Hyperinflations and the Signal Extraction Problem

Judit Temesvary
Assistant Professor of Economics
Department of Economics, Hamilton College

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Abstract: This paper builds on the overlapping generations “island” model to show that in the presence of a Taylor–rule following monetary authority, the Signal Extraction Problem can lead to hyperinflations. Producers lean about economic shocks through a signal in the price function they observe. When producers know the exact monetary policy targets for inflation and unemployment, monetary shocks have no effects on output: monetary neutrality holds. However, uncertainty about monetary policy means that producers cannot disentangle monetary shocks from real shocks in the price signal, leading them to alter production. This paper presents simulation exercises to show that there are certain monetary policy objectives (weights) such that hyperinflations will result even if the price function is concave in the signal. Finally, the paper presents simulated comparative statics exercises to examine the relationship between the evolution of the price level and monetary policy weights.

Keywords: Rational expectations, Neutrality of Money, Signal Extraction Problem, Loss function, Hyperinflations, High inflations

JEL classification: E3, E4, E5

Correspondent:
Judit Temesvary
Assistant Professor of Economics
Department of Economics, Hamilton College, 198 College Hill Road, Clinton NY, 13323 USA. jtemesva@hamilton.edu
1 Introduction

Models of high inflations and hyperinflations can be separated into two groups. The first group examines the hyperinflationary episodes of the European economies in the first half of the twentieth century, which generally occurred in the framework of economic crises following major wars. The most infamous episode of such hyperinflations occurred in Germany in the 1920s, when monthly inflation reached $3.25 \times 10^6$. Hungary also experienced hyperinflation between the two world wars, when inflation rates were as high as $4.19 \times 10^{16}$ per month. Models of these episodes are generally explanatory in nature. \textasteriskcentered builds a model of adaptive expectations to analyze hyperinflations. \textasteriskcentered develop an elegant rational expectations theory of hyperinflations, where they show that under certain conditions (such as the feedback from inflation to future rates of money creation), the adaptive expectations model of \textasteriskcentered can be rational as well. They make the interesting argument that high rates of inflation cause high rates of money creation, and not the other way around, as was believed by many researchers at the time. \textasteriskcentered further contributes to the understanding of hyperinflations with his detailed empirical study of four central European countries’ experience with “big inflations”. The second group of high inflation models examines inflationary outcomes as functions of various types of monetary policy rules in the context of developed economies, estimating rule parameters and making policy recommendations. The most influential papers in this group were written by \textasteriskcentered and \textasteriskcentered. These papers are mostly focused on policy implications as opposed to the explanatory focus of the first group of models. The model described below belongs to this second group. The modeling context of this paper is the “island” model. The “island parable” literature was developed in the early 1970s, first invented by \textasteriskcentered and further developed by \textasteriskcentered. The island model provides a framework for studying how the signal extraction problem faced by fully rational agents results in real effects of monetary policy, while fully consistent with the classical idea of the neutrality of money. \textasteriskcentered develops an overlapping
generations model where young producers and old consumers are allocated across two islands. Young producers are uncertain of their allocation across the two islands (the output shock), but they observe a signal through the equilibrium price function. The signal they receive depend on monetary policy as well as the output shock. When monetary policy objectives are clear, output shocks are clear: monetary shocks have no effect on producers and monetary neutrality holds. When monetary policy objectives are unclear, however, producers’ inability to decipher the output shocks from the price signal (the signal extraction problem or SEP) leads to real effects. In the ? model, the rate of money creation is assumed strictly exogenous. Further, the model extends to two periods only and is not aimed at studying the evolution of the price level over time. This paper ties the high–inflation and the ”island” literature together by developing a multi–period ”island” model with responsive monetary policy. Accordingly, the model described below differs from the ? model along several lines. The active monetary authority determines the rate of money creation based on the minimization of a loss function (a weighted sum of squared inflation and output deviations). The allocation of young producers across the two islands is exogenous, and follows a dynamic stochastic AR(1) process. Without loss of generality, the model examines the first island’s economy only. Young producers observe a signal about the output shock and the monetary policy weights from the price function, which has constant elasticity with respect to the signal. The optimal monetary policy depends on the signal elasticity of the price and the weights on the inflation and output deviations in the loss function. Based on this model, it is shown that there are certain specifications for the parameters so that hyperinflations will develop. It is shown that hyperinflations can develop for concave-in-the-signal price functions as well, for certain values of the loss function weights. The model in this paper is consistent with rational expectations in the sense of ? and the classical quantity theory of money. The paper is structured as follows. Section 2 describes the model and derives the optimal production rule, equilibrium price function and monetary policy rule. We develop the
monetary policy and derive the inflation rate. Section 3 presents conditions for hyperinflations to develop. In Section 4, specific distributional assumptions are made to derive a price function that is convex in the signal. This price function is used to describe the simulated comparative statics of the price function. Section 5 summarizes the results and concludes. Detailed derivations and calculations are shown in the Appendix.

