thus AD to fall in both markets. The share of income going to salaried workers increases, while the share falls for production labor, due to the reduction in employment of the latter. This shift in income shares will be matched by a shift in consumption shares, but will also have the effect of shifting profit from market 1 to market 2. This should, in turn lead to both a relative increase in R&D investment in market 2 and the relocation of firms from market 1 to market 2 over time. The latter will eventually wipe out the supernormal profits in market 1 but will also permanently increase the relative variety of consumer goods available to salaried workers. Meanwhile the period of supernormal R&D investment should temporarily boost product innovation that caters to the consumption of salaried workers.[13] So in principle, we may get a triple wedge between the utility of the two classes in the form of divergent levels, variety, and qualities of consumption each of which persists after profits have normalized.

As noted in section 11, we would not expect the same to be the case for an increase in salary rates (or overhead labor requirements, or a decrease in wages), as once output and employment expand in response, income shares and relative market sizes will return to their previous equilibrium if the markup has not changed. We would expect this to happen on a much faster time scale than the relocation of firms across markets which is required in the aftermath of a change in the markup. However, increases in these overhead employment and compensation rates may impact innovation both directly, via increases in the overhead costs of R&D, and indirectly via their effects on turnover. Turnover rates can have positive indirect effects on innovation if entrants imitate firms

[12] This skewness is limited by the variety loving aspect of consumer preferences and the ratios m/n_1 and m/n_2 which characterize the opportunities in each market for firms to niche in the product space.

[13] Note that R&D and innovation are fixed and uniform across firms in the representative agent version of the model in section 11, so this innovation effect is suppressed there.

with higher quality than firms that are exiting.

Finally, it is worth noting that in an adaptive environment, profit provides incentives for individual firms to engage in R&D and select the market to exit or enter, but these incentives can be clouded by both noise and general disequilibrium feedback and so may not be entirely straightforward. Ultimately the outcomes are emergent.

13 A Computational Experiment

Here we conduct a computational experiment in the full model by simultaneously increasing the markup η and the salary rate s. Specifically, in period 2000 the markup is increased from 2.0 to 3.0 and the salary rate from 1.0 to 2.0. This precipitates a shift in income shares from production labor to overhead labor and a movement of production and production labor from market 1 to market 2 but is aggregate employment neutral in the representative agent benchmark.

Results are illustrated first via a representative run in Figures 1-5 below. The increase in product markups η lowers the real wage, while the parallel reduction in the real salary rate is more than offset by the increase in the nominal salary rate. We see then demand falling immediately in market two and rising in market one. Production and the employment of production labor shifts from market one to market two, while the utility of salaried overhead workers rises and the utility of wage earning production workers falls.

At the same time, profits that had come into equality across the two markets now increase in market 2 and decrease in market 1. This has two effects. Firms that fail in market 1 begin to shift to re-enter in market 2. Similarly, R&D investment responds to the changes in profit by tending to increase in market 2 and decrease in market 1. Thus, we have two further effects on relative utility – a variety effect and an innovation effect. The movement over time of firms from market 1 to market 2 offers overhead labor greater product variety and reduces product variety for production laborers. And the temporary shift

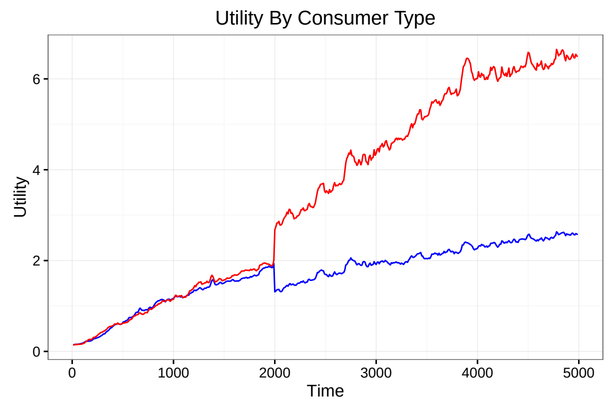

Figure 1: Utility of production workers (U_1, blue) and overhead workers (U_2, red) following a simultaneous increase in the common markup η from 2.0 to 3.0 and the salary rate s from 1.0 to 2.0 in time period 2000. The total number of firms in this simulation is 100.

