Productivity Growth and Agricultural Out-Migration in the United States

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Abstract

In the twentieth century U.S., the average annual decline in the farm share of employment relative to non-farm employment was approximately 3.6 percent. Despite this rapid reallocation of labor, there was nevertheless a large, persistent wage gap between the farm and non-farm sectors that declined only slowly over time. We show that there were three significant sources of farm out-migration during this period: (i) absolute farm productivity growth in conjunction with subsistence food consumption, (ii) relative productivity growth in conjunction with a low elasticity of substitution between farm and non-farm goods, and (iii) endogenously declining wage gaps. Quantitative features of the model accord well with the U.S. experience.

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1 Introduction

Throughout the world, gradual but massive migration from the farm to the non-farm sector has accompanied industrialization and capitalist development. The U.S. experience is especially remarkable. In the twentieth century, coinciding with an acceleration of farm productivity growth, the average annual decline in the farm share of employment relative to non-farm employment was approximately 3.6 percent. Yet despite considerable and steady off-farm migration, the twentieth century U.S. economy was characterized by a significant gap between farm and non-farm real wages that declined only gradually over time. Figure 1 presents these striking trends.[1] A key puzzle involves how to explain the relationship between these rapid labor flows and the (slow) change in the farm–non-farm wage gap over time. We demonstrate how both the changes in the wage gap and the rate of migration are ultimately determined by two factors common to both trends: absolute and relative farm productivity growth. In particular, we show that the pattern of absolute and relative farm productivity growth observed in the twentieth century U.S. can simultaneously account for the rapid off-farm migration and gradually declining wage gaps. The slow but secular decline in wage gaps arises because rising incomes move individuals further away from subsistence, lowering their aversion to risk. Our main contribution is therefore two-fold: (i) we develop endogenously determined, persistent wage gaps that experience a secular (and gradual) decline and, simultaneously, (ii) we account for the reallocation of labor from the farm to the non-farm sector in an internally-consistent fashion.

The traditional account of off-farm migration has emphasized *absolute* farm produc-

[1]We discuss our data sources underlying this figure and our estimates throughout the paper in a detailed appendix on Data Sources, available from the authors upon request. We shall use "off-farm labor reallocation" and "off-farm migration" interchangeably. Although, in general, they are distinct, both measures of structural change point in similar directions after 1920; compare employment share versus population based estimates in Figure 1. See Gardner (2002) for a recent account of the off-farm migration trends, and Olmstead and Rhode (2000) for an account of structural transformation in northern U.S. agriculture.

tivity growth in conjunction with the subsistence consumption of agricultural goods.[2] This explanation is quite intuitive. As productivity in agriculture rises, supply outstrips demand due to the low income elasticity of demand for farm goods. As a result, labor moves out of agriculture. This explanation is empirically appealing as well. Absolute farm productivity growth in the U.S. accelerated at the same time that farm out-migration accelerated, and the low income elasticity of demand for farm goods is one of the few undisputed facts in economics. This is the first source of farm out-migration that we identify.

The second source of farm out-migration is relative farm–non-farm productivity growth, which has also been a remarkably important feature of the U.S. data in the twentieth century. This observation hinges critically on a non-unitary elasticity of substitution between farm and non-farm products. In the special case of unitary elasticity, a productivity growth differential across sectors leads to offsetting income and substitution effects, and so does not influence the relative demand for farm goods. As a result, relative productivity growth by itself does not create any incentives for labor to move out of the farm sector. By contrast, in the empirically relevant case of a low demand elasticity of substitution between farm and non-farm goods (i.e., the goods are gross complements), relatively high technological progress in the farm sector results in a relative increase in the demand for non-farm goods, which ultimately leads to unfavorable shifts in the agricultural terms-of-trade and exerts additional pressure on labor to move out of farming.

To gauge the quantitative importance of both absolute and relative productivity growth for structural change, we provide a detailed reinterpretation of the changes in the sectoral labor shares in the twentieth century U.S. In particular, we use a baseline general equilibrium model to calibrate sectoral labor shares. The baseline model allows for instantaneous sectoral reallocation of labor in response to a change in relative wages. We decompose the changes in these calibrated series into the contributions of absolute and relative farm

[2] See, e.g., Nurkse (1953), Lewis (1954), Timmer (1988), and Kongsamut et al. (2001).

productivity growth. We find that, depending on the relative importance of subsistence food consumption, between one-fifth to one-third of the observed reallocation of labor from the farm to the non-farm sector is due to relative productivity growth combined with a low elasticity of substitution between farm and non-farm goods.

The third source of farm out-migration that we study is related to the "transfer problem" (of moving labor out of agriculture and into industry) as well as recent accounts of regional convergence.[3] The focal point of this literature has been persistent sectoral wage and income gaps which were interpreted as indicative of a significant misallocation of labor.[4] Specifically, despite the substantial transfer already taking place, many contemporary commentators felt that the twentieth century U.S. off-farm migration rate was too low based primarily on the size and persistence of the wage gap in favor of industry.

The baseline model with zero migration costs is unable to account for the transfer problem. We therefore extend the model so that workers have to incur fixed and sunk costs when they switch sectors (and move). This creates a certain amount of inertia in sectoral allocations of labor. In this case, migration decisions follow (S, s) rules whereby relocation is triggered only when sectoral wage gaps exceed S (or s) percentage points. We characterize these relocation thresholds analytically. In particular, we show that, as incomes increase and as the share of food consumption in total expenditures declines, the wage gaps in an economy also decline *endogenously*. The ultimate reason for the decline in wage gaps is non-homothetic preferences due to subsistence nature of food consumption.

[3] See, e.g., Shultz (1945), Johnson (1956), Heady (1962), the 1960 AEA session on "Facilitating Movements of Labor Out of Agriculture" in the *Proceedings of the American Economic Association Meetings*, and a more recent assessment by Mundlak (2000, p. 264): "The off-farm labor migration is a universal phenomenon that continues over a long time. Why does it take such a long time before it comes to an end? If the non-farm sector is more attractive then all farm labor should leave it at once." For a survey of rural–urban migration models that address related issues see Williamson (1988).

[4] We should note that the real wage and income gaps were significant even after they were adjusted to take into account skill and educational differentials, purchasing power and income tax differences; see, e.g., Johnson (1956), Hatton and Williamson (1992), and Olmstead and Rhode (2000, pp. 718–19). There is general agreement that the farm and non-farm wage differential had declined considerably only by the 1990's. However, we recognize that estimates of these wage gaps for the late nineteenth and early twentieth century vary considerably across alternative sources. Caselli and Coleman (2001) discuss these differences.

At higher levels of income, individuals are less susceptible to slipping below subsistence for a given pattern of shocks and, as such, are less risk averse. Declining risk aversion as the economy develops implies that workers can better accommodate the risks associated with migration thereby reducing the wage gaps.[5]

This framework allows us to assess whether the wage gaps observed in the U.S. (and elsewhere) in fact imply a "large misallocation" of labor across sectors. Our calibration results suggest that the "shortfall" in farm out-migration was in fact relatively small. However, the closing of that shortfall comprised a third, albeit transitory, source of farm out-migration.

Although our analysis is related to a voluminous literature on structural change (as surveyed in Syrquin [1988]), let us briefly mention here two of the most recent complementary studies that explicitly attempt to match the declining employment share of agriculture. Kongsamut et al. (2001) model very long-run economic growth with three sectors in which relative employment shares vary over time. They assume a unitary elasticity of substitution across goods and invoke identical productivity growth rates across sectors. As a consequence of these two assumptions, all structural change is driven by absolute productivity growth in their theoretical model. Caselli and Coleman (2001) model regional convergence over the medium- to long-run, and allow for differential productivity growth rates across farm and non-farm sectors. They also assume a unitary elasticity of substitution between farm and non-farm goods. So again all structural change is driven by absolute productivity growth in their analysis. Although the economic significance of these assumptions is ultimately an empirical matter (as we discuss in detail below), our simulations indicate that they are important for the twentieth century U.S. experience.

The rest of our paper is organized as follows. Section 2 outlines a two-sector model. Section 3 discusses the baseline model without fixed costs of moving, presents its quantita-

[5] Of course, with fixed costs of migration and fluctuating productivity, *on average* wage gaps will always be non-zero (but they can also be economically insignificant). Exogenous changes in these costs, due to social and institutional progress may further reduce the observed mean income differentials.

tive implications for changes over time in sectoral labor shares and links these to absolute and relative productivity growth rates. Section 4 integrates equilibrium wage gaps into the two-sector model with fixed costs. Section 5 concludes.

2 A Two-Sector Model

We present a tractable two-sector model which disentangles the independent contributions to labor reallocation of: (i) absolute farm productivity growth in conjunction with the subsistence consumption of food, and (ii) relative sectoral productivity growth in conjunction with a low elasticity of substitution between farm and non-farm goods. We also augment this model to address the stylized fact of farm–non-farm (*sectoral*) wage gaps. We think of the two sectors as corresponding to distinct locations, and the wage differentials among otherwise identical workers as the result of migration costs which lead to endogenous and partial labor mobility.[6] Specifically, we adopt the two-sector model advanced by Dixit and Rob (1994). We extend their framework, which involves a combination of dynamic uncertainty and (exogenous) fixed costs, by allowing for subsistence consumption, and derive closed form solutions for migration thresholds and triggering wage gaps. In what follows, we describe this basic environment.

2.1 Preferences

Time is continuous, $t \in [0, \infty)$. There is a continuum of workers indexed by i, and the measure of the entire set of workers in the economy is normalized to unity, $i \in [0, 1]$, with each worker of measure zero. The set of workers is fixed over time.

There are only two sectors, farm (or "agriculture") (A) and non-farm ("manufacturing") (M), and each individual works in either the A sector or the M sector. We use the

[6] As we noted above these gaps were large and persistent even within identical skill categories. Search dynamics may be viewed as an alternative source of friction, but one which is more appropriate for wage inequality within each *occupation* (Moscarini, 2000). We do not model search dynamics as they are not an important ingredient of the structural transformation addressed in this paper.

convention that non-farm jobs are located in the city, and agricultural jobs on the farm.

Each worker lives forever. Workers have preferences over a composite consumption good (C) and inelastically supply one unit of indivisible labor. Workers can change sectors at any time as relative wages fluctuate (stochastically). In the baseline model there are no migration costs. In the general case, workers incur a fixed and sunk cost when they relocate.

To jointly determine the sectoral employment and consumption decisions, we assume that each worker maximizes the following expected utility function:

$$\mathrm{E}\left[\int_0^\infty e^{-\rho t}\ln C(t)\,dt - \sum_j e^{-\rho t_j} c\right], \tag{1}$$

subject to

$$\begin{aligned} C(t) &= \left[\eta^{\frac{1}{\nu}} c_M^{\frac{\nu-1}{\nu}}(t) + (1-\eta)^{\frac{1}{\nu}}\left(c_A(t)-\gamma_A\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu-1}{\nu}}, \\ w(t) &\geq p_M(t)c_M(t) + p_A(t)c_A(t), \end{aligned}$$

where c_M and p_M represent consumption and the unit price of the non-farm good; c_A and p_A represent consumption and the unit price of the farm good; and w represents nominal earnings, which depends solely on the sectoral wage rate of w_M in the M sector or w_A in the A sector.[7] E is the conditional expectations operator. The parameters are interpreted as follows: $\gamma_A \geq 0$ is the subsistence consumption of food, $\eta \in (0,1)$ measures the consumption weight of the manufactured good; $\nu > 0$ is the elasticity of substitution between manufacturing and agricultural goods; and $0 < \rho < 1$ is the subjective discount rate. If $\nu < 1$, goods are said to be "gross complements," and otherwise they are "gross substitutes." In this paper, we maintain that farm and non-farm goods are gross complements. (We present some evidence for this in Appendix B.)

[7]In the model labor is indivisible. In the development literature, the relevant unit of analysis is sometimes taken to be a household or family and depending on the context this simplification would matter. For the U.S. and the long-run dynamics that we are examining, however, we feel that this issue is less important.

The second term in the utility function captures the fixed cost $c \geq 0$ of changing the sector of employment (i.e., the cost of migration). Fixed costs of migration have long been recognized in the literature and are important ingredients of the accounts of the historical episode we study; see, e.g., Johnson (1944), and Maddox (1960). These migration costs can be viewed as a combination of transportation, training, adjustment, and "psychic" costs. We postpone a more precise interpretation of the potential magnitude of these costs to Section 4 where we empirically link them to historically observed wage gaps and earlier studies that provide an "accounting" of these costs. What matters for our purposes is that these costs can be modeled as a fraction of *current* income. To reduce the burden on notation, we also assume that costs are not sector specific. When $c > 0$, job switches take place in discrete instances indexed by t_j. Setting $c = 0$ gives the baseline model, which has no migration costs.

2.2 Production

A fraction L_A of the labor force is employed in the agricultural sector, leaving $L_M = 1 - L_A$ employed in the manufacturing sector. We use constant returns to scale technology with labor as the sole factor of production in both sectors:

$$Y_M = z_M L_M, \qquad Y_A = z_A L_A, \tag{2}$$

where z_M and z_A measure labor productivity in manufacturing and agriculture. To ensure subsistence consumption, we assume that the economy is always sufficiently productive: $\gamma_A < z_A$. Stochastic variability in z_M and z_A are the two ultimate sources of uncertainty, and we postpone the specification of these processes until Section 4. In what follows, we will also find it convenient to work with manufacturing productivity relative to that of agriculture, $z = z_M / z_A$.

In anticipation of some of the discussion ahead, let us mention two aspects of our setup. First, when there are no migration costs, the timing of moves is not an issue:

workers relocate when relative prices tend to change, and by doing so they instantaneously eliminate any wage differentials that might otherwise arise. As such, there is a one-to-one mapping from the realizations of productivity growth to endogenous variables, such as expenditure shares and relative prices. We exploit this aspect of the (baseline) model without fixed costs in Section 3. However, with fixed costs timing of migration matters. Thus, there is a "zone of inaction" where no migration takes place. Within this zone there is no longer a unique mapping from the realizations of productivity to the endogenous variables. Second, uncertainty, induced by the stochastic component of productivity growth, matters for the the timing of moves. Stochastic productivity growth also allows for short- to medium-run variations in the migration rate during structural change.

2.3 Intratemporal Equilibrium

First, consider the demand side. The following first-order conditions characterize the optimal resource allocation by each worker:

$$\frac{c_M}{c_A - \gamma_A} = \left(\frac{\eta}{1-\eta}\right)\left(\frac{p_A}{p_M}\right)^{\nu}. \tag{3}$$

Henceforth, we will work with the relative price, $p = p_M/p_A$, and assume that $(c_A - \gamma_A) > 0$ is always satisfied. Further, let $C_A = \int_0^1 c_A(i)\mathrm{d}i$ and $C_M = \int_0^1 c_M(i)\mathrm{d}i$ denote the *aggregate* consumption of farm and non-farm goods, respectively. Of course, the optimal consumption ratios in equation (3) must hold in the aggregate, as well as for each worker.

Second, consider the market clearing conditions:

$$\begin{gathered} C_A = Y_A, \qquad C_M = Y_M, \\ L_A + L_M = 1. \end{gathered} \tag{4}$$

Finally, we assume that factor and product markets are competitive, and that the wage rate in each sector is equal to the sectoral marginal revenue product of labor. Net labor flows between agriculture and manufacturing depend on the sectoral wage differential. It is useful to represent this differential as a ratio:

$$\frac{w_M}{w_A} \equiv w = z \cdot p. \tag{5}$$

In our general equilibrium framework, w is endogenous and depends on the exogenously given value for relative productivity, z, and on the endogenously determined value of the relative price, p. When $w > 1$, A sector workers will have an instantaneous incentive to migrate to the M sector. Historically, this has been the dominant tendency. When $w < 1$, however, there will be an instantaneous incentive to migrate to the A sector, or an incentive for "on-farm migration".

Using equations (2), (3) and (4), the expression for p is:

$$p(z_A, z, L_M) = \left[\left(\frac{1}{z}\right)\left(\frac{\eta}{1-\eta}\right)\left(\frac{1-L_M}{L_M}\right)\left(1 - \frac{\gamma_A}{z_A(1-L_M)}\right)\right]^{1/\nu}. \tag{6}$$

When $w = 1$, there is an inverse relationship between z and p; i.e., $z = 1/p$, and this expression can be simplified further.

Our task is to find equilibrium values of L_M for given realizations of z and z_A. In the next two sections, we calibrate the changes in sectoral labor shares under different assumptions about migration costs. We consider the baseline model (which has no migration costs) first, as it provides insights about the relative contribution of subsistence consumption to long-run structural change.