2 Model

2.1 Setup and Notation

The modeling framework in this paper is an extension of the overlapping generations “island” economy model. There are two islands in this economy, with a total of $2K$ consumers living on the two islands. $K$ consumers are young, and $K$ consumers are old. At time $t$, fraction $\theta_t$ of the young people live on the first island and fraction $1 - \theta_t$ live on the second island. Without loss of generality, it is sufficient to analyze equilibrium on the first island. The realization of $\theta_t$ is unknown to both generations, and its value must be inferred from the price function. The old people are allocated across the two islands so as to equate money demand across the islands: $K/2$ old people live on both islands. The assignment of activities is as follows. The young people consume and produce at the same time, while the old people only consume. Production takes place in each period, and storage of the consumption good is not possible. Young people produce the single consumption good in the form of home production (according to a linear production function). They consume a fraction of their output, and sell the remaining output to the old for money, in order to secure old-age consumption. Young consumers solve a two-period optimization problem, deciding (1) how much output to produce, and (2) how much of the output to consume – which also pins down how much of the output will be sold to the old for money. The old’s decision problem is trivial: they use all their money to buy the good from the young generation, using the money they brought with them from their youth.
Since only the young people produce in this model, $\theta_t$ is an indicator of aggregate output in the economy. Consecutive realizations of $\theta_t$ are interpreted as an output shock process. The government (or monetary authority) plays an active role in this model: money enters the economy as the government distributes money to the old people in each period (in the form of welfare payments), proportional to their existing per-person money holding $M_{t-1}$, in the spirit of "helicopter money":

$$M_{t+1} = M_t \ast x_t$$

where $x_t$ is the rate of money creation in period $t$. The government determines the rate of money creation $x_t$ based on the minimization of a loss function, as shown below. From producers’ perspective, the random variables of the model are the fraction of young people on a given island at time $t$, i.e. the “real shock” given by $\theta_t$, and the weights the government places on the inflation and output deviation terms in the loss function (see below). The weights $(\phi_{1t}, \phi_{2t})$ are chosen by the monetary authority each period, and citizens’ uncertainty about the monetary policy is the result of unclear communication of the monetary policy objectives (\?). As a result, citizens also do not know the exact value of the rate of money growth $x_t = f(\phi_{1t}, \phi_{2t})$. New values of the random variables are realized each period according to their respective distributions, and all past values of the random variables are public knowledge.

Figure 1 describes the sequence of events at time $t$. In the beginning of the period, the government chooses the rate of monetary expansion $x_t$, based on the minimization of the loss function. Second, the old and the young people form their demand and supply functions based on utility maximization. For the young people, these demand and supply functions are conditional on the signal they expect to observe through the price. Immediately following this is the period of trading between the old and the young people. As a
result, the price is formed according to market clearing. The signal \( z_t = x_t/\theta_t \) is observed through the price function. Finally, at the end of the period, the contemporaneous values of the random variables are observed, so that they are public knowledge for the generations of the next time period \( t+1 \). It is important to emphasize that the young people can only observe a signal from the price, instead of the exact contemporaneous values of the random variables. Since this signal is a ratio, the young people do not know how much of the price change comes from \( x_t = f(\phi_{1t}, \phi_{2t}) \) or the real shock \( \theta_t \). This is the Signal Extraction Problem (SEP). As a result of SEP, changes in the monetary variable \( x_t = f(\phi_{1t}, \phi_{2t}) \) cause real changes in the economy. As we will show below, the price function is a function of the signal \( z_t \) only, and its evolution depends on the constant signal elasticity of the price. Finally, the real shock process (the share of your producers allocated to the island) is an AR(1) process:
\[
\theta_t = \nu \theta_{t-1} * (1 + \psi_t)
\] (2.2)

where \(\psi_t\) has mean zero and support \([1 - \eta; 1 + \eta]\).

The solution of the model is as follows. First, the optimal monetary policy formation is described. Next, the solutions to the old and young generations’ time \(t\) utility maximization problem are presented. Based on these results, the certainty (known monetary policy) and uncertainty (unknown monetary policy) cases are described separately.