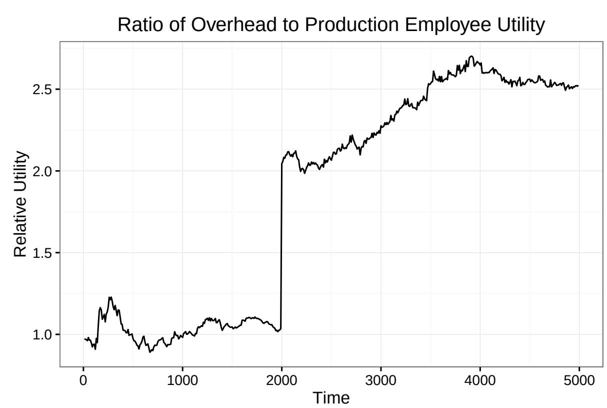

Figure 2: Relative utility of overhead workers to production workers: U_2/U_1.

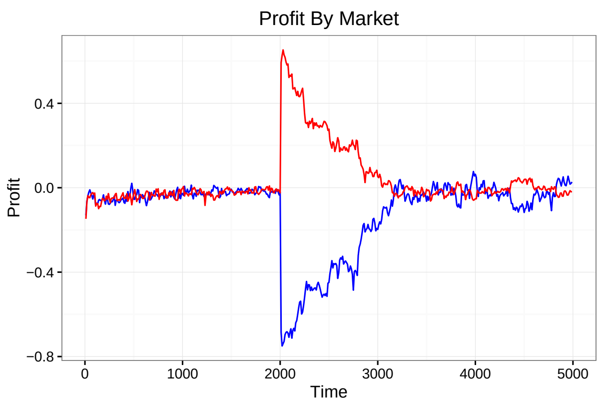

Figure 3: Average profit of firms in each market.

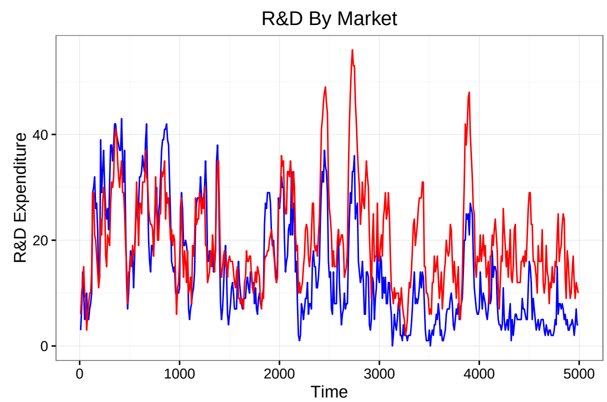

Figure 5: RandD spending in each market.

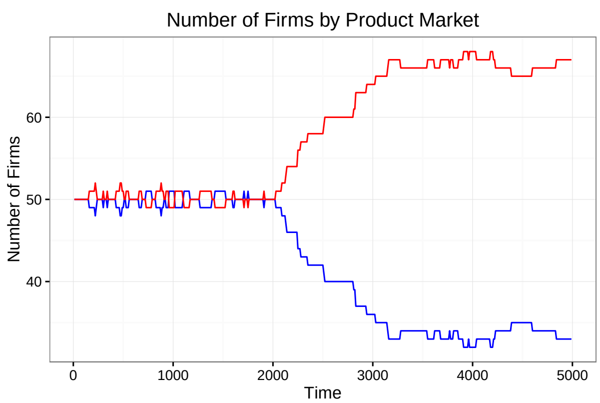

Figure 4: Number of firms in each market.

in R&D investment causes a tendency for relatively greater product innovation in market 2 for some time.