3 Equilibrium without Fixed Costs

3.1 Employment Shares and Engel's Law

When migration decisions do not entail any costs to the individual, job switches across sectors will eliminate any relative wage differentials that may arise due to fluctuations in z. In this case $w = 1$. Equations (3)–(4), together with this no-arbitrage condition, determine the equilibrium labor share L_M^f in terms of z, z_A and the model's parameters:

$$L_M^f(z_A, z) = \left[1 + \left(\frac{1-\eta}{\eta}\right) z^{1-\nu}\right]^{-1} \left(1 - \frac{\gamma_A}{z_A}\right). \tag{7}$$

Notice from equation (7) the independent contributions of relative and absolute productivity growth to changes in the sectoral allocation of labor. For future reference, also note that in this model real output (measured in terms of the farm good) is proportionate to productivity in the agricultural sector ($Y = z_A$), and agricultural output C_A is given by:

$$C_A(z_A, z) = \frac{z_A + \gamma_A \left(\frac{\eta}{1-\eta}\right) z^{\nu-1}}{1 + \left(\frac{\eta}{1-\eta}\right) z^{\nu-1}}. \tag{8}$$

3.1.1 Absolute Farm Productivity Growth

Consider now the role of absolute farm productivity growth in isolation. To keep things simple, assume that $z = 1$ implying that manufacturing productivity growth is identical to that of agriculture, such that growth in z_A is synonymous with growth in income. The initial impact of an increase in z_A on the terms-of-trade is favorable to manufacturing (given a low elasticity of substitution between farm and non-farm goods; see equation [6]). But this shift in the terms-of-trade, by raising the relative wage in manufacturing, causes labor to shift into manufacturing (equation [7]), which in turn lowers the terms-of-trade to its initial level of $1/z$. Also, from equation (8), an increase in aggregate income causes the consumption of agricultural goods to rise, but less than proportionally (implying that the consumption of manufacturing goods increases more than proportionally). In sum, the asymmetry in the income expansion paths for demand of each good creates incentives for labor reallocation when aggregate income rises. Absolute productivity growth is thus capable in isolation of explaining a shift of labor out of agriculture and into manufacturing.

3.1.2 Relative Farm Productivity Growth

Let us turn now to the role of relative productivity growth, z. To keep things simple, assume now that farm productivity z_A is constant. A decrease in z, meaning relatively faster growth in agricultural productivity (caused in this case by a decrease in z_M), will lower the terms-of-trade of the farm sector proportionately (see equation [6]).[8] This leads to a reallocation of labor *away* from the agriculture (see equation [7]), and also contributes to the Engel's Law effect.

We summarize the combined influences of absolute and relative productivity growth effects for the expenditure share of farm goods, θ_A, as follows:

$$\begin{aligned}\theta_A(z_A, z, L_M) &= \frac{C_A}{C_A + pC_M} \\ &= \left[1 + \left(\frac{\eta}{1-\eta}\right) z^{\nu-1} \left(1 - \frac{\gamma_A}{z_A(1 - L_M^f)}\right)\right]^{-1}. \end{aligned} \tag{9}$$

We have used equations (3) and (5) to derive this expression. Recall that in equilibrium $C_A = z_A(1 - L_M^f)$. With economic development one would expect γ_A/C_A to decrease at a diminishing rate over time. As long as this is the case and $\nu < 1$, this formulation establishes an inverse relationship between the share of expenditure on agricultural output and the productivity ratio. As z falls (i.e., farm productivity increases relative to non-farm productivity), the relative price of agricultural goods falls and the share of expenditure on agriculture declines as well. Indeed, the share of food in total expenditures in the U.S. has exhibited a secular decline. Thus, both absolute productivity growth and relative productivity growth contribute independently to the Engel's Law.[9]

[8] We have kept z_A constant for expositional purposes. However, a decrease in z will more likely be driven by rapid growth in z_A which will bring about both effects simultaneously.

[9] When $\nu = 1$, Engel's Law operates only through absolute productivity growth in agriculture, with the (controversial) implication that the farm terms of trade should remain constant over time. Olmstead and Rhode (2000, pp. 717–18) discuss the declining relative farm prices in the twentieth century U.S. and Mundlak (2000, p. 3) presents similar international evidence for the period 1967–1992.

3.2 Structural Change

We link Engel's Law to structural change by examining its effect on the change in the distribution of labor across sectors, L_M.[10] Specifically, we measure structural change by:

$$\mathrm{d}\ln L_M^f(z_A, z) = \left[\frac{\varrho(z_A)}{1-\varrho(z_A)}\right]\frac{\mathrm{d}z_A}{z_A} + (\nu - 1)\left[\frac{\vartheta(z)}{1+\vartheta(z)}\right]\frac{\mathrm{d}z}{z},$$

where

$$\varrho(z_A) = \frac{\gamma_A}{z_A}, \qquad \text{and} \qquad \vartheta(z) = \left(\frac{1-\eta}{\eta}\right) z^{1-\nu}.$$

Let $\tilde{\mu}_A \mathrm{d}t = \mathrm{d}z_A/z_A$ be the (realized) productivity growth in the farm sector, and $\tilde{\mu}\mathrm{d}t = \mathrm{d}z/z$ be the relative productivity growth rate. Working with the *realized* productivity growth rates is appropriate because in the baseline model, there is a one-to-one relationship between labor shares and realizations of productivity levels. This approach shows that the rate of change over time in the share of non-farm–farm employment is shaped by two contributions:

$$\frac{\mathrm{d}\ln L_M^f(z, z_A)}{\mathrm{d}t} = \underbrace{[\ \varrho(z_A)/(1-\varrho(z_A))\ ] \times \tilde{\mu}_A}_{\text{absolute farm prod. growth}} + \underbrace{(\nu-1)\,[\vartheta(z)/(1+\vartheta(z))] \times \tilde{\mu}}_{\text{relative farm prod. growth}}. \quad (10)$$

Recall that $\nu < 1$. Clearly, the absence of subsistence food consumption, $\gamma_A = 0$, implies that a higher *relative* farm productivity growth rate would be the only determinant of labor reallocation and off-farm migration. Even allowing for subsistence consumption of agricultural goods, i.e., $\gamma_A \neq 0$, as long as $\varrho \to 0$ over time (which occurs eventually since γ_A is constant and z_A grows over time), only relative productivity growth matters in the long run.

In the intermediate case, $\gamma_A \neq 0$ and $\varrho \neq 0$, the rate of labor reallocation into the non-farm sector depends on the relative strengths of these two factors. In the early stages of industrialization, absolute productivity growth in agriculture is likely to be the

[10] The change in the ratio of sectoral labor shares ($(1 - L_M)/L_M$) is another candidate for measuring structural change, but the main arguments are not sensitive to this alternative measure.

dominant factor behind structural change (because $\varrho/(1-\varrho)$ would be large). However, in periods when $\tilde{\mu}_A$ is positive and $\tilde{\mu}$ is negative, the relative contribution of each type of productivity growth to labor reallocation depends on which of the two terms, $\varrho(z_A)$ or $\vartheta(z)$, goes to zero at a faster rate. Further, if relative productivity is roughly constant $\tilde{\mu} = 0$ (on average), then L_M increases at a decreasing rate with rising z_A, and because $\varrho(z_A) \to 0$, the share of labor in each sector will eventually stabilize at a steady-state as both components of equation (10) will tend to zero.

3.3 Quantitative Performance of the Baseline Model

We now examine the quantitative performance of the baseline model (without fixed migration costs) in terms of its ability to account for the long-term rate of labor reallocation from the farm to the non-farm sectors. While the baseline model rules out wage gaps, it illustrates the influence of the subsistence–consumption ratio on the analysis, as well as the relative magnitudes that are involved.

To calibrate the sectoral labor shares, we need to (i) parameterize the baseline model, and (ii) have data on the realizations of relative and absolute farm productivity levels, $\tilde{z}, \tilde{z}_A$. The level of z is crucial for our exercise (due to the subsistence element involved in food consumption). We first obtained an estimate of the relative productivity series $\hat{\tilde{z}}$ using the share of food in total expenditures, as suggested by our model (see equation [9]). We also ensured that these implied productivity series were (broadly) consistent with the farm share of labor in 1991. We then used these productivity series and our parameter choices in equation (7) to obtain the calibrated sectoral employment shares.[11] Our parameter choices and overall procedure is documented in more detail in Appendix B.

Figure 2 shows the results of this calibration. The baseline model with $\gamma_A = 0$ is illustrative and we consider it first. This version of the model assigns no role to subsistence

[11] Although we did not explicitly impose this restriction in our calibration, we checked whether all annual calibrated values of γ_A/C_A were strictly less than one. This was the case in the parameter choices and productivity series implied by the expenditure share of food reported below.

consumption, so the overall rate of structural transformation is determined solely by the interaction between ν and implied relative productivity $\tilde{\mu}$; this can be seen by setting $\varrho(z_A) = 0$ in equation (10). The sample mean of the calibrated value of $\tilde{\mu}$ implied by expenditure shares is -1.44 percent for 1900–91, and -1.53 percent for 1920–91. While the calibrated relative productivity growth indeed favors the farm sector (and implies an acceleration in the farm productivity growth rate), the calibrated off-farm rate of labor reallocation (growth rate in $L_M/(1-L_M)$) is 1.3 percent, considerably smaller than the actual rate of 3.5 percent.

The performance of the baseline model can be improved by introducing the subsistence consumption of food. Figure 2 demonstrates the increase in the rate of structural change driven by absolute productivity growth in the farm sector. Assumptions about the initial subsistence–consumption (γ_A/C_A) ratio have a significant impact on the calibrated rate of labor reallocation out of the farm sector. Three examples are considered: $\gamma_A/C_A = 0.5, 0.7$ and 0.8 at the beginning of the calibration period. All series show convergence to actual labor shares by the 1990s. For initial $\gamma_A/C_A = 0.5$, the growth rate in $L_M/(1-L_M)$ between 1900–91 would be 1.7 percent, for initial $\gamma_A/C_A = 0.7$, it would be about 2.1 percent, and for $\gamma_A/C_A = 0.8$, it would be 2.5 percent. Thus, the discrepancy between the actual and calibrated series declines considerably as we increase the value of the initial subsistence–consumption ratio.

Taken literally, our imputed labor shares in Figure 2 suggest that there were relatively "too many" workers in the farm sector for about two decades beginning in the mid-1930's (regardless of our choice of initial γ_A/C_A). In this sense, the concerns raised by many commentators about an "insufficient transfer of labor" seem to be justified by a general equilibrium analysis of the U.S. economy. However, most of these commentators arrived at this conclusion indirectly after witnessing what appeared to be "large" and persistent farm and non-farm wage earnings differentials, which lie outside the scope of this baseline model. The natural next step, therefore, is to allow for sectoral real wage gaps in an equilibrium setting.

4 Equilibrium with Fixed Costs

In the baseline model, a critical assumption is that real wages are continuously equalized across sectors, $w = 1$. Before we model the recorded real wage differentials between farm and non-farm employment, it is useful to illustrate how departures from this assumption might affect the allocation of labor across sectors. To this end, we use the definition of relative wages ($w = z \cdot p$) and equation (6) to obtain:

$$\begin{aligned}\ln w &= \frac{\nu - 1}{\nu} \ln z + \frac{1}{\nu} \ln \left[\left(\frac{1 - L_M}{L_M} \right) \left(\frac{\eta}{1 - \eta} \right) \left(1 - \frac{\gamma_A}{z_A(1 - L_M)} \right) \right] \\ &= \frac{1}{\nu} \left[\ln \frac{1 - L_M}{L_M} - \ln \frac{1 - L_M^f}{L_M^f} \right]. \end{aligned} \tag{11}$$

Equation (11) shows that, in a general equilibrium framework, wage gaps contain significant information about labor "misallocation." Intuitively, the impact of the wage gap (in favor of the non-farm sector) on implied labor shares is as follows. Throughout the entire period we examine, farm wages have lagged behind non-farm wages, implying $w > 1$. For this to be compatible with our stated equilibrium conditions, the non-farm price has to be relatively high, which can only happen when there are relatively "too few" workers in the non-farm sector.

To get a feel for the magnitudes, consider the (absolute value of):

$$\frac{L_M^f - L_M}{L_M},$$

as a measure of discrepancy between actual and desired labor shares. Using the relative actual employment share and wage data (from *Historical Statistics*) from about 1920s to 1970, we solved equation (11) with $\nu = 0.1$. The resulting employment gaps ranged from a maximum of 2.5 percent to 0.2 percent. These estimates are indicative of relatively small misallocations of labor associated with large (upwards of 75 percent) relative wage gaps in favor of non-farm sector.[12]

4.1 Uncertainty and Relocation Decisions

To study equilibrium under uncertainty with fixed migration costs, we also need to specify the stochastic processes upon which workers condition their decisions. In the absence of fixed costs, uncertainty plays no role in the analysis because workers can respond to incentives instantaneously. With non-trivial costs, however, they have to consider the current and expected wage gaps, as well as the value of waiting before engaging in a costly move. In what follows, we will specify the stochastic processes that underlie the productivity growth rates, and then outline the basic solution. A detailed specification of these processes and solution of the model is contained in the Technical Appendix.

Unpredictable variability in the z_i's $(i = A, M)$ is the ultimate source of uncertainty, and we assume that the z_i's can be represented as geometric Brownian motion processes:

$$\frac{dz_i}{z_i} = \mu_i \, dt + \sigma_i \, d\omega_i, \tag{12}$$

where μ is the trend of the diffusion process, σ^2 is the instantaneous conditional variance, and $d\omega$ is a standard Weiner increment.[13] Given that both z_A and z_M follow geometric Brownian motion processes, it is also true that relative productivity $z = z_M/z_A$ follows a geometric Brownian motion process. Our analysis for the no-subsistence case is conducted solely on the basis of this relative productivity series. In the case with food subsistence, we must consider both z and z_A.

[12] Adjustments for cost of living and income tax differences between farm and city tend to increase real farm wages. For instance, following Hatton and Williamson (1992), if we apply an upward adjustment of 25 percent to the farm wages uniformly over our sample period, the bounds decline to 1.9 and 0.1 percent respectively. On the other hand, larger values of ν would increase the estimates reported here.

[13] This representation is also empirically quite plausible. We think of the term "trend" as a way to capture the medium- to long-run mean growth rate. Our view is that a century long structural transformation can involve long but transitional periods of relative and absolute productivity growth that differ from the steady-state rates. One could also allow for a mean-reverting process for relative productivity growth. Although this case is analytically more difficult, the arguments by Metcalf and Hassett (1995) suggest the thresholds we analyze below are not likely to be sensitive to this specification.

We determine workers' migration decision rules by comparing the benefits of staying versus relocating. In each period they choose a sector of employment, and incur a fixed cost should they relocate. Given these migration costs, there will be periods during which some workers will choose not to migrate (i.e., periods of "inaction"), despite fluctuations in productivity and relative wages. The zone of inaction corresponds to a range of wage differentials (conditional on the productivity values and L_M) that can be sustained up to a maximum value. The maximum sustainable wage gaps encapsulate each worker's evaluation of the benefits relative to costs. Benefits are always measured against real income in the destination sector. Once one of these wage-gap maxima is reached or exceeded, workers will relocate.

In particular, each worker's decision involves three components: (i) pricing the net option value of waiting (U_O), (ii) calculating the present value of consumption differentials (U_Δ) that arise when the worker is employed in one sector rather than the other, and (iii) comparing these to the cost of migration, c, to compute relocation thresholds that mark the boundaries of the zone of inaction. Both U_Δ and U_O are determined by productivity levels and L_M. Jointly they determine the desired direction of migration. Let positive values of U_Δ correspond to higher M sector wages in present value terms. Then, workers relocate from the farm sector A to the non-farm sector M when:

$$U_\Delta(z_A, z, L_M) + U_O(z_A, z, L_M) > c.$$

And workers relocate from sector M to A when:

$$U_\Delta(z_A, z, L_M) + U_O(z_A, z, L_M) < -c.$$

The triplets (Z_A, Z, L_M) at which the above expressions are satisfied with equality determine the relocation thresholds. Once these thresholds are crossed, workers relocate until the relative wage converges to the relevant maximum sustainable wage gap.

4.2 Equilibrium Wage Gaps

In this section we relate the relocation thresholds to maximum sustainable wage gaps, and show how observed wage gaps may endogenously decline over time as incomes increase. We start with the analytically more tractable case in which subsistence consumption is zero, and then discuss the implications of $\gamma_A > 0$ for our analysis.

4.2.1 The Case of No-Subsistence Consumption

In the case of logarithmic instantaneous utility and $\gamma_A = 0$ (unitary CRRA), the maximum sustainable wage gaps only depend on the parameters governing relative productivity z, and the decision rules turn out to be remarkably intuitive:

$$\begin{aligned} &\text{move from } A \text{ to } M \text{ if: } \ln w = \left(\frac{\nu - 1}{\nu}\right) s > 0, \\ &\text{move from } M \text{ to } A \text{ if: } \ln w = \left(\frac{\nu - 1}{\nu}\right) S < 0. \end{aligned} \tag{13}$$

In words, migrants use simple rules: When the (adjusted) wage gap between non-farm and farm wages, $\nu/(\nu-1) \cdot \ln w$, reaches or exceeds s percentage points, there is farm out-migration. As a result, the non-farm share of employment increases. Conversely, when this gap reaches S percentage points, there is farm in-migration and agriculture's share of employment rises. Within the (S, s) bands, workers are immobile despite current and expected wage gaps. The expressions for (S, s) are functions of the model parameters. Moreover, the maximum allowable wage gaps are time invariant, and the actual wage gap will tend to a stationary distribution (see the Technical Appendix).