### 2.2 Monetary Policy

Let \(\pi_t\) denote inflation at time \(t\) and \(N_t\) is output at time \(t\). The goal of the monetary authority is to minimize expected deviations from the inflation target \(\bar{\pi}\) and output target \(\bar{N}\). This policy is essentially the Taylor Rule (\(?\)) of the “leaning against the wind” type:

\[
\begin{align*}
    r_t &= h(x_t) = \phi_{1t}(\pi_t - \bar{\pi})^2 + \phi_{2t}(\log N_t - \log \bar{N})^2 \\
&= \phi_{1t}(\pi_t - \bar{\pi})^2 + \phi_{2t}(\log N_t - \log \bar{N})^2
\end{align*}
\] (2.3)

The monetary authority chooses the rate of money expansion \(x_t\) to solve:

\[
\min_{x_t} E_t[L_t] = E_t[\phi_{1t}(\pi(x_t) - \bar{\pi})^2 + \phi_{2t}[\log N_t(x_t) - \log \bar{N}]^2]
\] (2.4)

The chosen policy weights \((\phi_{1t}, \phi_{2t})\) are unknown to the young people. With the “leaning against the wind” policy, it is sufficient to focus only on positive values of the weight terms.
2.3 Money Market Equilibrium

There are $K\theta_t$ young people on the first island, and $K/2$ old people. Let $\lambda_t$ denote each young person’s demand for nominal money balances at time $t$. Money market is in equilibrium when the old’s total money supply equals total money demand by the young:

$$KM_t = KM_{t-1}x_t = \left(\frac{K\theta_t}{2}\right)\lambda_t$$

(2.5)

Solving for the per-young-person money demand $\lambda_t$:

$$\lambda_t = \frac{M_{t-1}z_t}{2}$$

(2.6)

where $z_t = x_t \theta_t$ is the signal that each citizen observes through the price.

2.4 Consumption and Production Choices

Let the superscript $o$ denote the old generation, and $y$ denotes the young generation. The decision problem of the old generation is trivial; the old do not produce, and they use all the money they hold to purchase the single consumption good from the young. Since the old’s utility is strictly increasing in the consumption good, they spend all their money on the good. The old people’s utility maximization problem is:

$$\max_{c_t^o} U(c_t^o) = (c_t^o)^\alpha$$

(2.7)

s.t. $p_t c_t^o = M_t$

(2.8)
where $M - t$ is the per-old-person money holding at time $t$. Since no inheritance is possible in this model, the old people spend all their money on the consumption good:

$$[c_t^o]^* = \frac{M_t}{p_t} \quad (2.9)$$

Next, the young producers’ optimal choices are described. The young people decide how much to produce, and how much of this production to consume today or sell to the current old generation for money (in order to secure old-age consumption). The young consumer’s two-period utility maximization problem can be written as:

$$\max_{c_t^y, n_t^y, \lambda_t} U(c_t^y; n_t^y; \lambda_t) = (c_t^y)^\alpha - n_t^y + E_t[(c_{t+1}^y)^\alpha] \quad (2.10)$$

s.t. 

$$p_t(n_t^y - c_t^y) - \lambda_t \geq 0 \quad (2.11)$$

$$c_{t+1}^o = \frac{\lambda_t x_{t+1}}{p_{t+1}} \quad (2.12)$$

The following conditions are assumed to hold:

$$\frac{\partial U}{\partial c_t} > 0 \quad \frac{\partial^2 U}{\partial c^2_t} < 0 \quad \frac{\partial U}{\partial n_t} < 0 \quad 0 < \alpha < 1 \quad \lim_{c_t \to \infty} \frac{\partial U}{\partial c_t} = 0 \quad \lim_{c_t \to 0} \frac{\partial U}{\partial c_t} = \infty \quad (2.13)$$

Rearranging the first-order optimality conditions, the expressions for the optimal choices of young age consumption, production and money demand are:
\[ c_i^y = (\alpha)^{\frac{1}{1-\alpha}} \]  
\[ n_i^y = \frac{\lambda_i}{p_t} + (\alpha)^{\frac{1}{1-\alpha}} \]  
\[ E_t[\lambda_t^{\alpha-1}(\frac{x_t+1}{p_{t+1}})^\alpha] = \frac{1}{\alpha p_t} \]  

Using Equation 2.6, Equation 2.16 can be rewritten as:

\[ p_t = \frac{(M_{t-1})^{1-\alpha}(z_t)^{1-\alpha}}{\alpha E_t[(\frac{x_t+1}{p_{t+1}})^\alpha | z_t]} \]  

There are two interesting cases for the evolution of the price level \( p_t \) over time. The first case is when there is no signal extraction problem, i.e. when the exact value of \( x_t \) is announced. The second case is where only \( z_t \) is known, and the signal extraction problem prevails. These two cases are analyzed sequentially.