More generally, these effects are conditioned by a number of factors. The increase in the salary rate makes $R\&D$ investment more expensive, and so less profitable in the short term. The increase in overhead costs also drives unprofitable firms to failure more quickly, increasing turnover rates. Depending on the degree of own market bias in imitation for reentering firms, there can me more or less drift in average product quality across the two markets. If reentering firms can imitate firms in the other market, then drift in product quality is limited and the high levels of innovation in market 2 ultimately trickle down to product quality in market 1. In the present experiment, there is a high degree of own market bias and thus relatively slow diffusion of innovation across markets.

14 Monte Carlo Experiment

The qualitative results in the last section are robust over multiple simulations and simulations with larger numbers of firms. Here we replicate figures 1-5 increasing the number of firms to 1000 and plotting average outcomes over 1000 runs with different random seeds. We also include bands indicating one standard deviation above and below the average for each time step.[14]

To get some sense of the relative importance of the three factors driving consumption inequality, we can consider a benchmark prediction from the representative agent case. With initial markup $\eta = 2.0$ and $n_1 = n_2$, we would have $q_2^*/q_1^* = 1.0$ in steady state. A simultaneous increase in η from 2.0 to 3.0 and the salary rate s from 1.0 to 2.0, holding product quality and the number of firms in each market constant, would simply shift income from wage labor to

[14]Note that the average utility is higher in the MC experiment below than in the individual run illustration above due to the larger number of firms, which yields greater variety and thus utility.

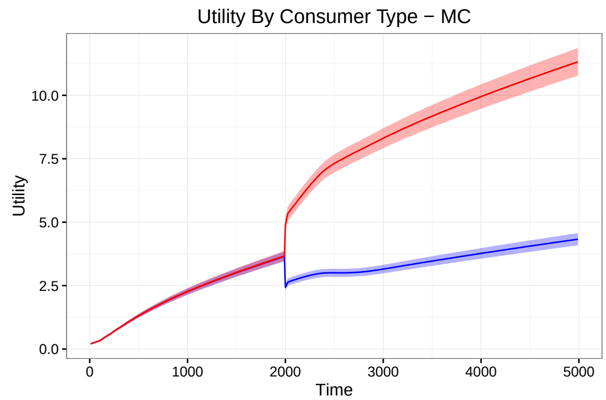

Figure 6: Utility for production workers (U_1, blue) and overhead workers (U_2, red). Average across 1000 runs for each time period with band indicating one standard deviation above and below the average. The markup η is increased from 2.0 to 3.0 and the salary rate s from 1.0 to 2.0 in time period 2000. The total number of firms in the two markets is 1000.

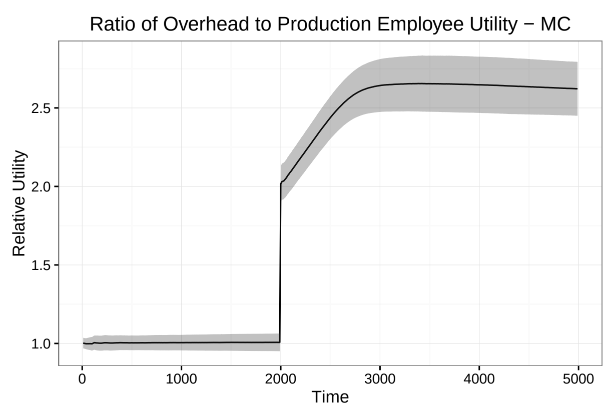

Figure 7: Ratio of utility of overhead workers to utility of production workers: U_2/U_1.