4.2.2 The Case with Subsistence Food Consumption

When $\gamma_A > 0$, then the above analysis must be modified to allow for the absolute productivity effects discussed in Section 3, as well as to account for the influence of non-homothetic preferences and the resulting decrease in risk aversion on choices. The modification is complicated by the fact that one minus the subsistence–consumption ratio in equation (11) follows a diffusion process that does not lend itself to straightforward manipulation in the expected utility calculations. However, under an analytically convenient assumption, the migration decisions can be summarized in an analogous way to the case of $\gamma_A = 0$:[14]

$$\begin{aligned} &\text{move from } A \text{ to } M \text{ if: } \ln w = \left(\frac{1}{\nu}\right) s_{ws}(t) > 0, \\ &\text{move from } M \text{ to } A \text{ if: } \ln w = \left(\frac{1}{\nu}\right) S_{ws}(t) < 0, \end{aligned} \tag{14}$$

where $s_{ws}(t)$ and $S_{ws}(t)$ are the instantaneous (S, s) band parameters corresponding to the subsistence case.

The expressions in (14) have an important feature: with growth in incomes the maximum sustainable wage gaps narrow *endogenously*. To see how the interaction between the fixed costs of moving and subsistence consumption leads to declining risk aversion, first consider the impact of subsistence consumption on the wage gap process. As subsistence consumption γ_A approaches zero, preferences become homothetic, and the perceived riskiness of moves will not vary with income. In this case, the thresholds collapse to those given in the no-subsistence case above. By contrast, with subsistence food consumption, preferences are non-homothetic, and the risk associated with costly migration decisions declines as incomes increase. Consider, for instance, off-farm migration. At low levels of income, the relocation of labor to the non-farm sector is very risky because γ_A/C_A is very high, and fixed costs have a disproportionately large negative impact on the marginal benefit of relocating, if it is undertaken. Initially, then, moves to the city require larger wage gaps. As incomes rise, however, the fixed cost of migration becomes less significant, once adjusted for risk, and the maximum sustainable wage gaps will shrink endogenously

[14]This involves approximating the optimal thresholds by assuming that at each instant workers treat current and future values of instantaneous drift and conditional variance as fixed. However, they are allowed to update drift and conditional variance estimates as new information arrives. See the Technical Appendix for details.

adding further impetus to off-farm migration.

We demonstrate the economic significance of this mechanism in Figure 3, which shows the influence of productivity growth on the off-farm migration threshold for different values of the cost of migration, c. The parameter values for this example are: $\mu_A = .016$, $\sigma_A = .026$, $\mu_M = .004$, $\sigma_M = .016$, $\eta = .5$, $\nu = .01$, $\rho = .15$, $\gamma_A/C_A = .8$, and $c = \{0.05, 2, 5, 10\}$. Note how the maximum sustainable wage-gap for off-farm migration declines over time in the case of subsistence. Initially, potential off-farm migrants must observe between a 106 percent (for $c = .05$) to a 118 percent (for $c = 10$) wage premium in manufacturing before they will migrate. These thresholds decline and flatten such that, at the end of the period, off-farm migrants will relocate given a relatively modest 27 percent (for $c = .05$) to 39 percent (for $c = 10$) wage premium.

Figure 4 shows the picture that emerges from a sample path of our model economy. The model delivers some of the key features of the historic migration experience: there is a shift of labor out of farm and into non-farm sectors, a decline in the relative wage premium in favor of the non-farm sector, and a secular increase in the relative price of non-farm output.

4.3 Quantitative Performance of the Model with Fixed Costs

To illustrate the quantitative performance of the model with fixed costs, we use a variety of parameter values and calculate the two key variables that our model has been designed to address jointly: (i) the average percentage growth rate of the relative wage differential ($\Delta \ln w$), and (ii) the rate of structural transformation ($\Delta \ln L_M$). We focus on the (annualized) average percentage decline in wage differentials to highlight the potential significance of the internal mechanisms that lead to the fall in the observed wage gap.[15] We then compare these with historical trends. We should emphasize that the results presented here are meant merely to illustrate the sensitivity of the model to changes in

[15]Note that the range of wage premia the model is capable of has already been partly addressed in Figure 4.

parameter values, and that we are not attempting to "match" the historical means.

Consider first the historical means of $\Delta \ln w$ and $\Delta \ln L_M$, which tend to vary across subperiods and data sources (see Appendix A, Table A.1). The average annual growth rate for the share of labor in non-farm production was 0.57 percent for the period 1900 to 1990, and about 0.41 percent from 1920 to 1990. We used the former as our benchmark. Estimates for the rate of decline of the prevailing wage gap vary considerably depending on the data source. Data reported in Caselli and Coleman (2001) correspond to a range from -1.37 percent for 1900–1990 to -1.15 percent for the sub-period 1920–1990. However, their observed wage gap data are considerably higher than those reported in the *Historical Statistics*. If we (informally) combine these, and use the latter source for the initial year and Caselli and Coleman's estimate for the final year, this rate of decline drops to as little as 0.20 percent (for the period 1920–1990). Given this range, we used annual percentage growth in (w_M/w_F) of (negative) one percent as our benchmark.

In addition, the following parameter values were used in the simulations. As in the baseline model, we allowed the degree of subsistence, γ_A/C_A to take four different values: 0.8, 0.7, 0.5 and 0 (the no subsistence case). For each of these values, we examined three examples to demonstrate the strength of the mechanisms identified: (i) $\mu_A \gg \mu_M$ with $\mu_A = 0.03$ and $\mu_M = 0.004$; (ii) $\mu_A > \mu_M$ with $\mu_A = 0.016$ and $\mu_M = 0.004$; and (iii) $\mu_A = \mu_M = 0.01$. To check the sensitivity of the results to different values of the cost of migration, we considered the cases: $c = 0.05$, 2, 5, and 10. We simulated each of these 64 examples 1000 times and calculated the average values for $\Delta \ln w$ and $\Delta \ln L_M$.[16]

Table 1 presents the results for $c = 0.05$. The main conclusions are robust to all choices of c so we relegate the results based on these alternative costs to Appendix G. The results suggest that a relatively high initial subsistence consumption ratio as well as significant absolute *and* relative farm productivity growth rates are required to account for the historical record. For the parameter values $\gamma_A/C_A = 0.8$ or $\gamma_A/C_A = 0.7$ and

[16]To isolate the influence of parameters, we used the same random number generator seed for each of these examples. We also dropped a few (less then 10 per 1000) extreme cases.

$\mu_A \gg \mu_M$, the model performs reasonably well (after accounting for sampling error) on both accounts. However, the example using $\mu_A > \mu_M$, with $\gamma_A/C_A = 0.8$ also yields plausible results. For the labor reallocation process, these results are entirely consistent with our earlier conclusions based on the baseline model.

The empirical evidence, which we discuss in Appendix A, further suggests that relative productivity growth was much higher after 1948 compared to the earlier periods. The fact that the decline in the reported farm–non-farm wage gap and rapid labor reallocation out of agriculture coincided with an acceleration in relative farm productivity growth indicates the strength of the internal mechanisms that we stress.

4.4 Other Endogenous Mechanisms

The subsistence nature of food consumption combined with farm productivity growth, as we argued above, is a powerful internal mechanism that can lead to a decline in the observed wage gaps. There are of course other potentially important endogenous mechanisms, and we consider two here.

First, in the case of unitary CRRA (the logarithmic case), the income and substitution effects of relative productivity growth on the risk-valuation of migration costs cancel each other out. This leads to decision rules that are independent of the sectoral allocation of labor (L_M). When the CRRA differs from one, the relocation thresholds depend on L_M, as well as relative productivity growth. For example, with a relatively higher tolerance for the risk of staying (i.e., setting the CRRA parameter to less than one), the relocation thresholds are wider at low levels of L_M compared to the unitary case. The thresholds decline not only as incomes increase, but also as labor relocates to the non-farm sector.[17]

Second, wealth accumulation may reduce the relative significance of the fixed costs of migration. However, we conjecture that the existence of precautionary saving does not change the basic intuition about the endogenously declining wage gaps. Our conjecture

[17]A formal proof is available from the authors upon request.

is based on the reasoning that precautionary saving introduces two competing forces: if workers are risk averse, savings allow them to "delay" migration (acting as a buffer stock) and this widens the relocation thresholds. At the same time, the downside risk associated with a move would be less significant, which would narrow the thresholds.[18]

5 Conclusion

This paper makes a case for three important mechanisms that together can account for the key stylized facts of the U.S. off-farm labor reallocation experience of the twentieth century. Although this paper deals with the U.S. experience, the structural changes discussed have occurred and will continue to occur in developing countries. We believe that the mechanisms we identify are general and significant.

A Data

A separate Data Appendix, available from the authors, contains more detail about the variables used in the study. Here we list our main data sources.

Farm Share of Employment.— We parsed several series from the *Historical Statistics* (series D5, D6, D15 and D16). We used the same data to construct the series on the rate of labor reallocation out of agriculture.

Farm and Non-Farm Wages.— Census based estimates shown in Figure 1 are from Caselli and Coleman (2001). Annual series are from the *Historical Statistics* following Hatton and Williamson (1992). For the farm wage rate we used series K179. For the non-farm wage (lower skilled manufacturing) we used series D778 and D804. The main differences between these two data sources are demonstrated in Table A.1.

Expenditures on Food and Non-Food.— Personal consumption expenditures and implicit price deflators are from the Bureau of Economic Analysis. These are used to compute the growth rate of expenditures on food. The share of food in total expenditures is from Costa (2001) and is based on NIPA.

Farm and Non-farm Productivity Growth.—Our measures of productivity are the indices of employee output in the total private economy based on "farm and non-farm output per man-hour" (*Historical Statistics*, series D684 and D686). The relative sectoral

[18] One could also consider endogenously declining costs. For instance, Carrington et al. (1996) argue in a deterministic setup that the fixed (informational) cost of migration may decline due to a network externality formed in the destination sector, leading to accelerating migration.

productivity growth rate is the productivity growth rate in the non-farm sector minus that of the farm sector. Our maximum likelihood estimates of relative productivity growth (μ) and its standard deviation (σ) for the period 1920–1966 were, respectively, -0.0090 and 0.0645. These are broadly consistent with the TFP growth estimates in Jorgenson and Gallop (1992), and the former is consistent with Caselli and Coleman (2001). We also calculated relative labor productivity using "non-farm business sector output per hour index, 1992=100" from the Bureau of Labor Statistics, series PRS85006093 and "farm output per unit of farm labor index, 1992=100" from the *Economic Report of the President, 2002*, Table B–99. These are only available after 1948. The two series are not directly comparable due to changes in methodology, and the correlation between the overlapping portions of the two series is 0.42. These series show that the relative productivity growth in the farm sector accelerated in the post-war era, exceeding non-farm labor productivity growth by about 2.3 percent. This is the upper limit we use in our sensitivity analysis.

Relative Price of Food.—We used the wholesale price index for all commodities divided by the wholesale price index for farm goods, series E23 and E25. We also constructed a relative price series using the Bureau of Labor Statistics' consumer price indexes for non-food and food items.

B Parameter Choices

To calibrate the different versions of our model we need: (i) estimates of ν, η, and γ_A/C_A, (ii) an estimate of the level of relative productivity, z (for the baseline model), and (iii) trend and volatility parameters for farm and non-farm productivity growth (for the model with fixed costs). We discuss each of our choices in turn.

ν: We estimated this parameter using equation (3), and cointegration techniques. The results are available upon request, and here we summarize our main findings. To maximize consistency of data and sample period we used real per capita consumption *expenditure* data from 1929 to 2001 (NIPA), and relevant relative price series on food and non-food items, as described in Appendix A. (We used total civilian population from the *Survey of Current Business.*) To control for WWII and the Nixon price controls (August 1971–April 1974), we introduced separate dummy variables. While we were *not* able to reject the null of no cointegration, Park's (1992) $H(1,q)$ with $q \geq 1$ statistic typically provided no evidence for stochastic cointegration. Saikkonen's cointegrated regressions estimate ν was statistically different from zero, and the point estimate was between .1 and .2. We adopted the lower range and set $\nu = .1$. This choice of gross complementarity between food and non-food items is consistent with estimates reported in Brown and Heien (1972) that use micro level data.

η: We set η equal the long-run share of expenditure on non-farm goods, $\theta_A = 1 - \eta$. To see this, consider equation (9) under the plausible long-run conditions: $z = 1$, and $\gamma_A/C_A = 0$. Accordingly, we set this parameter equal to .8.

γ_A/C_A: To calibrate the ratio of subsistence expenditure out of agricultural consumption, we follow Orshansky (1965, pp. 6–8), and for 1960, we assume that the average per-capita weekly "subsistence cost" for January 1964 was \$240 per capita per annum for a 4-person family, that the share of income spent on food was 25 percent for an urban family of four, and that the per capita income of such a family during this period was \$1,854.[19] Then, $(\gamma_A/C_A)_{1960}$ is calculated as: $\$240/[(.25)\times\ \$1,854] = 0.51$. We use this estimate of the subsistence-consumption ratio in 1960 to back out the initial ratio for the beginning of our period, 1900. Under the plausible assumption that from 1900 to 1960 consumption expenditures on food increased at a compounded rate of 0.8 percent, this implies that the subsistence ratio should have *shrunk* at the same rate. The initial subsistence consumption ratio, $(\gamma_A/C_A)_0$, that solves $(\gamma_A/C_A)_{1900}e^{-(.008)\times 60} = .51$, is $(\gamma_A/C_A)_{1900} = \exp[.008 \times 60 + \ln(.43)] \approx .82$. In our simulations, we use 0.8 as the initial subsistence ratio.

z values: Our calibration results are sensitive to the *level* of relative productivity z. Although index-number-based relative productivity data can be used to obtain the drift and standard deviation of the productivity growth rate, they are not appropriate for levels estimates as these series arbitrarily set $z = 1$ in the base year. In order to approximate the level of z, we begin with the assumption that, by 1991, farm income and labor shares had converged (see Fig. 1) so that L_M was sufficiently "close" to L_M^f. Thus, we used the actual L_M at the end of our sample period to "back out" the corresponding equilibrium level of z (z^*):

$$z^* = \left[\left(1 - \frac{\gamma_A}{C_A}\right)\left(\frac{1 - L_M}{L_M}\right)\left(\frac{\eta}{1-\eta}\right)\right]^{1/(1-\nu)}.$$

Using the share of food in total expenditure, θ_A, in equation (C.2), we computed the implied relative productivity series ($\hat{\hat{z}}$) from for the baseline model with $\gamma_A = 0$:

$$\hat{\hat{z}} = \left[\left(\frac{\eta}{1-\eta}\right)\left(\frac{\theta_A}{1-\theta_A}\right)\right]^{1/(1-\nu)}.$$

We scaled the series to ensure that it converged to z^* at the end of the sample.

Drift and volatility parameters: Given the closeness of our relative labor productivity estimates (based on consumption data) and the relative TFP estimates (reported in our detailed Data Appendix), for the example $\mu_A > \mu_M$ we used the following: $\mu_A = 0.016$, $\sigma_A = 0.022$, $\mu_M = 0.004$, $\sigma_M = 0.016$, and $\rho_{M,A} = 0.12$. The drift (μ_i) parameters are from Jorgenson and Gallop (1992), and the volatility parameters ($\sigma_i, \rho_{M,A}$) are from Jorgenson et al. (1987). For the example $\mu_A \gg \mu_M$, we used the same volatility and correlation parameters, but used a higher relative drift. This is based on "non-farm

[19]Oshansky's share and income estimates are based on a BLS consumer expenditure survey. Her estimates on the lowest-cost food plan that can provide all of the recommended nutrients and vitamins as well as caloric intake ("subsistence consumption") relies on an earlier study by Cofer et al. (1962).

business sector output per hour index, 1992=100" from the Bureau of Labor Statistics, series PRS85006093 and "farm output per unit of farm labor index, 1992=100" from the *Economic Report of the President, 2002*, Table B–99. These series, which are only available after 1948, show that the relative productivity growth in the farm sector exceeded non-farm labor productivity growth by about 2.3 percent.

The fixed cost of moving, c: There are two possible approaches: direct and indirect. The direct approach uses the "accounting" approach of Maddox (1960). The indirect approach attempts to gauge the costs of migration using information on wage gaps. We consider the direct approach here, and our calibrated wage gap series provides collaborative evidence from the indirect approach. Maddox attempts to put dollar amounts on three of the four costs that he deems relevant for migration (from the migrant's perspective). He asserts that the opportunity cost of lost wages over the interim is trivial and can be safely ignored. He estimates the expenditures for transportation, lodging, and food assuming that it takes on average 10 days to find a job at the destination. He presents estimates for training costs, and assumes that the skills learned on the farm are entirely transferable. He also argues that while for children under 15 these education costs are borne entirely by the parents or family, there are reasons to believe that some of these transfers are eventually paid back to the family therefore treats training costs as a form of credit. For our purposes, we are interested in the portion of credit that is paid back to the family above and beyond what would have been paid back if the person had stayed on the farm, but there is no way to determine this. Maddox also stresses the significance of the subjective (or psychic) cost, but does not attempt to measure its significance relative to other costs. To conclude, although Maddox's accounting is promising, it leaves many degrees of freedom. Maddox provides the following figures:

- Transportation, food and lodging (10 days to find a non-farm job, and wait a week for the first pay check): $100 per person (1960 prices).