### 2.5 Certainty Case: The Exact Monetary Policy is Known

In the certainty case, the money demand function reduces to \( \lambda_t = \frac{M_{t-1} x_t}{2\theta} \). Then iterating Equation 2.16 over time, the general form of the price function in the certainty case becomes:

\[ \log p_t = \log x_t - \frac{\log \alpha}{1-\alpha} + \log M_{t-1} - 2(1-\alpha)^2 \sum_{t=k}^{\infty} \alpha^{(k-i)} \log \theta_k \]  

From Equation 2.18, it is clear that the classical monetary neutrality result holds in the certainty case:
\[
\frac{\partial \log p_t}{\partial \log x_t} = 1 \quad \frac{\partial \log n_t^y}{\partial \log x_t} = 0 \tag{2.19}
\]

From Equation 2.19, it follows that the inflation rate is \(\pi_{t+1}\) is given by:

\[
\pi_{t+1} = \log p_{t+1} - \log p_t = \log x_{t+1} \tag{2.20}
\]

Therefore in the certainty case, the monetary authority can achieve the inflation target \(\bar{\pi}\) by choosing the rate of money creation \(\bar{x}\) such that

\[
\bar{\pi} = \bar{x} \tag{2.21}
\]

The more complicated case of unknown monetary policy is analyzed next.

### 2.6 Uncertainty Case: The Exact Monetary Policy is Not Known

Before analyzing the uncertainty case, it is useful to summarize the information structure of the model in Table 1

<table>
<thead>
<tr>
<th>Past Value of Random Variables</th>
<th>Loss Function Weights ((\phi_1; \phi_2))</th>
<th>Current Output Shock  (\theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young People</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Old People</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Monetary Authority</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>
Based on Equation 2.17, the general form of the price function is

\[ p_t = \frac{M_{t-1}z_t}{\alpha^{1-\alpha}} \Gamma(z_t) = \left[ \frac{M_{t-1}}{\alpha^{1-\alpha}} \frac{z_t}{\Gamma(z_t)} \right] \tag{2.23} \]

The signal elasticity of the price determines how the price function reacts to changes in the value of the signal. This elasticity is assumed constant:

\[ \frac{\partial \log p_t}{\partial \log z_t} = \epsilon_{p,z} \tag{2.24} \]

Given this price function and its constant signal elasticity \( \epsilon_{p,z} \), the monetary authority’s optimal choice of monetary expansion is

\[
\log x_t \approx \frac{\epsilon_{p,z} \phi_1 \bar{\pi} - \phi_2 (1 - \epsilon_{p,z}) \left[ \frac{1}{1-\alpha} \log \alpha + \epsilon_{p,z} \log \theta_{t-1} - \log \bar{N} \right] + \left[ \epsilon_{p,z}^2 \phi_1 - \phi_2 (1 - \epsilon_{p,z}) \epsilon_{p,z} \right] \log \nu}{\phi_1 \epsilon_{p,z}^2 + \phi_2 (1 - \epsilon_{p,z})^2} \tag{2.25} \]

Note that the loss functions weights (\( \phi_1; \phi_2 \)) are unknown to the young generation. Therefore, the young producers cannot solve for the exact value of \( x_t \).

For constant weights (\( \phi_1; \phi_2 \)), Equation 2.25 can be used to express the rate of inflation as:

\[
\pi_t = \frac{\epsilon_{p,z}^2 \phi_1 \bar{\pi} - \phi_2 (1 - \epsilon_{p,z}) \epsilon_{p,z} \left[ \frac{1}{1-\alpha} \log \alpha + \epsilon_{p,z} \log \theta_{t-1} + \log \nu - \log \bar{N} \right]}{\phi_1 \epsilon_{p,z}^2 + \phi_2 (1 - \epsilon_{p,z})^2} \tag{2.26} \]

---

1For detailed derivation of this price function, see the Appendix.
2The condition on the probability distributions that ensures this form of the price function is

\[ E[(1 + \psi_{t+1})^\alpha \Gamma(z_{t+1})^\alpha | z_t] = \Gamma(z_t) \tag{2.22} \]

This condition is satisfied for values of \( \alpha \) close to 1.