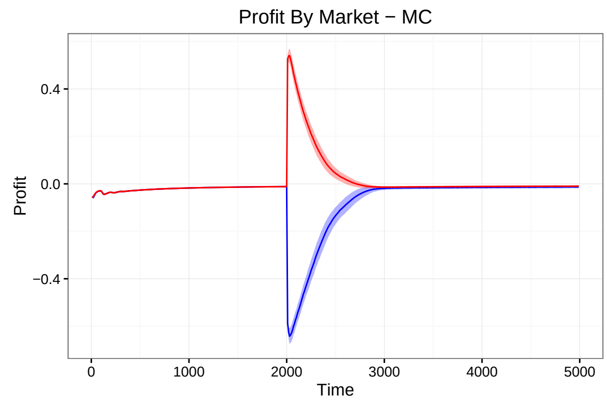

Figure 8: Average profit of firms in each market.

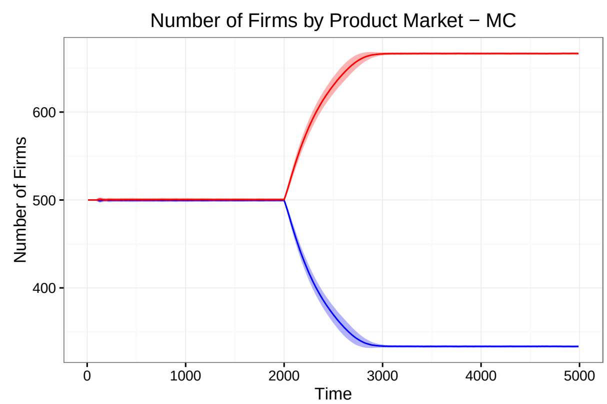

Figure 9: Number of firms in each market.

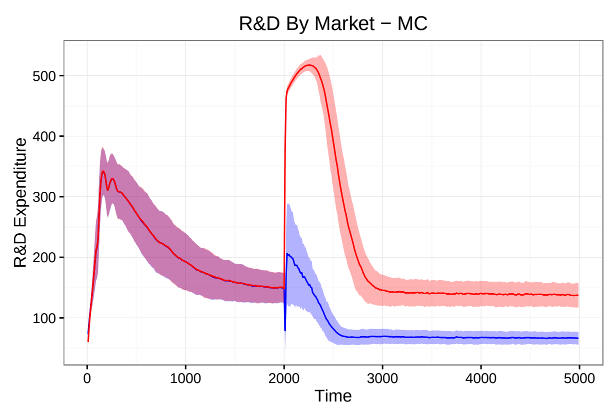

Figure 10: RandD spending in each market.

overhead labor in equilibrium, increasing Y_2/Y_1 and U_2/U_1 from 1.0 to 2.0. We see this prediction closely matched in both the individual run in Section 13 and the Monte Carlo experiment in Section 14, with rapid increases in U_2/U_1 following the distributional shock in period 2000.

Returning to the representative agent benchmark, over time, firms would respond to the increase in relative profit π_2/π_1 by migrating from market 1 to market 2, and n_2/n_1 would eventually rise from 1.0 to 2.0, at which point q_2^*/q_1^* would have returned to 1.0. Thus, the relative increase in consumption for overhead workers ultimately would express itself entirely as an increase in product variety in the consumption basket of overhead workers (and a decrease in variety for production workers), rather than a change in the quantities consumed of individual goods. In the special case in which all firms' product qualities are fixed and identical, in the sense that all characteristic magnitudes $z_{i,j}$ are identical across both i and j, we would see U_2/U_1 increase to approximately 2.38 in equilibrium, so an additional 38% increase due to the product variety effect. Adding in temporarily different rates of innovation would then add a third wedge between utility in the two markets.

We can see that in both the single run shown in Figures 1-5 and the Monte Carlo experiment shown in Figures 6-10, the utility ratios U_2/U_1 peak well above 2.38, at approximately 2.70 for the single run and 2.66 for the average of the MC runs, respectively. Following the distributional shock in period 2000, profit in market 2 rises above that in market 1 for an extended time driving both $n_2/n1$ to increase permanently (and thus the variety effect) and $\frac{R\&D_2}{n_2}/\frac{R\&D_1}{n_1}$ to rise above 1 during that time (and thus an innovation effect). A back of the envelope measure of the average innovation effect in the MC experiment is the difference between 2.66 and 2.38, or an additional 28% increase in U_2/U_1 due to the innovation effect.