- Costs of education (from age 8 through 15): $52 (1954 prices); net cost of rearing and educating a farm child through age fifteen was more than $11,000, of which $5,000 was spent on food, clothing, and medical care. The average cost of rearing and educating through age seven was about $5,000. Assuming that only children aged between 8 and 15 contribute to work on farm, we arrive at $4,000 as a modest estimate of the educational costs. Assuming that sector specific education takes place between ages 8 and 15, how much of this can be viewed as sunk, and therefore wasted once one migrates? Since not all children left after the age of 7, we can use the age distribution of movers – 36 per cent of those who left were under 15 – and, assuming a uniform distribution, we are left with 18 percent of the population who have not invested in sector specific skills. Thus, the *probability* of incurring this cost is (1-.18) = .82. Suppose these sector specific skills amount to one month's worth of education. Then: 0.82×$4,000/(7 years of education× 9 months of education per year)= $52.) ≈ $60 (1960 prices). In other words, each month's worth of farm specific education represents a sunk (and wasted) cost of \$60.

- Subjective or psychic costs: no estimate.

To convert this to a utility "tax" on current aggregate consumption index, recall the consumption index:

$$\begin{aligned} C &= \left[\eta^{1/\nu} c_M^{(\nu-1)/\nu} + (1-\eta)^{1/\nu} (c_A - \gamma_A)^{(\nu-1)/\nu}\right]^{\nu/(\nu-1)} \\ &= \left[\eta^{1/\nu} \left(\frac{c_M}{c_A}\right)^{(\nu-1)/\nu} + (1-\eta)^{1/\nu} \left(1 - \frac{\gamma_A}{c_A}\right)^{(\nu-1)/\nu}\right]^{\nu/(\nu-1)} c_A \\ &= x_A c_A. \end{aligned}$$

Assume that in 1960 $\gamma_A/c_A = .51$ (see above), and $c_M/c_A = (1-.25)/.25 = 3$ (from expenditure shares). Therefore, for $\nu = .1$ and $\eta = .8$, we have $x_A = 2.77$. Now note that the *dollar* value of consumption $\$C = x_A \$ c_A = \theta_A \times$ (\$1,854 /4) $\times$ 2.77 = \$322 per person per annum. Even in the absence of psychic costs, \$100 is a significant fraction of *monthly* expenditures, and would be a binding constraint if workers cannot borrow against their future labor income. In other words, actual dollar based figures suggest that the ratio of payable migration costs (\$100) relative to annual consumption (\$322) is approximately 33%, and the ratio of total migration costs inclusive of sunk human capital (\$160) relative to annual consumption is approximately 50%, and is roughly 6 times monthly expenditures. We also calculated the theoretical value of C using the 1960 values for $L_M = 0.917$ (from the data), $z = 0.156002$ (from our baseline model simulations), $c_A = 1$ (normalization), $\gamma_A/c_A = .51$ (see above), $\nu = .1$ and $\eta = .8$. These choices give us $x_A = 1.77$, and given our normalization, $C = 1.77$. The lower value of x_A in the theoretical calculations relative to the dollar based estimates may indicate that non-farm productivity might be slightly underestimated. For instance, setting $c = 1$ suggests that $(C/\exp[c]) = 0.65$; i.e., migration costs would be the equivalent of 1.5 times monthly consumption (in 1960). Note that C is increasing over time because: (i) income (c_A) is increasing and (ii) γ_A/c_A is decreasing.

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Table 1
The Effects of Subsistence Consumption and Productivity Growth on Changes in Wage Gap and Labor Reallocation, $c = .05$

	$\Delta \ln$ Wage Gap	Deviation	$\Delta \ln L_M$	Deviation
Historical values	$-.0100$	–	.0057	–
$\gamma_A/C_A = .8$				
$\mu_A \gg \mu_M$	$-.0133$	.0033	.0049	.0008
$\mu_A > \mu_M$	$-.0125$	.0025	.0041	.0016
$\mu_A = \mu_M$	$-.0097$	.0003	.0027	.0030
$\gamma_A/C_A = .7$				
$\mu_A \gg \mu_M$	$-.0100$	.0000	.0043	.0014
$\mu_A > \mu_M$	$-.0090$	.0010	.0035	.0022
$\mu_A = \mu_M$	$-.0043$	.0057	.0022	.0035
$\gamma_A/C_A = .5$				
$\mu_A \gg \mu_M$	$-.0056$	.0044	.0032	.0025
$\mu_A > \mu_M$	$-.0048$	.0052	.0025	.0032
$\mu_A = \mu_M$	.0026	.0126	.0014	.0043
$\gamma_A/C_A = 0$				
$\mu_A \gg \mu_M$	-3.5926	3.5826	.0008	.0049
$\mu_A > \mu_M$	-1.4499	1.4399	.0006	.0051
$\mu_A = \mu_M$	-0.8841	0.8741	$-.0001$	.0058

Note: The following parameters are used for the four examples: (i) $\mu_A \gg \mu_M$ with $\mu_A = 0.03$ and $\mu_M = .004$; (ii) $\mu_A > \mu_M$ with $\mu_A = .016$ and $\mu_M = .004$; and (iii) $\mu_A = \mu_M = 0.01$. We set $c = .05$. We simulate each of these 16 examples 1000 times and we report the average values in the table. Deviations are reported as absolute values. See the text for further details.

Table A.1
Growth Rates of Non-farm Share of Employment and Relative Wage

	L_M	w_M/w_F	
Period	[1]	[2]	[3][a]
1900–1990	0.0057	−0.0137	−0.0021
1920–1990	0.0041	−0.0115	−0.0020
1940–1990	0.0042	−0.0144	−0.0129

Source: Column 1 from *Historical Statistics* as explained in Appendix A. Column 2 from Caselli and Coleman (2001). Column 3 from *Historical Statistics* as explained in Appendix A and Caselli and Coleman.
Notes: L_M is non-farm share of employment. Caselli and Coleman (2001) parse per employee service income (1900–1920) with per worker wage income (1940–1990), both from decennial population censuses. In column 3 first year relative wages (w_M/w_F) are from *Historical Statistics* and last year wages are from Caselli and Coleman. Growth rate in L_M is annual averages. For w_M/w_F average growth rate is calculated using only the endpoints and finding the implied exponential growth rate of $\ln(w_M/w_F)$.
[a] 1902–1990.

Figure 1:

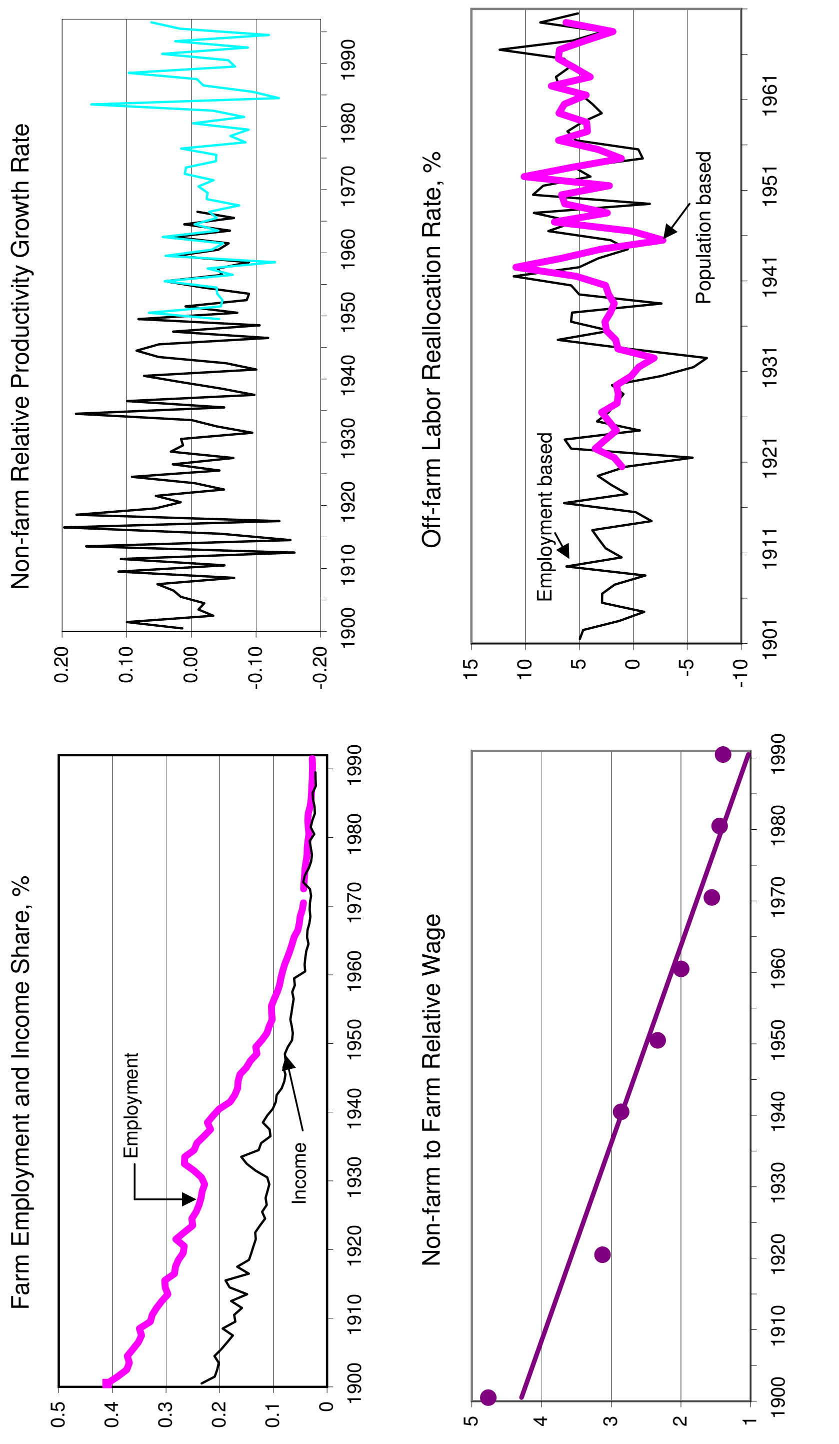

Figure 1: U.S. Farm Data

Figure 2:

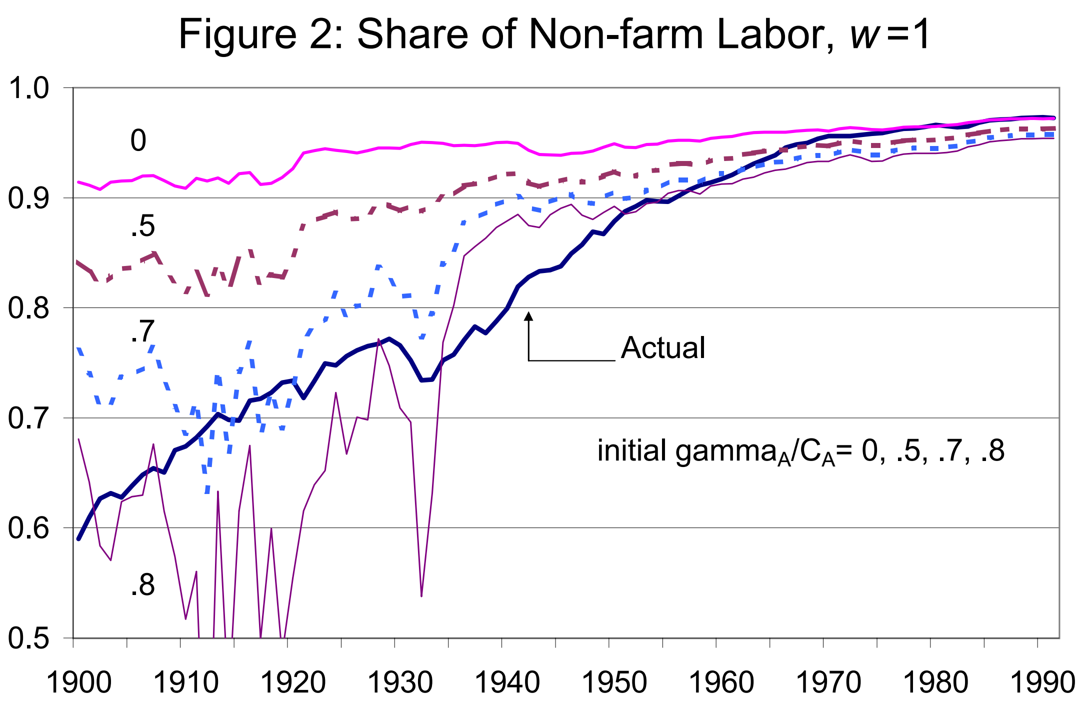

Figure 2: Share of Non-farm Labor, w =1
1.0
0.9
0.8
0.7
0.6
0.5
0
.5
.7
.8
Actual
initial gamma$_A$/C$_A$= 0, .5, .7, .8
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990

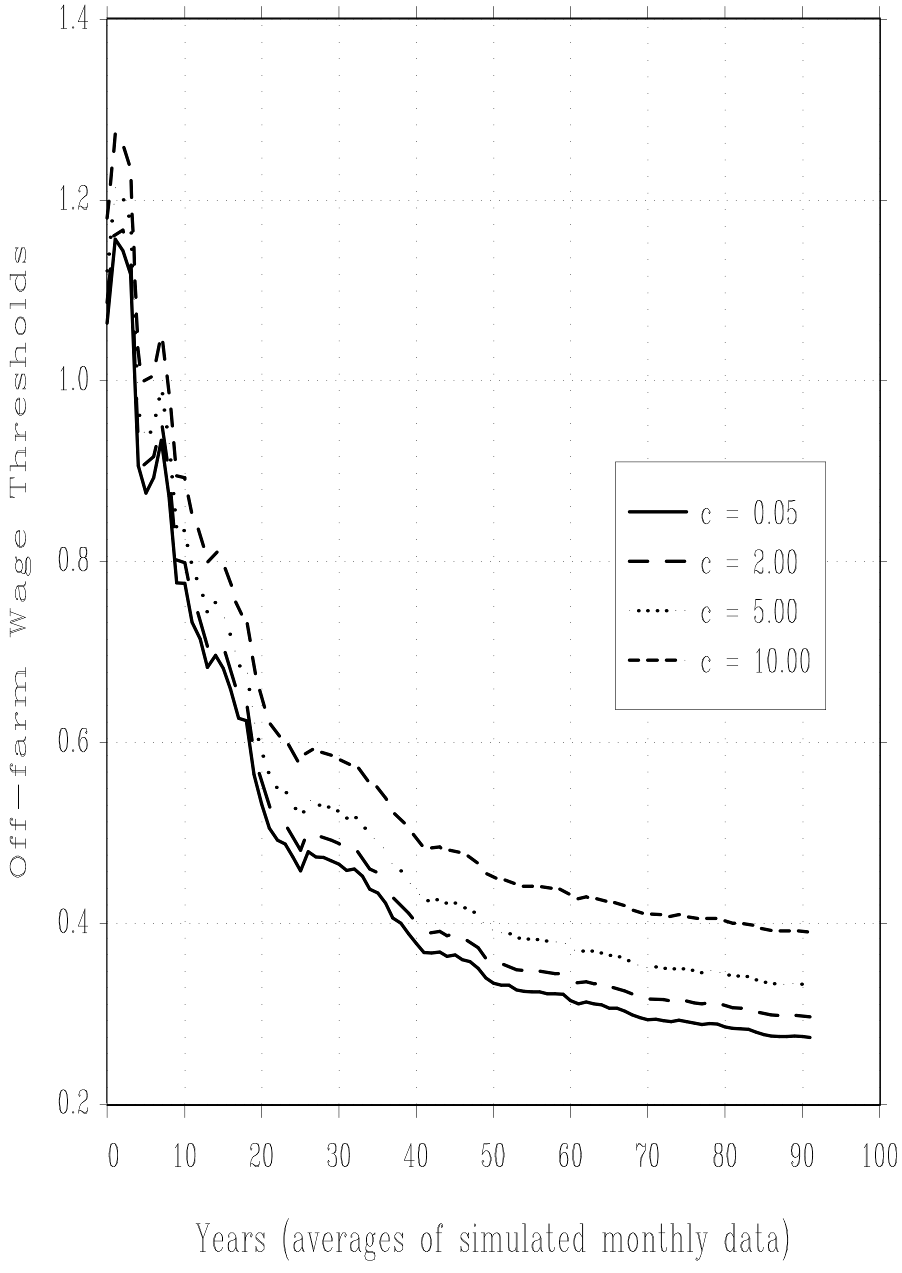

Figure 3: Relocation Thresholds

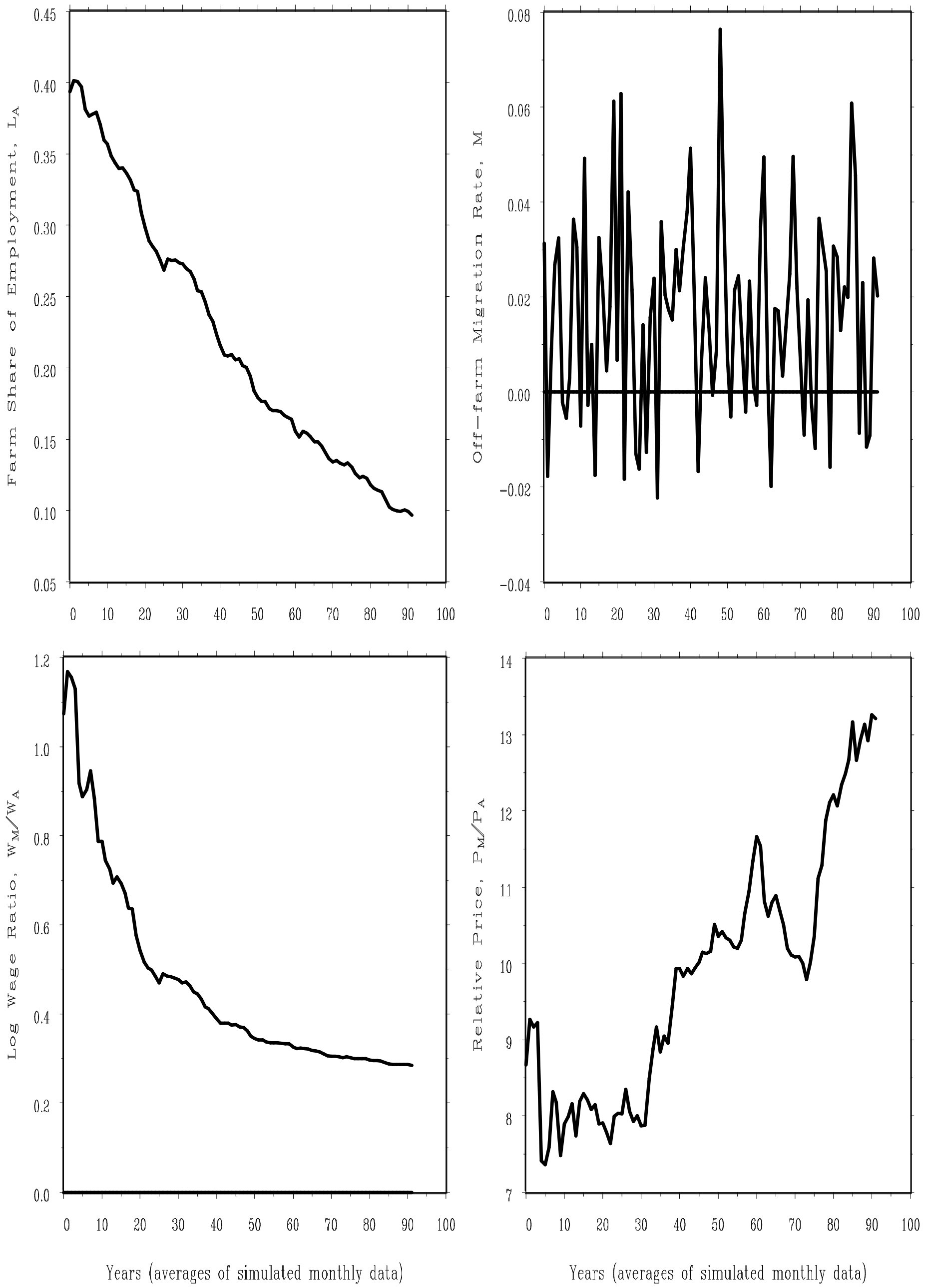

Figure 4: A Sample Path

Technical Appendix for Productivity Growth and Agricultural Out-Migration in the United States

Benjamin N. Dennis Talan B. İşcan

C Solution of the Frictionless Model

C.1 Expenditure and Labor Shares

To derive the expenditure and labor share equations given in the text, we first distinguish between aggregate consumption C_A, C_M and the quantity demanded by worker i, $c_A(i), c_M(i)$. Then using equation (3):

$$\frac{c_M(i)}{c_A(i)-\gamma_A} = \left(\frac{\eta}{1-\eta}\right)\left(\frac{1}{p}\right)^{\nu}.$$

Let:

$$w^j(i) = \begin{cases} w_M \text{ if } i \in [0, L_M) \\ w_A \text{ if } i \in (L_M, 1] \end{cases}.$$

The budget constraint is:

$$w^j(i) = p_A c_A(i) + p_M c_M(i).$$

Solving for the demand for farm goods gives:

$$c_A(i) = \frac{w^j/p_A + \gamma_A\left(\frac{\eta}{1-\eta}\right)p^{1-\nu}}{1+\left(\frac{\eta}{1-\eta}\right)p^{1-\nu}}.$$

The market clearing equations are:

$$\begin{aligned}
&\int_0^1 c_A(i)\mathrm{d}i = C_A = z_A(1-L_M), \\
&\int_0^1 c_M(i)\mathrm{d}i = C_M = z_M L_M, \\
&z_A(1-L_M) + p z_M L_M = p L_M w_M + (1-L_M) w_A = Y.
\end{aligned}$$

The last line is real output measured in farm-goods prices. Noting that:

$$C_A = L_M\left[\frac{w_M/p_A + \gamma_A\left(\frac{\eta}{1-\eta}\right)p^{1-\nu}}{1+\left(\frac{\eta}{1-\eta}\right)p^{1-\nu}}\right] + (1-L_M)\left[\frac{w_A/p_A + \gamma_A\left(\frac{\eta}{1-\eta}\right)p^{1-\nu}}{1+\left(\frac{\eta}{1-\eta}\right)p^{1-\nu}}\right],$$

and using the definition of real output measured in units of farm goods Y, as well as the expenditure share of farm goods:

$$\theta_A = \frac{C_A}{C_A + pC_M} = \frac{C_A}{Y}$$

we obtain:

$$C_A = \frac{Y + \gamma_A \left(\frac{\eta}{1-\eta}\right) p^{1-\nu}}{1 + \left(\frac{\eta}{1-\eta}\right) p^{1-\nu}}. \tag{C.1}$$

Consequently, we can rewrite the expenditure share of farm goods:

$$\theta_A = \left[1 + \left(\frac{\eta}{1-\eta}\right)\left(\frac{w}{z}\right)^{1-\nu}\left(1 - \frac{\gamma_A}{C_A}\right)\right]^{-1}. \tag{C.2}$$

To determine L_M, we start with equation (C.1) for aggregate farm good consumption:

$$Y = C_A \left[1 + \left(\frac{\eta}{1-\eta}\right) p^{1-\nu}\left(1 - \frac{\gamma_A}{C_A}\right)\right]$$

We use the market clearing condition $C_A = z_A(1 - L_M)$, the definition of $Y = z_A(1 - L_M) + pz_M L_M$, and $w = zp$ to obtain the expression for L_M:

$$L_M = \left[1 + z\left(\frac{1-\eta}{\eta}\right)\left(\frac{w}{z}\right)^{\nu}\right]^{-1}\left(1 - \frac{\gamma_A}{z_A}\right). \tag{C.3}$$

When $w = 1$, we obtain equation (7).

Now consider the remaining endogenous variables, p, C_A, and aggregate income Y. From equation (5) we have $p = 1/z$, and C_A and Y are given by:

$$Y = z_A\left[1 + (p\ z - 1)L_M^f\right] = z_A, \tag{C.4}$$

$$C_A = \frac{Y + \gamma_A \left(\frac{\eta}{1-\eta}\right) p^{1-\nu}}{1 + \left(\frac{\eta}{1-\eta}\right) p^{1-\nu}}. \tag{C.5}$$

C.2 The Consumption Based Price Index

We want to find P which is the minimum expenditure $Z = C_A + pC_M$ such that $C = \Omega(C_A, C_M) = 1$, given p. Note that consumption C is:

$$C = \left[\eta^{1/\nu} c_M^{(\nu-1)/\nu} + (1-\eta)^{1/\nu}\left(c_A - \gamma_A\right)^{(\nu-1)/\nu}\right]^{\nu/(\nu-1)}.$$

subject to $Z = C_A + pC_M$. Maximizing C subject to the constraint on Z, we obtain $P = [(1-\eta) + \eta p^{1-\nu}]^{\frac{1}{1-\nu}} + \gamma_A$. Thus, real GDP is given by:

$$\begin{aligned} Y_{CPI} &= \frac{Y}{[(1-\eta) + \eta p^{1-\nu}]^{\frac{1}{1-\nu}} + \gamma_A}, \\ &= \frac{z_A}{[(1-\eta) + \eta z^{\nu-1}]^{\frac{1}{1-\nu}} + \gamma_A} \end{aligned}$$

and it is clear that either an increase in z_A *or* an increase in z will, ceteris paribus, raise real GDP.

C.3 Structural Change

We calculate the trend rate of labor reallocation analytically, and use this to match the mean rate of structural change in our sample period. While we calculate the trend from the equilibrium labor shares when there are no fixed costs, they are also informative about the case with fixed costs. Even with fixed costs, in the long-run the historical mean log labor shares deviate from the baseline model's log labor shares by a constant value [see equation (E.10) below]. In other words, since fixed costs only influence the short-run dynamics, the steady-state change in labor shares in the baseline model still appropriately gives the prevailing long-run rate of structural transformation. We treat the no-subsistence and subsistence cases separately below.

C.3.1 The Case of No-Subsistence Consumption

Recall that, in the no-subsistence case without fixed costs, the labor allocation is given by equation (7) with γ_A set to zero. Solving for the baseline model's (log) labor shares (agriculture relative to manufacturing), we obtain:

$$\ln\left(\frac{1-L_M^f}{L_M^f}\right) = \ln\left(\frac{1-\eta}{\eta}\right) + (1-\nu)\ln z. \tag{C.6}$$

Since z follows geometric Brownian motion, total differentiation gives

$$\begin{aligned} \mathrm{d}\ln\left(\frac{1-L_M^f}{L_M^f}\right) &= (1-\nu)\mathrm{d}\ln z \\ &= \mu_* \mathrm{d}t + \sigma_* \mathrm{d}\omega. \end{aligned} \tag{C.7}$$

In the expression above the long-run rate of decline in the agricultural labor share is given by the deterministic part:

$$\mu_* = (1-\nu)\mu_z.$$

Notice that this is negative when $0 < \nu < 1$ and $\mu_z < 0$.

C.3.2 The Case with Subsistence Food Consumption

In this case, we begin with equation (7), manipulate, take logs, and differentiate to obtain:

$$\mathrm{d}\ln\left(\frac{L_M^f}{1-L_M^f}\right) = \left(\frac{\varrho(z_A)}{1-\varrho}\right)\left[\frac{1+\vartheta(z)}{\varrho(z_A)+\vartheta(z)}\right]\frac{\mathrm{d}z_A}{z_A} - (1-\nu)\left[\frac{\vartheta(z)}{\varrho(z_A)+\vartheta(z)}\right]\frac{\mathrm{d}z}{z}. \quad \text{(C.8)}$$

where $\varrho(z_A) = \frac{\gamma_A}{z_A}$, and $\vartheta(z) = \left(\frac{1-\eta}{\eta}\right) z^{1-\nu}$. The rate of structural transformation over time is again made up of two contributions:

$$\underbrace{[\varrho(z_A)/(1-\varrho(z_A))]\,[(1+\vartheta(z))/(\varrho(z_A)+\vartheta(z))]\times\mu_A}_{\text{absolute farm prod. growth}} - \underbrace{(1-\nu)\,[\vartheta(z)/(\varrho(z_A)+\vartheta(z))]\times\mu}_{\text{relative farm prod. growth}}.$$

This measure has the following nice properties: if $\gamma_A = 0$, then the rate of structural change only depends on $\mu(1-\nu)$. Even in the case of subsistence consumption of agricultural goods, i.e., $\gamma_A \neq 0$, as long as $\varrho(z_A) \to 0$ over time (which occurs eventually since γ_A is constant and z_A grows on a positive trend), only relative productivity growth matters for structural change.

D Productivity Processes

This section specifies two of the key stochastic processes used in our analysis.

D.1 The Stochastic Process for z

First note that $z = z_M z_A^{-1}$ follows a correlated Brownian motion, where:

$$\mathrm{d}z_A = \mu_A z_A \mathrm{d}t + \sigma_A z_A \mathrm{d}\omega_A, \quad \text{(D.1)}$$
$$\mathrm{d}z_M = \mu_M z_M \mathrm{d}t + \sigma_M z_M \mathrm{d}\omega_M, \quad \text{(D.2)}$$
$$\mathrm{E}[\mathrm{d}\omega_M \mathrm{d}\omega_A] = \rho_{M,A}\mathrm{d}t.$$

Noting that: $\frac{\partial z}{\partial z_M} = z_A^{-1}$, $\frac{\partial z}{\partial z_A} = -z_M z_A^{-2}$, $\frac{\partial^2 z}{\partial z_M \partial z_A} = -z_A^{-2}$, $\frac{\partial^2 z}{\partial z_M 2} = 0$, and $\frac{\partial^2 z}{\partial z_A 2} = 2 z_M z_A^{-3}$, the process for z is therefore:

$$\frac{\mathrm{d}z}{z} = \left(\mu_M - \mu_A + \sigma_A^2 - \rho_{M,A}\sigma_M\sigma_A\right)\mathrm{d}t + \sigma_M \mathrm{d}\omega_M - \sigma_A \mathrm{d}\omega_A,$$

with:

$$\text{the drift rate of } \mu_z = \left(\mu_M - \mu_A + \sigma_A^2 - \rho_{M,A}\sigma_M\sigma_A\right),$$
$$\text{the volatility of } \sigma_z^2 = \left(\sigma_M^2 + \sigma_A^2 - 2\rho_{M,A}\sigma_M\sigma_A\right).$$

D.2 The Stochastic Process for γ

Consider the following subsistence variable:

$$\gamma = \left[1 - \left(\frac{\gamma_A}{1 - L_M}\right) z_A^{-1}\right]. \tag{D.3}$$

The elementary source of stochastic variation over the planning horizon is due to the evolution of z_A as γ_A and $1 - L_M$ are considered to be constant in the case with frictions (from the potential migrant's perspective). Given equations (D.1) and (D.3), we can calculate the stochastic path of γ using Ito calculus as follows:

$$\mathrm{d}\gamma \;\; = \;\; (1-\gamma)\left[\mu_A - \sigma_A^2\right] \mathrm{d}t + (1-\gamma)\sigma_A \mathrm{d}\omega.$$

where we have used the fact that $\left(\frac{\gamma_A}{1-L_M}\right) z_A^{-1} = 1 - \gamma$. Now let:

$$\mu_\gamma(\gamma) = \left(\frac{1-\gamma}{\gamma}\right)\left[\mu_A - \sigma_A^2\right]; \quad \text{and } \sigma_\gamma(\gamma) = \left(\frac{1-\gamma}{\gamma}\right)\sigma_A,$$

and we can re-identify the process driving γ as:

$$\frac{\mathrm{d}\gamma}{\gamma} \;\; = \;\; \mu_\gamma(\gamma)\mathrm{d}t + \sigma_\gamma(\gamma)\mathrm{d}\omega. \tag{D.4}$$

The coefficients in this process are time-varying, and we evaluate them using numerical methods. However, note that the subsistent coefficients depend on the moments of the stochastic process for z_A, consistent with our discussion of the role of absolute productivity in the presence of subsistence.

E Solution with Fixed Costs

E.1 Relocation Thresholds

This section presents the derivations for the relocation thresholds. Our arguments are essentially identical to those in Dixit and Rob (1994, especially pp. 60–66), with the difference being that they consider the case $\theta \neq 1$ and $\nu = 1$, whereas we solve for $\theta = 1$ and $\nu \neq 1$ (see also their Fig. 1).

In what follows, for ease of exposition, we refer to the non-farm sector as "city" and the farm sector as "farm", and consider U_Δ and U_O in turn.

First consider $U_\Delta(z, L_M)$, which is the present discounted utility of consumption differentials assuming that the worker will never reallocate in the future. If this expression is positive the worker has an instantaneous incentive to switch from F to N. For an initial level of productivity, z_0, and for a given L_M (since each worker takes this as a constant in competitive equilibrium), we have:

$$U_\Delta(z, L_M) \;=\; \mathrm{E}\left[\int_0^\infty e^{-\rho t} \ln w(z, L_M)\, \mathrm{d}t\right]. \tag{E.1}$$

We will treat the no-subsistence case first, then move to the case with subsistence.

E.1.1 The Case of No-Subsistence Consumption

Rewrite the expression for $\ln w$ given in equation (11) as:

$$\ln w \;=\; \frac{1}{\nu}\ln B(L_M) + \frac{\nu-1}{\nu}\ln z.$$

where:

$$B(L_M) = \left(\frac{1-L_M}{L_M}\right)\left(\frac{\eta}{1-\eta}\right).$$

As given by Harrison (1985, pp. 44–45), the expected discounted value of Δ is the solution $U_\Delta(z, L_M)$ to the following differential equation:

$$\rho U_\Delta(z, L_M) - \mu_z z \frac{\partial U_\Delta(z, L_M)}{\partial z} - \frac{\sigma_z^2}{2} z^2 \frac{\partial U_\Delta(z, L_M)^2}{\partial^2 z} = \Delta(z, L_M).$$

Solving equation (E.1) for the function U_Δ, we obtain:

$$U_\Delta \;=\; \frac{1}{\rho\nu}\left[\ln B(L_M) + (\nu-1)\ln z_0\right] + \frac{\nu-1}{\rho^2\nu}\left(\mu_z - \frac{\sigma_z^2}{2}\right). \tag{E.2}$$

Note that U_Δ is calculated on the basis of a permanent migration to the other sector. However, farm return migration is a well-documented phenomenon which we wish to allow for. This entails a tradeoff of options. When migration actually takes place, the worker gives up the "option to stay" in the sector of origin and in return acquires the "option to stay" in the destination sector. The first derives its value from waiting before engaging in a costly move, and the second derives its value from the possibility of switching back. In each period, a rational worker considers the net value of these two options, as well as U_Δ, so that the thresholds are actually higher than they otherwise would be if the worker acted in a myopic way and ignored the value of waiting.