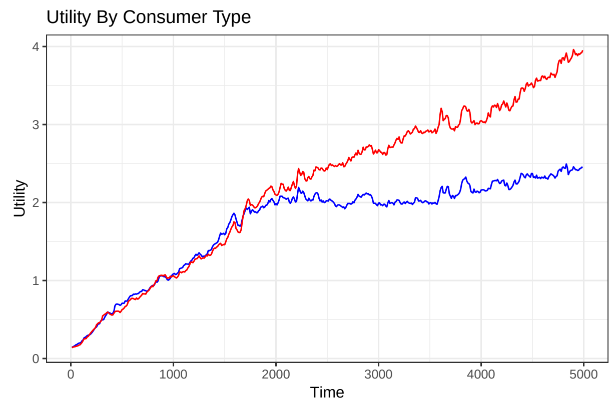

Figure 11: U_1 (blue) and U_2 (red) in a run in which μ is increased from 2.0 to 2.4 gradually during periods 2000-4000. The number of firms is 100.

15 Competition, Distribution, and Economic Dynamism

Considering the markup in isolation, increasing the markup in both markets tends to increase prices and reduce demand, production and utility for both classes of workers in the short term. Innovation will continue over time, working against the level effect loss of utility from the loss of competition. It is straightforward then that if the markup rises slowly over a period of time, utility will be pulled down by the rising markups while being pushed up by product innovation. This is illustrated in Figures 11-12, in which the common markup μ rises gradually between periods 2000-4000. We see that between the negative effect of the rising markup and the ongoing positive effect of the background innovation, the utility of production workers stagnates during the transitional period, while impact on the growth of utility of salaried workers is more muted. Production workers are hit harder by the increase in markups as they face the same price increases and thus reduced purchasing power of a dollar of income as salaried workers, but also face reduced employment due to the effect on aggregate demand. Figure 11 illustrates for a specific run of the model, while Figure 12 provides Monte Carlo results.

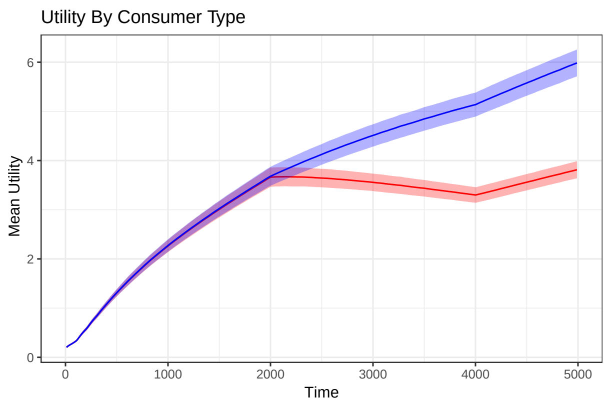

Figure 12: Same as Figure 12, but MC averages and one standard deviation band over 1000 runs with 1000 firms.

Overall, the utility (U_2) of salaried workers rises 57.2% over the utility (U_1) on average in the MC experiment. We can again perform a back of the envelope decomposition of this overall effect on relative utility between the two classes into three components with the aid of the representative agent version of the model. Holding both the number of products (firms) in each market and all product qualities constant, the effect of the increase in the markup on prices would cause U_2/U_1 to increase by 40% in the model. As firms move from market 1 to market 2 and increase their R%D in that market over time in response to the temporary increased relative profitability of market 2, the variety effect contributes an additional 12%, and the innovation effect accounts for an additional 5%, calculated as a residual between the 57% observed in the simulation and the 52% predicted by the model.