We define U_O as the option of staying in the non-farm sector minus the option of staying in the farm sector, and we value it as a non-dividend paying "asset" measured in utility terms, whose value depends purely on the "capital gains" that may result from fluctuations in z. Over a time interval dt, the expected return on this net option (ρU_O) is equal to the expected capital gain on the option:

$$\rho U_O(z, L_M) = \frac{1}{\mathrm{d}t}\mathrm{E}\left[\mathrm{d}U_O(z, L_M)\right].$$

We use Itô's Lemma to expand the right hand side of this equation and obtain:

$$\rho U_O(z, L_M) - \mu z \frac{\partial U_O(z, L_M)}{\partial z} - \frac{\sigma^2}{2} z^2 \frac{\partial U_O(z, L_M)^2}{\partial^2 z} = 0.$$

The general solution to this equation takes the form [see, e.g., Dixit and Pindyck (1994, pp. 140–144)]:

$$U_O(z, L_M) = K_1(L_M) z^{\beta_1^z} + K_2(L_M) z^{\beta_2^z}, \tag{E.3}$$

where K_1 and K_2 are constants to be determined, and $\beta_1^z > 0$ and $\beta_2^z < 0$ are the roots of the quadratic equation:

$$\mathcal{Q}(\beta^z) \equiv \frac{\sigma^2}{2} \beta^z(\beta^z - 1) + \mu \beta^z - \rho. \tag{E.4}$$

Together equations (E.2) and (E.3) give the total utility gains associated with migration. Our next task is to evaluate these terms at the relocation thresholds Z_{OFF} and Z_{ON}, and find analytic expressions for them.

Migration from Farm to City For a worker considering whether to migrate from the farm to the non-farm sector, the present discounted value of the wage gap between the non-farm and farm sector $(\ln(w_N) - \ln(w_A))$ plus the net option value of migrating at Z_{OFF} is:

$$U_\Delta(Z_{\text{OFF}}) + U_O(Z_{\text{OFF}}) = c.$$

The first term is given by equation (E.2). For the option value of migration our choice of which root to use depends on the value of ν. Consider the plausible case where $\nu < 1$. For farm-to-city migration the net options value is the value of the option to stay in the city (destination) sector minus the value of waiting in the farm (origin) sector (which is forfeited in the event of a move). For $\nu < 1$, within the zone of inaction, a higher value of z should reduce the option value of waiting to switch to the non-farm sector: the combination of higher non-farm productivity and low elasticity of substitution between farm and non-farm products depresses the prices of the non-farm sector, hence relative non-farm wages, and makes migration less likely. As the value of z approaches the threshold Z_{OFF} (from above), the option of waiting, which is to be given up, increases in value, so:

$$\lim_{z \downarrow Z_{\text{OFF}}} U_O > 0.$$

Setting $K_1 = 0$, and using the negative root, β_2^z, satisfies this, and we note that $K_2 < 0$.

The two optimality conditions can now be stated. Value-matching:

$$\frac{1}{\rho\nu} \left[\ln B(L_M) + (\nu - 1) \ln Z_{\text{OFF}}\right] + \frac{\nu - 1}{\rho^2 \nu} \left(\mu_z - \frac{\sigma_z^2}{2}\right) + K_2 Z_{\text{OFF}}^{\beta_2^z} = c.$$

and smooth-pasting:

$$\left(\frac{\nu-1}{\nu\rho Z_{\text{OFF}}}\right)+K_2\beta_2^z Z_{\text{OFF}}^{\beta_2^z-1}=0.$$

Solving these for Z_{OFF}, the threshold value for off-farm migration gives:

$$Z_{\text{OFF}}=\exp\left\{s+\frac{1}{1-\nu}\ln B(L_M)\right\},$$

where:

$$\begin{aligned}
s &= \frac{\nu\rho c}{\nu-1}+\frac{1}{\rho}\left(\frac{\sigma_z^2}{2}-\mu_z\right)+\frac{1}{\beta_2^z},\\
\beta_2^z &= \frac{1}{2}-\frac{\mu_z}{\sigma_z^2}-\sqrt{\left(\frac{\mu_z}{\sigma_z^2}-\frac{1}{2}\right)^2+\frac{2\rho}{\sigma_z^2}}<1.
\end{aligned}$$

Migration from City to Farm The relevant move is from city to farm. We can use the same logic as above to find:

$$U_\Delta(Z_{\text{ON}})+U_O(Z_{\text{ON}})=-c.$$

At the relocation threshold, the present discounted value of the wage gap ($\ln w$) is:

$$U_\Delta = \frac{1}{\rho\nu}\left[\ln B(L_M)+(\nu-1)\ln Z_A\right]+\frac{\nu-1}{\rho^2\nu}\left(\mu_z-\frac{\sigma_z^2}{2}\right).$$

We continue to consider the case $\nu<1$. The relevant move is from city to farm, and the net options value is the value of waiting in the city (origin) minus the value of staying in the farm (destination).

For $\nu<1$, within the zone of inaction, as z decreases, the value of waiting in the city should decrease due to a less favorable relative farm wage. As the value of z approaches the threshold Z_{ON}, the option of waiting, which is to be given up, increases in value:

$$\lim_{z\uparrow Z_{\text{ON}}} U_O>0.$$

Setting $K_2=0$ and using the positive root β_1^G satisfies this, and we note that $K_1>0$.

Using the two optimality conditions – value-matching and smooth-pasting – which are obtained in the same manner as above, we can determine the threshold value for on-farm migration as:

$$Z_{\text{ON}}=\exp\left\{S+\frac{1}{1-\nu}\ln B(L_M)\right\},$$

where:

$$S = \frac{-\nu\rho c}{\nu - 1} + \frac{1}{\rho}\left(\frac{\sigma_z^2}{2} - \mu_z\right) + \frac{1}{\beta_1^z},$$

$$\beta_1^z = \frac{1}{2} - \frac{\mu_z}{\sigma_z^2} + \sqrt{\left(\frac{\mu_z}{\sigma_z^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma_z^2}} > 0.$$

Substituting the expressions for Z_{OFF} and Z_{ON} in the wage gap expression (11) gives the rules shown in the text.

E.1.2 The Case with Subsistence Food Consumption

In this case, rewrite equation (11) as:

$$\ln w = \frac{1}{\nu}\left[\ln B(L_M) + \ln G\right].$$

where $G = \gamma z^{\nu-1}$ and $\gamma = \left[1 - \frac{\gamma_A}{z_A(1-L_M)}\right]$. When $\gamma > 0$, for analytical tractability, we approximate the optimal thresholds by assuming that at each instant workers treat $\mu_\gamma > 0$ and σ_γ in equation (D.4) as fixed. (However, when we numerically simulate our model economy, we allow workers to update these values continuously as new information arrives.) Then at each instant G can be treated as following a geometric Brownian motion. Using techniques given by Dixit and Pindyck (1996, pp. 81-83):

$$\frac{\mathrm{d}G}{G} = \mu_G \mathrm{dt} + \sigma_G \mathrm{d}\omega_G.$$

where:

$$\mu_G(t) = \left[\mu_\gamma + (\nu-1)\mu_z + \frac{1}{2}(\nu-1)(\nu-2)\sigma_z^2 + (\nu-1)\rho_{\gamma,z}\sigma_z\sigma_\gamma\right], \tag{E.5}$$

$$\sigma_G(t) = \left[\sigma_\gamma^2 + (\nu-1)^2\sigma_z^2 + 2\rho_{\gamma,z}\sigma_\gamma\sigma_z(\nu-1)\right]^{1/2}. \tag{E.6}$$

As described above, the potential migrant updates μ_γ and σ_γ in each period but nonetheless *treats the updated values as fixed* . Thus, μ_G and σ_G are implicitly updated as well. We will suppress these time subscripts when no confusion is likely to arise. Solving equation (E.1) for the function U_Δ, we obtain:[20]

$$U_\Delta = \frac{1}{\rho\nu}\left[\ln B(L_M) + \ln G_0\right] + \frac{1}{\rho^2\nu}\left(\mu_G - \frac{\sigma_G^2}{2}\right). \tag{E.7}$$

[20]This function reduces to the "no-subsistence" fundamental value. If we set $\gamma = 1$ and $\mu_\gamma = \sigma_\gamma = 0$, it is straightforward to show that:

$$G = z^{\nu-1}, \qquad \mu_{Gns} = (\nu-1)\mu_z + \tfrac{1}{2}(\nu-1)(\nu-2)\sigma_z^2,$$
$$\sigma_{Gns} = \left[(\nu-1)^2\sigma_z^2\right]^{1/2}, \qquad \mu_{Gns} - \tfrac{\sigma_{Gns}^2}{2} = (\nu-1)\left[\mu_z - \tfrac{1}{2}\sigma_z^2\right].$$

where G_0 is a composite of the fixed initial level of productivity and subsistence $\gamma_0 z_0^{\nu-1}$. Note that γ in the last expression is evaluated at the fixed initial values of z_A and L_M.[21] When the subsistence consumption parameter $\gamma_A = 0$, $\gamma = 1$, and we obtain the results given in the case with no subsistence consumption. The options value of postponing migration is identical to that of the no-subsistence-consumption case, save for replacing z with G.

Migration from Farm to City This analysis is identical to that of the no-subsistence-consumption case, so we highlight only the key differences. For $\nu < 1$, within the zone of inaction, a higher value of z and hence a *lower* value of G should reduce the option value of waiting to switch to the non-farm sector. As the value of G approaches the threshold G_{OFF} (from below), the option of waiting, which is to be given up, increases in value, so:

$$\lim_{G \uparrow G_{\text{OFF}}} U_O > 0.$$

Setting $K_2 = 0$, and using the positive root, β_1^G, satisfies this, and we note that $K_1 < 0$. Solving the value-matching and smooth-pasting conditions for G_{OFF}, the threshold value for off-farm migration gives:

$$G_{\text{OFF}}(t) = \exp\left\{s_{ws}(t) - \ln B(L_M)\right\},$$

where:

$$\begin{aligned} s_{ws}(t) &= \nu\rho c + \frac{1}{\rho}\left(\frac{\sigma_G(t)^2}{2} - \mu_G(t)\right) + \frac{1}{\beta_1^G(t)}, \\ \beta_1^G(t) &= \frac{1}{2} - \frac{\mu_G(t)}{\sigma_G(t)^2} + \sqrt{\left(\frac{\mu_G(t)}{\sigma_G(t)^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma_G(t)^2}} > 1. \end{aligned}$$

It can be verified that when $\gamma = 1$ and $\sigma_G 2$ and $(\mu_G - \sigma_G 2/2)$ are given by the equations in footnote (17), the expressions above collapse to the no-subsistence-consumption case.[22]

Migration from City to Farm For $\nu < 1$, within the zone of inaction, as G increases, the value of waiting in the city should decrease due to a less favorable relative farm wage.

[21] To ensure economically plausible results, we assume that two times the trend decline in subsistence–consumption ratio exceeds its volatility, $\mu_\gamma > .5\sigma_\gamma$.

[22] In the case where $\gamma = 1$, β_2^G can be written strictly in terms of the parameters governing the diffusion process for z:

$$\beta_2^{Gns} = \frac{1}{\nu - 1}\left[\left(\frac{1}{2} - \frac{\mu_z}{\sigma_z^2}\right) - \sqrt{\left(\frac{\mu_z}{\sigma_z^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma_z^2}}\right] = -\beta_2^z > 0.$$

As the value of G approaches the threshold G_{ON}, the option of waiting, which is to be given up, increases in value:

$$\lim_{G \downarrow G_{\text{ON}}} U_O > 0.$$

Setting $K_1 = 0$ and using the negative root β_2^G satisfies this, and we note that $K_2 < 0$.

Using the two optimality conditions of value-matching and smooth-pasting, the threshold value for on-farm migration is:

$$G_{\text{ON}}(t) = \exp\left\{S_{ws}(t) - \ln B(L_M)\right\},$$

where:

$$\begin{aligned}
S_{ws}(t) &= -\nu\rho c + \frac{1}{\rho}\left(\frac{\sigma_G(t)^2}{2} - \mu_G(t)\right) + \frac{1}{\beta_2^G(t)}, \\
\beta_2^G(t) &= \frac{1}{2} - \frac{\mu_G(t)}{\sigma_G(t)^2} - \sqrt{\left(\frac{\mu_G(t)}{\sigma_G(t)^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma_G(t)^2}} < 0.
\end{aligned}$$

Substituting the expressions for G_{OFF} and G_{ON} in the wage gap expression (11) gives the rules shown in the text.

E.2 Maximum Sustainable Wage Gaps

Begin with the definition of the labor share in the baseline model with subsistence, i.e., equation (7), and we can solve to find:

$$\frac{1 - L_M^f}{L_M^f} = \frac{(1 - L_M^f)\left(\frac{1-\eta}{\eta}\right) z^{1-\nu}}{1 - L_M^f - \frac{\gamma_A}{z_A}} = \frac{\left(\frac{1-\eta}{\eta}\right) z^{1-\nu}}{1 - \frac{\gamma_A}{z_A(1-L_M^f)}}.$$

Use this result and rewrite equation (11) to obtain:

$$\begin{aligned}
\ln w &= \frac{1}{\nu}\ln\left(\frac{1 - L_M}{L_M}\right) - \frac{1}{\nu}\ln\left[\left(\frac{1-\eta}{\eta}\right) z^{1-\nu}\left(1 - \frac{\gamma_A}{z_A(1 - L_M)}\right)^{-1}\right], \\
&= \frac{1}{\nu}\left[\ln\left(\frac{1 - L_M}{L_M}\right) - \ln\left(\frac{1 - L_M^f}{L_M^f}\right)\right].
\end{aligned}$$

E.3 The Long-run Distribution of Wage Gap

As is clear from equation (11) the log wage gap, $w_\Delta = \ln w$, will follow a stochastic process determined by $\ln G$. Let $F = \ln G$. Using the same technique (from Dixit and Pindyck

[1994]) as we used to develop dG above, it is straightforward to determine that F follows the *simple* Brownian motion process:

$$\mathrm{d}F \;=\; \mu_F \mathrm{dt} + \sigma_F \mathrm{d}\omega_F.$$

where:

$$\mu_F = \left(\mu_\gamma + (\nu-1)\mu_z - \frac{\sigma_\gamma^2}{2} - (\nu-1)\frac{\sigma_z^2}{2}\right),$$
$$\sigma_F = \left(\sigma_\gamma^2 + (\nu-1)2\sigma_z^2 + 2\rho_{\gamma,z}(\nu-1)\sigma_\gamma\sigma_z\right)^{1/2}.$$

Thus, the log wage gap is governed by a simple Brownian motion:

$$\mathrm{d}w_\Delta \;=\; \mu^* \mathrm{d}t + \sigma^* \mathrm{d}\omega_F. \tag{E.8}$$

where $\mu^* = \mu_F/\nu$ and $\sigma^* = \sigma_F/\nu$.

Equation (E.8) suggests that, in the absence of reflecting boundaries, $w_\Delta(t)$ would be normally distributed with mean, $\mu^* t$, and variance, $\sigma^{*2} t$. So, the wage gap fluctuates freely without leading to migration unless it hits the upper threshold $\overline{w} \equiv w_\Delta(G_M)$ or a lower threshold $\underline{w} \equiv w_\Delta(G_A)$, where:

$$\overline{w} = \left(\frac{1}{\nu}\right) s_{ws}(t), \quad \text{and} \quad \underline{w} = \left(\frac{1}{\nu}\right) S_{ws}(t).$$

We have shown that these thresholds are a function of time. However, as income increases, the subsistence effect becomes increasingly irrelevant and the thresholds approach stable long-run values, i.e.: $s_{ws}(t) \rightarrow \bar{s}_{ws}$ and $S_{ws}(t) \rightarrow \bar{S}_{ws}$ (see, e.g., Figure 3). It is these stable long-run values that matter to us in determining the long-run stable wage gap. Although the wage gap w_Δ follows a simple Brownian motion off-threshold due to endogenous migration, it nevertheless converges to a stable long-run distribution as we shall show. The next task is to characterize this probability density function for w_Δ, f.

To this end we use the standard result from Dixit and Pindyck (1994, pp. 83–84, 89) and obtain the following long-run stationary probability density function:

$$f(w_\Delta) = \frac{\alpha e^{\alpha w_\Delta}}{e^{\alpha\overline{w}} - e^{\alpha\underline{w}}},$$

with boundary conditions $f(\underline{w}) = 0$, and $f(\overline{w}) = 0$, and $\alpha \equiv 2\mu^*/\sigma^{*2}$. This suggests that the average wage gap W_Δ prevailing in the economy is:

$$W_\Delta \equiv \int_{\underline{w}}^{\overline{w}} w_\Delta f(w_\Delta) dw_\Delta.$$

The integral in this expression (solved using integration by parts where $u = e^{\alpha w_\Delta}$, $du = \alpha e^{\alpha w_\Delta} dw_\Delta$, $v = w_\Delta$, and $dv = dw_\Delta$) yields:

$$W_\Delta = \left\{ \frac{(\overline{w} - 1/\alpha) \exp[\alpha \overline{w}] - (\underline{w} - 1/\alpha) \exp[\alpha \underline{w}]}{\exp[\alpha \overline{w}] - \exp[\alpha \underline{w}]} \right\}. \tag{E.9}$$

We can also calculate the differences in relative sectoral labor shares with and without frictions:

$$L_\Delta \equiv \left[\ln \left(\frac{1 - L_M^f}{L_M^f} \right) - \ln \left(\frac{1 - L_M}{L_M} \right) \right] = -\nu W_\Delta.$$

Consequently, the average difference in sectoral labor shares L_Δ is:

$$L_\Delta = -\nu W_\Delta = -\nu \left\{ \frac{(\overline{w} - 1/\alpha) \exp[\alpha \overline{w}] - (\underline{w} - 1/\alpha) \exp[\alpha \underline{w}]}{\exp[\alpha \overline{w}] - \exp[\alpha \underline{w}]} \right\}.$$

References

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Harrison, Michael. *Brownian Motion and Stochastic Flow Systems.* New York: John Wiley & Sons Publishers, 1985.

Data Appendix for
Productivity Growth and Agricultural Out-Migration in the United States

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F Data Sources

All of our data, unless otherwise stated, come from the *Historical Statistics of the United States* (supplemented by *Datapedia of the United States*). The definitions of variables used and data sources are as follows.

F.1 The Farm Share of Employment

In our model, the farm share of employment corresponds to $1 - L_M$. Unfortunately, we do not have consistent time series data on the share of non-farm employment, L_M and so we formed the farm employment data underlying Figure 1 as shown in Table F.1.

F.2 The Off-farm Migration Rate

The off-farm migration rate M using model based variables is defined as:

$$M(t) = \frac{L_M(t) - L_M(t-1)}{1 - L_M(t)}. \tag{F.1}$$

and is based on changes in the farm share of employment. This measure is distinct from the off-farm migration rate M, which has also been used to capture the pace of structural change:

$$M(t) = \frac{N_M(t) - N_M(t-1)}{N_A(t)}, \tag{F.2}$$

where N_i is population in sector A or M. For comparison, we also examine *net* off-farm migration and farm population data covering the period 1920 to 1970. These migration data, however, pertain to the entire farm population, and cover the period from April 1st of one year to March 31st of the next. In the source material, the migration numbers given for, say, 1921, refer to the period from April 1, 1920 to March 31, 1921. This makes it difficult to align the migration data with the rest of our variables. Given these constraints, we compute the off-farm migration rate in two alternative ways. In the first method ($M1$), our migration rate for 1920 is net off-farm migration from April 1920 to March 1921 divided by the total farm population in April 1920, and so on. In the second

Table F.1
Data Sources for the Farm Share of Employment

Period (frequency)	L_M	Explanation	Series
1800–1860 (Decennial)	$1 - \frac{\text{emp. in ag.}}{\text{labor force}}$	10 years old and over	D167–181
1870–1890 (Decennial)	$\frac{\text{emp. in non-ag.}}{\text{emp. in ag.} + \text{non-ag.}}$	16 years old and over	D11–25
1900–1947 (Annual)	$\frac{\text{non-farm emp.}}{\text{total emp.}}$	14 years old and over	D1–10
1948–1970 (Annual)	$\frac{\text{emp. in non-ag.}}{\text{total emp.}}$	16 years old and over	D1–10
1972–1991 (Annual)	$1 - \frac{\text{emp. in ag.}}{\text{labor force}}$	10 years old and over	D167–181

Note: All series are from the *Historical Statistics*.

method ($M2$), the starting year for the series is 1921, and it measures off-farm migration from April 1920 to March 1921 divided by the total farm population in April 1921, and so on.

Aside from the problem of overlapping observations, there is another issue regarding the calculated off-farm migration rates. Our model concerns labor flows, but these alternative data pertain to persons who *reside* on farms. The data may overstate the net off-farm migration rate if the number of household members under working age in the out-migration population exceeds that of those involved in in-migration.

In any event, we have computed the correlation coefficients between these three definitions, and they are given in Table F.3. In summary, the correlations are relatively low, but there is a closer match between the M and $M1$ definitions.

Note that in our empirical work we primarily use data from 1900 to 1991 and there is a change in definition in 1948. However, broader trends in the relative employment share should not be affected. Based on these data, we first calculate the actual long-run rate of labor reallocation in the U.S. In particular, assuming that $\left(\frac{1-L_M}{L_M}\right)$ follows a diffusion process with drift $\tilde{\mu}$ and standard deviation $\tilde{\sigma}$, the maximum likelihood estimates of $\tilde{\mu}$ and $\tilde{\sigma}$ are:

$$
\begin{aligned}
\hat{\tilde{\mu}} &= \frac{1}{T}\sum_{1}^{T} \ln\left(\frac{(1-L_M(t))/L_M(t)}{(1-L_M(t-1))/L_M(t-1)}\right), \\
\hat{\tilde{\sigma}} &= \frac{1}{T}\sum_{1}^{T} \ln\left[\left(\frac{(1-L_M(t))/L_M(t)}{(1-L_M(t-1))/L_M(t-1)}\right) - \hat{\tilde{\mu}}\right]^2.
\end{aligned}
$$

The term $-\hat{\tilde{\mu}}$ gives our measure of the long-run rate of structural change. Table F.2 presents these estimates of the rate of reallocation towards the non-farm sector for dif-

Table F.2
Estimates of long-run rate of off-farm labor reallocation

Period	Drift ($\tilde{\mu}$)	Standard Deviation ($\tilde{\sigma}$)
1800–1991	0.0292	—
1900–1966	0.0377	0.0455
1900–1991	0.0358	0.0429
1920–1966	0.0396	0.0497
1920–1991	0.0368	0.0451

Source: Authors' calculations as explained in the text.

Table F.3
Correlation Matrix for M, M1 and M2

	M1	M2
M	.475	.376
M1		.424

Source: Authors' calculations as explained in the text.

ferent periods in our sample and suggests that the rate of structural transformation has *accelerated* in later periods.[23]

F.3 The Relative Wage

In calculating the relative wage, the theoretically appropriate variable to use is labor earnings in the farm sector relative to labor earnings in the non-farm sector. We compute the relative wage as follows.

The farm wage rate.—In the absence of reliable labor earnings from farm production, we used the "farm wage rate per month with house" (series K179). There are several issues involved with these data. First, farm operators heavily rely on own and unpaid family labor, and only about a fourth of total farm labor is hired. Due to well-known monitoring problems in agriculture, the wage rates for farm workers are likely to be lower than the return to family labor. An alternative would be to infer the return to family

[23]Since the earlier data are irregularly sampled, we simply fit the following regression:

$$\ln \frac{1 - L_M}{L_M} = \text{constant} + \text{slope} \times \text{time},$$

where "time" varies from 1800 to 1970, and the estimated value of the "slope" coefficient is the sample counterpart of $\tilde{\mu}$ in the model for this extended sample.

labor from the net earnings of farm operators. However, since farm income includes the return on land and farm machinery and equipment, and measuring the revenue share of these inputs is very difficult, we did not pursue this approach. Schultz (1953, pp.101–02) reports that during the first half of the 20th century, the *ratio* between net farm income per family worker and wage income per hired farm worker has stayed fairly constant, except during the war years (when the net farm income increased relative to the wage income of hired farm workers), and during the periods 1920–1923 and 1930–1933 (when the relative net farm income fell).

Second, wage data, especially from the earlier part of the 20th century is highly unreliable. There are numerous measurement problems. First, during our sample period agricultural wages typically included either room and board or house, as well as some "in kind" payments. A comparison of data on (daily) wage rates across different payment schemes – i.e., with room and board, with house (no meals) and with no room and board – suggests that implied valuation of meals is about 15 to 25 percent of actual wage (especially in the early periods) and that of housing is about 20 percent.

The non-farm wage rate.—In the absence of comprehensive non-farm wage data, most authors (including us) use manufacturing wages to compute the relative farm wage rate. Three problems stand out. First, average skill levels and experience may differ across the agricultural and manufacturing sectors. Using manufacturing wages for "lower skilled" labor may only partly address some of these problems. However, we cannot control for other compositional effects due to differences across sectors in experience (age) and gender. Second, job creation and destruction rates may vary across sectors, leading to sectoral variations in (frictional) unemployment, which is something we do not model. Third, a manufacturing job, even with the lowest skill requirements, is not the only alternative to a farm job.

The non-farm wage data are constructed as follows. From 1890 to 1920, we used "lower skilled labor, full time weekly earnings" (series D778), and from 1921 to 1970, we used "average manufacturing wage per week" (series D804), both multiplied by four to convert them to monthly earnings. As an alternative we have also computed the manufacturing wage rate by "average weekly hours worked" times "average hourly pay" (series D802, D803). These series are only available after 1914. We find that these three series have a very high correlation (above .98) during the period (1914–1920) when the data overlap.

Another difficulty involves converting these nominal wage gaps into real wage gaps by adjusting for the farm–urban cost of living differential. Williamson and Lindert (1980, p. 121) observe that the standard benchmark estimate, by N. Koffsky, is about 25 percent for the year 1941. Furthermore, their own estimates (Appendix H) suggest that this differential has remained relatively constant over our sample period. Hatton and Williamson (1992) incorporate these cost living adjustments, as well as consider unemployment rate adjusted ("Todaro") relative wages.

Caselli and Coleman (2001) further discuss the problems encountered in estimating farm–city wage gaps and conclude that different data sources yield average wage gaps.

They use census data from 1940 to 1990, and combine it with an existing series by Lee et al. (1957) on relative farm–non-farm service income from 1880, 1900, 1920 and 1950. According to Lee et al. "service income is the sum of wages and salaries (excluding employee contributions to social insurance and 'other labor income' such as cash sickness compensation, etc.) and proprietors' income, with imputed rents of farm dwellings included in the agricultural component of service income" (p. 703).[24] Their scaled estimates suggest that relative farm earnings has increased from about 0.2 in 1880 (Table 1) to 0.72 in 1990 (data obtained from http://www.economics.harvard.edu/faculty/caselli/caselli.html). Any empirical analysis of historic wage gaps should thus be interpreted under the caveat that we lack good quality high-frequency relative labor earnings data. These different data sources and adjustments are summarized in Figure F.1.[25]

F.4 Farm and Non-Farm Productivity Growth

Our primary variable of interest is labor productivity because our model does not include other fixed or quasi-fixed factors of production. Section B discusses our method of imputing relative productivity. Here we discuss the alternative measures that are available. First, we used indices of employee output in the total private economy based on "farm and non-farm output per man-hour" (Series D683–688, columns 684, 686) to compute the relative sectoral productivity growth rate (defined as the productivity growth rate in the non-farm sector minus that of the farm sector). There are two disadvantages associated with these series: (i) they are indices and both productivity series are set to 100 in 1958, and (ii) they end in 1966. Therefore, to estimate the trend productivity growth rate, we first took the log ratio of these series.

To estimate the parameters of equation (12) we specified its empirical counterpart:

$$d \log z = \alpha dt + \sigma d\omega.$$

We estimated the mean α ($\hat{\alpha}$) and standard deviation σ ($\hat{\sigma}$) of the log of relative productivity using maximum likelihood estimates:

$$\begin{aligned} \hat{\alpha} &= \frac{1}{T}\sum_{1}^{T} \ln\left(\frac{z(t)}{z(t-1)}\right), \\ \hat{\sigma} &= \frac{1}{T}\sum_{1}^{T} \ln\left[\left(\frac{z(t)}{z(t-1)}\right) - \hat{\alpha}\right]^2. \end{aligned}$$

[24] Since this definition is different from wage income, Caselli and Coleman apparently parse these two series. The Lee et al. data set implies an average farm–non-farm relative service income of 0.74 for 1950, but Caselli and Coleman's census based relative wage estimates are 0.43. Caselli and Coleman re-scale the estimates for relative earnings in 1880, 1900 and 1920 using 4/7 as a scale.

[25] Margo (1995) places the farm–non-farm wage gap in the 1850s at between 30–40%, and the real wage gap at between 10–20%. Whether between 1850s and 1880 the wage gap has increased as much as the Caselli and Coleman (2001) data suggests is an open issue.

Table F.4
Estimates of Non-farm versus Farm Relative Productivity Growth

Period	Mean (α)	Standard Deviation (σ)	Drift (μ)
1889–1966	−0.0007	0.0782	0.0023
1900–1966	−0.0026	0.0789	0.0005
1920–1966	−0.0111	0.0645	−0.0090
1949–1996	−0.0246	0.0572	−0.0230

Source: Authors' calculations based on data from the *Historical Statistics of the United States*, series D683–688, except the last row which is based on Bureau of Labor Statistics, series PRS85006093 and *Economic Report of the President*, Table B–99.

Note that $\hat{\alpha} \equiv \hat{\mu} - \frac{1}{2}\hat{\sigma^2}$, where the $\hat{\mu}$ is the empirical counterpart of the drift parameter in equation (12). Table F.4 shows the results. Although the estimates are subject to qualifications, they show the acceleration of productivity growth in the agricultural sector after the 1920s relative to productivity growth in the non-agriculture sector. Furthermore, the estimates are broadly consistent with out imputed series. Evidently, both the increased mechanization of U.S. agriculture starting in the early 1920s and the "chemical revolution" of the 1950s-60s partly account for these trends.

The second relative labor productivity measure we used is based on "non-farm business sector output per hour index, 1992=100" from Bureau of Labor Statistics, series PRS85006093 and "farm output per unit of farm labor index, 1992=100" from *Economic Report of the President, 2002*, Table B–99. The first and second series are not directly comparable due to changes in methodology, and the correlation between the overlapping portions of the two series is 0.42. In any event, as shown in the last row of Table F.4, the relative productivity growth in the farm sector has clearly accelerated in the post-war era, exceeding non-farm labor productivity growth by about 2.3 percent.

Admittedly, there is considerable uncertainty surrounding the exact magnitude of agricultural productivity growth relative to non-farm productivity growth during this period. However, the findings of the existing total factor productivity (TFP) literature are comparable to our estimates. For example, Jorgenson and Gallop (1992) the relative TFP growth at about 1.2 percentage points in favor of the farm sector relative to private non-farm sector between 1947 and 1985. For the period 1949–79 Jorgenson, Gallop, and Fraumeni (1987, Table 9.3 and Table D.1) also give estimates for agricultural and aggregate TFP growth, which are respectively 1.5 and 0.8 percent; again favoring the farm sector.

There is also considerable evidence that farm labor productivity growth accelerated during the twentieth century. Using index numbers of output and inputs, Schultz (1953) estimates the multi-factor productivity index for 1910–1950. Although Jorgenson and Gallop's estimates are not directly comparable with those of Schultz's, a comparison of these estimates indicates an acceleration in the farm productivity growth rate over the course of this period. There is also considerable evidence which suggests that the structural break in farm productivity growth took place around 1930. In particular, using a number of alternative farm productivity estimates, Meiburg and Brandt (1962) show that the acceleration in farm productivity started around 1930.

Our stance on the relative farm productivity is also consistent with other studies. For instance, in their calibrations of the U.S. economy from 1880 to 1990, Caselli and Coleman (2001, p. 614) use double the value of the non-agricultural TFP growth rate as an estimate of the agricultural productivity growth rate (which roughly corresponds to a .8 percentage point gap in productivity growth per annum).

In addition, according to data reported by Mundlak (2000, Figure 1.11), from 1960 to 1992, the growth rate of labor productivity in agriculture exceeded that of the non-farm sector in about 80 percent of the countries in a sample of 88 observations. The median value by which the growth rate in average agricultural labor productivity exceeded that of manufacturing was 1.58 percentage points [see also Mundlak (2000, p. 388)].[26]

F.5 Relative Prices

We used two series: one based on producer prices for farm relative to industrial goods, and the other consumer prices for food items relative to non-food items. These data are shown in Figure F.2, and discussed below.

Relative price of farm goods.—For the period 1913–1954 we used the wholesale price index for industrial commodities (series E23), divided by the wholesale price index for farm goods, (E25), obtained from the *Historical Statistics*. For the period 1955–2001, we used producer price index for total industrial commodities divided by producer price index for farm products, both from the *Economic Report of the President, 2002*, Table B–67.

Relative price of food to non-food items.—Our food–non-food consumer price indexes (CPI) are constructed using data from the Bureau of Labor Statistics (BLS, website www.bls.gov/data/home.htm). The complication is that although there is a CPI series of 'food' prices (series CUUR0000SAF1) for the period 1913 to 2002, the series for 'all items less food' (CUUR0000SA0L1) only covers the period 1935 to 2002. To extend the data for this category back to 1913, we first obtained the series that comprise all non-food items and their weights in the 'all items less food.'