In the example above, the increase in the markup is modest and impact on R&D is primarily transitional, so in the long run, there are long term level effects on the utilities of the two classes, but long run growth is eventually reestablished. ** Further, as noted above, the diffusion of product innovation across markets on a slow time scale will cause the relative innovation effects to eventually fade away – i.e., the boost in product quality in the high end market eventually will trickle down to the low end market – while the income share and variety wedges will persist.

More generally, the pace of long run growth in the model is conditioned by the endogeneity of both R&D and firm turnover. Due to the tension between the short and long run effects of a firm's R&D on it's profit, the relationship between long run dynamism and the distribution of income is more complex that that suggested by the representative agent benchmark in Section 11. For example, increasing the salary rate will raise the cost of R&D but also stimulate aggregate demand. Interestingly, the impact on long run wellbeing can be highly nonlinear, with successive increases in salary at first stimulating the economy and driving up the utility of production workers, but after a point causing R&D spending to rapidly decline, crashing the rate of innovation along with consumer welfare.

In Figure 13, we see the impact of the salary rate on the utility of production workers at two times in the simulation, averaged over 100 Monte Carlo runs.

For modest increases in salary, the demand effects outweigh the R&D effects (on average) at both of these time horizons. However, as salary is increased further, the number of firms engaging in R%D becomes less stable, rising and falling in waves in individual runs, and then ultimately collapses. In Figure 13, at high salary rates (over 2.5) R&D never gets a foothold, and U_1 becomes stuck close to it's low initial value (of approximately 0.2) in each run.

Interestingly, U_1 increases monotonically in salary in the equilibrium off the representative agent version of the model in section 11. There, all firms are assumed to engage in R&D, and that behavior is sustained by a macroeconomic equilibrium in which firms earn zero profit. However, under the discrete choice rules followed by firms in the heterogeneous agent model under general disequilibrium, R&D tends to becomes increasingly fragile as the salary rate rises to higher levels.

16 Conclusion

We extend the agent-based macroeconomic model in Georges (2011, 2018) to investigate consumption inequality. An increase in product markups and salary

Figure 13: Utility of production workers (U_1) in periods 5000 and 9000 as the salary rate is varied across simulations. Monte Carlo averages over 100 runs for each of 30 salary values.

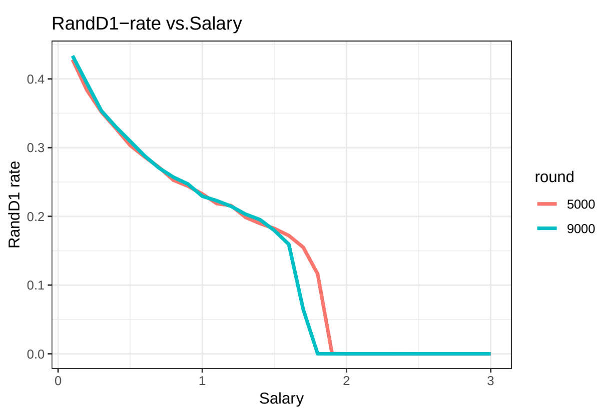

Figure 14: Fraction of firms in market 1 engaging in R&D in periods 5000 and 9000 in Figure 13. Monte Carlo averages over 100 runs for each of 30 salary values.

rates leads to an increase in rents going to salaried overhead labor. The increase in the share of income going to this class of consumers skews production, product variety and product innovation toward goods that cater to that class and thus widens consumption inequality to a greater degree than raw income inequality. We illustrate how, in response to rising markups, this process of macroeconomic gentrification can lead to a protracted period of stagnation of the welfare of production workers.

Of course the process will also work in reverse. A rising share of income accruing to production labor would shift product innovation and variety in favor of that group. Further, innovation that favors one group will eventually diffuse (or 'trickle down') to the other group. Thus, the innovation effect described here can be protracted, but is ultimately transitory, and can operate in different directions over the course of history.

We also show in the model that the relationship between the distribution of income and long run economic dynamism can be highly non-linear.

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