The weights are given in Table 113 of the 1983 *Handbook of Labor Statistics* published by the BLS. We use the earliest available weights (for the period 1935–39) which are as follows for the 'all items' series: food and beverages (35.4%), housing (33.7%), apparel and upkeep (11.0%), transportation (8.1%), medical care (4.1%), entertainment (2.8%),

[26] Of course, we are not claiming that productivity growth has consistently favored agriculture. For instance, estimates for the U.S. reported by Greenwood and Seshadri (2002, p. 156)) suggest relatively faster non-farm productivity growth in the 19th century. As well, Syrquin (1988) argues that the long-run total factor productivity trend has favored industry, at least in some industrial countries.

and other goods and services (4.9%). The 1950 *Handbook of Labor Statistics* provides the CPI data for 'housing and apparel' back to 1913, but does not provide the other series. However, the series 'miscellaneous' is defined by the 1950 *Handbook* (p. 97) as including: transportation, medical care, household operation, recreation, personal care, etc. We thus impute a weight for the category 'miscellaneous', of 30.8% by adding the weights (given above) for transportation, medical care, entertainment, and other goods and services. Using the 1950 *Handbook* series for miscellaneous (from Table D–1), the 1983 *Handbook* series for 'residential' and 'apparel and upkeep' (from Table 110), and the weights for housing, apparel and upkeep, and miscellaneous described above, we calculated a series for 'all items less food' from 1913 to 1950 (note that all series were re-calibrated to the same base year). For the period of overlap (1935–1950) between this series and the one given on the BLS website, the correlation coefficient was .9986. Using a minor scaling factor, we spliced the constructed series (through 1934) to the BLS website series (1935–2002) to obtain a price series from 1913 to 2002 for non-food. The resulting relative price series are shown in Figure F.2. The shaded areas are the two world wars, and the spike in the early 1970s corresponds to the Nixon price controls (August 1971–April 1974). In the absence of these major shocks, there is a secular downward trend in the relative price of food.

Of course, differential inflation rates at the farm gate and retail levels – the latter of which includes increasingly more non-farm inputs – may bias our estimates. There is ample evidence that in fact the latter has exceeded the former, suggesting that the decline in the relative price of farm goods has been more pronounced than that suggested by the relative price of food. Alternative estimates indicate more pronounced decreases in the relative price of food. For instance, Caselli and Coleman (2001) report that wholesale price index for farm goods divided by the CPI has fallen by about 20 percent between 1880 and 1990.

F.6 Expenditures on Food and Non-Food

Personal consumption expenditures on all items and food (current dollars), and an implicit price deflator for final sales of domestic product for 1929–1991 are obtained from the website of the Bureau of Economic Analysis (www.bea.org, downloaded on 5/21/03). All these series are based on national income and product accounts (NIPA). To estimate the growth rate of expenditures on food for the period 1900–1929, we used the labor productivity growth rate in the farm sector (see below) minus the growth rate in non-farm employment share.

F.7 The Share of Food Expenditure

The share of food in total expenditures is taken from D. Costa (2001), and is based on NIPA (for 1929–91). Note that given our stylized theoretical model, these series are

TABLE F.5
EXPENDITURES ON FARM PRODUCTS AS A PERCENT OF NATIONAL INCOME, %

1870	34
1880	32
1890	22
1900	17
1910	19
1922	16.1
1925	15.4
1929	13.4
1934	12.8
1937	13.7
1939	11.6

NOTE: Expenditures on farm products are adjusted for agricultural exports and imports.
SOURCE: Schultz (1953), Table 5–6 and Table 5–7, p. 66, and p. 67.

preferable to alternatives such as the share of farm products in national income, which includes investment in fixed capital and government services. Schultz (1953, Tables 5–6 and 5–7) also provides estimates for expenditure share of farm products which share the same downward trend but are not in fact comparable to Costa's series (see Figure F.3 and Table F.5).

F.8 Agriculture's Share in National Income

For the period 1890 to 1959, we used Kendrick (1961, p. 298-301, table AIII), and Series F126–128 in *Historical Statistics*. These refer to gross domestic product originating from private farm sector divided by the sum of farm and non-farm sectors. For 1960–1989 we used Series F226–227, which correspond to the share of agriculture, forestry, and fisheries in national income.

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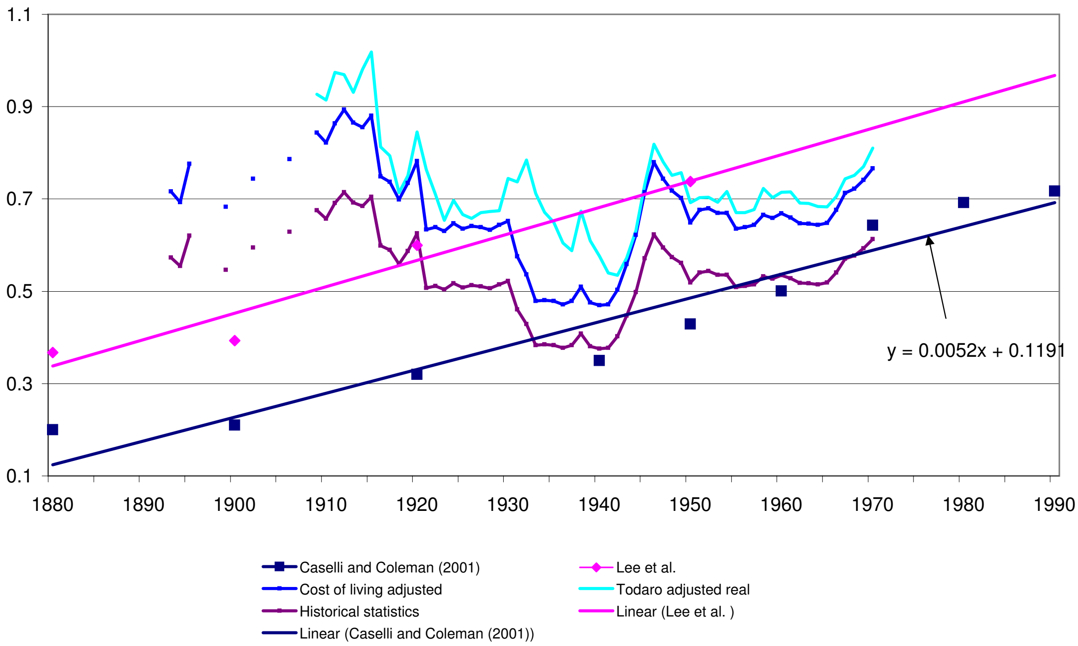
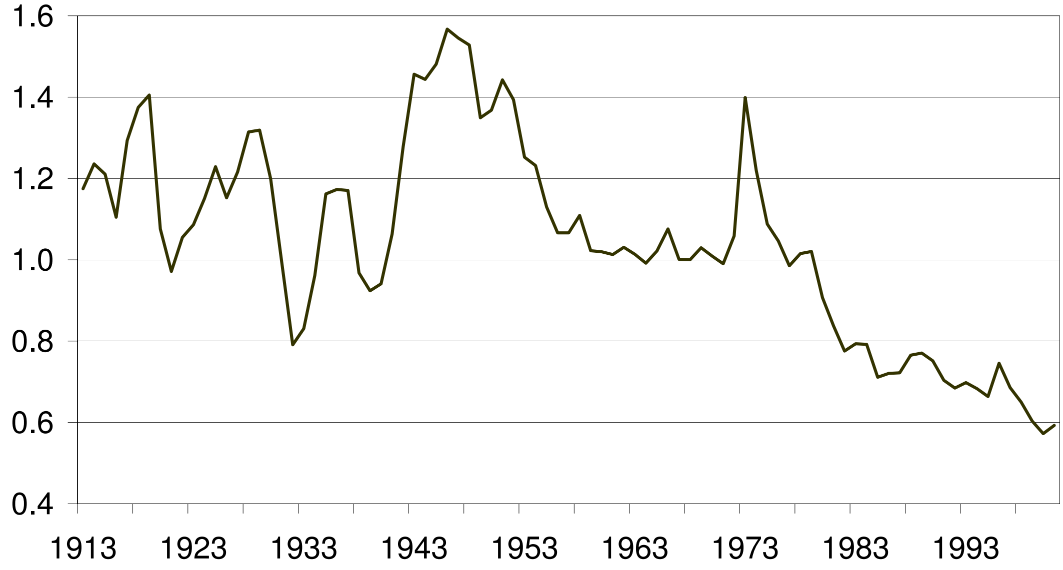
Figure F.1:

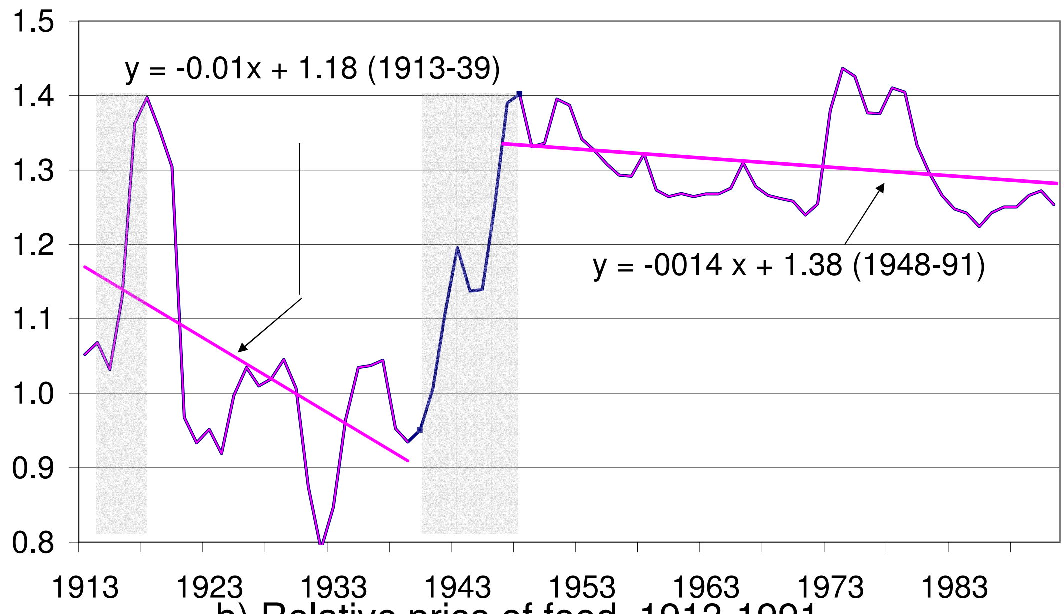


Farm-Nonfarm Wage Ratio

Figure F.2:

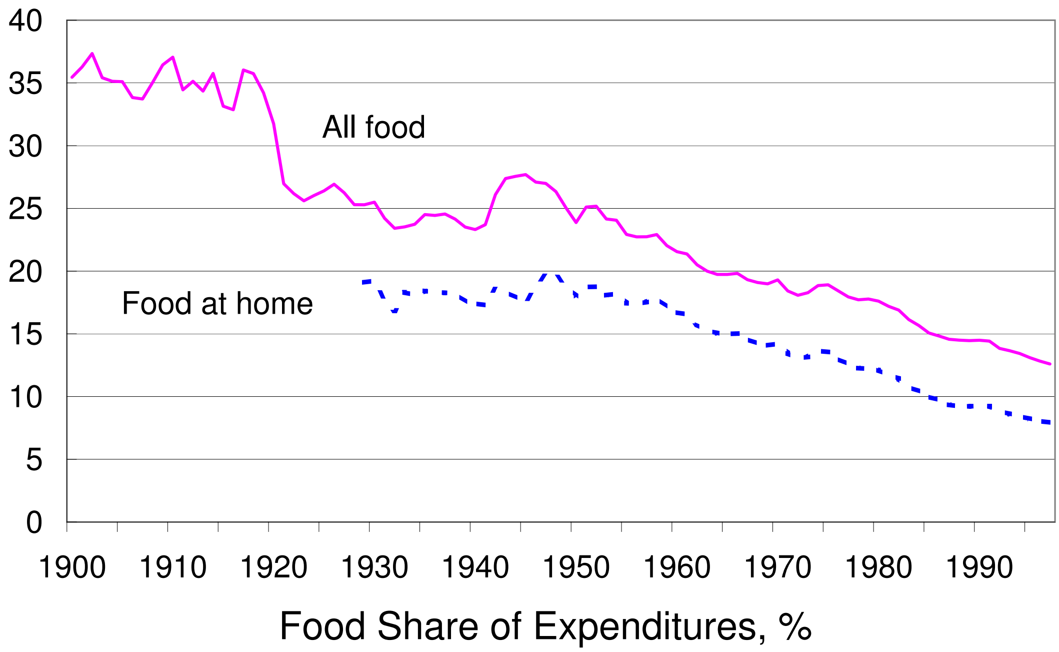


a) Relative price of farm goods, 1913-2001
(1967=100)



b) Relative price of food, 1913-1991
(1982-84=100)

Figure F.3:



40
35
30
25
20
15
10
5
0
All food
Food at home
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
Food Share of Expenditures, %

Sensitivity Analysis for Productivity Growth and Agricultural Out-Migration in the United States

Benjamin N. Dennis Talan B. İşcan

G The Effects of Subsistence Consumption and Productivity Growth on Changes in Wage Gap and Labor Reallocation for Alternative Values of c

TABLES TO FOLLOW

TABLE G.1
THE EFFECTS OF SUBSISTENCE CONSUMPTION AND PRODUCTIVITY GROWTH ON CHANGES IN WAGE GAP AND LABOR REALLOCATION, $c = 2$

	$\Delta \ln$ Wage Gap	Deviation	$\Delta \ln L_M$	Deviation
Historical values	-.0100	–	.0057	–
$\gamma_A/C_A = .8$				
$\mu_A \gg \mu_M$	-.0122	.0022	.0049	.0008
$\mu_A > \mu_M$	-.0124	.0024	.0041	.0016
$\mu_A = \mu_M$	-.0086	.0014	.0027	.0030
$\gamma_A/C_A = .7$				
$\mu_A \gg \mu_M$	-.0090	.0010	.0043	.0014
$\mu_A > \mu_M$	-.0086	.0014	.0035	.0022
$\mu_A = \mu_M$	-.0042	.0058	.0022	.0035
$\gamma_A/C_A = .5$				
$\mu_A \gg \mu_M$	-.0050	.0050	.0032	.0025
$\mu_A > \mu_M$	-.0045	.0055	.0025	.0032
$\mu_A = \mu_M$	-.0022	.0122	.0014	.0043
$\gamma_A/C_A = 0$				
$\mu_A \gg \mu_M$	-4.1450	4.1350	.0008	.0049
$\mu_A > \mu_M$	-1.5209	1.5109	.0006	.0051
$\mu_A = \mu_M$	-0.8841	0.8741	-.0001	.0058

NOTE: See notes to Table 1.

Table G.2

The Effects of Subsistence Consumption and Productivity Growth on Changes in Wage Gap and Labor Reallocation, $c = 5$

	$\Delta \ln$ Wage Gap	Deviation	$\Delta \ln L_M$	Deviation
Historical values	-.0100	–	.0057	–
$\gamma_A/C_A = .8$				
$\mu_A \gg \mu_M$	-.0108	.0008	.0050	.0007
$\mu_A > \mu_M$	-.0121	.0021	.0041	.0016
$\mu_A = \mu_M$	-.0167	.0067	.0027	.0030
$\gamma_A/C_A = .7$				
$\mu_A \gg \mu_M$	-.0079	.0021	.0043	.0014
$\mu_A > \mu_M$	-.0079	.0021	.0035	.0022
$\mu_A = \mu_M$	-.0044	.0056	.0022	.0035
$\gamma_A/C_A = .5$				
$\mu_A \gg \mu_M$	-.0042	.0058	.0032	.0025
$\mu_A > \mu_M$	-.0040	.0060	.0025	.0032
$\mu_A = \mu_M$	.0018	.0118	.0014	.0043
$\gamma_A/C_A = 0$				
$\mu_A \gg \mu_M$	-4.7396	4.7296	.0008	.0049
$\mu_A > \mu_M$	-1.6664	1.6564	.0006	.0051
$\mu_A = \mu_M$	-0.8841	0.8741	-.0001	.0058

Note: See notes to Table 1.

Table G.3

The Effects of Subsistence Consumption and Productivity Growth on Changes in Wage Gap and Labor Reallocation, $c = 10$

	$\Delta \ln$ Wage Gap	Deviation	$\Delta \ln L_M$	Deviation
Historical values	-.0100	–	.0057	–
$\gamma_A/C_A = .8$				
$\mu_A \gg \mu_M$	-.0092	.0008	.0050	.0007
$\mu_A > \mu_M$	-.0085	.0015	.0041	.0016
$\mu_A = \mu_M$	-.0061	.0039	.0027	.0030
$\gamma_A/C_A = .7$				
$\mu_A \gg \mu_M$	-.0065	.0035	.0043	.0014
$\mu_A > \mu_M$	-.0071	.0029	.0035	.0022
$\mu_A = \mu_M$	-.0040	.0060	.0022	.0035
$\gamma_A/C_A = .5$				
$\mu_A \gg \mu_M$	-.0034	.0066	.0032	.0025
$\mu_A > \mu_M$	-.0035	.0065	.0025	.0032
$\mu_A = \mu_M$	.0013	.0113	.0014	.0043
$\gamma_A/C_A = 0$				
$\mu_A \gg \mu_M$	–	–	.0008	.0049
$\mu_A > \mu_M$	-2.100	2.0900	.0006	.0051
$\mu_A = \mu_M$	-0.8842	0.8742	-.0001	.0058

Note: See notes to Table 1.