Reference Frames

HW #1, Due 9/3/04

Read T&R Ch.1 (read quickly), 2.1-2.4

1) Galelean Transformation. A train moving with a speed of 50 m/s passes through a railway station at time t=t'=0. Fifteen seconds later a firecracker explodes on the track 1.0 km away from the train station in the direction that the train is moving. Find the coordinates (position and time) of this event in both the station frame (Home Frame) and the train frame (Other Frame).
2) Briefly describe an inertial reference frame. Since it takes time for light to travel from an event to the origin, an observer at the origin "sees" an event a short time after the event happens. Describe one way that our reference frame can take this travel time into account. (This travel time is a nuisance, but it does <u>not</u> cause the problems with simultaneity that we will soon meet.)
3) Use Excel to graph $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$ for $0<\beta<0.995$. Since $\beta = v/c$, you are really plotting γ as velocity v approaches the speed of light. We will spend quite a bit of time working with γ and β in the next few weeks. Make sure that you use enough points to show the shape of the curve.
4) Use a Taylor expansion to show that $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} \approx 1 + \dfrac{1}{2}\beta^2 \cdots$ for v << c (small velocities as compared to the speed of light). You can make your life easier by considering β^2 as one entity.
5) The Michelson-Morley experiment: Assume that interferometer arm #1 is in the +x direction and interferometer arm #2 is in the +y direction. Each arm has length L in the lab frame. Further assume that the whole experiment is moving in the +x direction with respect to the 'ether' (snicker, snicker). Assume that light travels at c=3e8m/s with respect to the 'ether.'
   a) Find the velocity in the lab frame (speed along the ground) of the light in
      i) arm 2 (perpendicular to motion)
      ii) arm 1, first leg (in the direction of the motion of the lab, against the "flow" of ether)
      iii) arm 1, second leg (along with the "flow" of ether)
   b) Show that the total time traveled on arm 2 is given by $T_2 = \dfrac{2L}{c}\dfrac{1}{\sqrt{1-\beta^2}}$ and the total time in arm 1 is given by $T_1 = \dfrac{2L}{c}\dfrac{1}{1-\beta^2}$.
   c) Use Taylor expansions to show that $\Delta T = T_1 - T_2 = L\beta^2$. Note the following Taylor expansions: $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} \approx 1 + \dfrac{1}{2}\beta^2 \cdots$ and $\gamma^2 = \dfrac{1}{1-\beta^2} \approx 1 + \beta^2 \cdots$.
   d) Extra Credit: Imagine watching this experiment from the "ether" frame. Show that if you contract the length of arm 1 (in the direction of the motion) by $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} \approx 1 + \dfrac{1}{2}\beta^2$ you get no time difference. Lorentz made this argument 15 years before Einstein's theory of relativity.
6) T&R problem 2.6

Time Dilation, Lorentz Transformation

HW #2, Due 9/10/04

Read T&R Ch 2.4-2.10, Mr. Tompkins Ch. 1, Taylor 3.4-3.6, 3.8

1) At exactly noon (as measured on your watch) your mother thinks of you and wishes that you are doing well. At exactly 1 ms later, you spontaneously decide to do your laundry. Could these events be causally connected? Briefly explain, then call your mother.
2) Assume that the speed of light is constant in all frames. Draw a spacetime diagram showing a disagreement between two frames of reference on whether two events are simultaneous. In other words, pick two events that are simultaneous in one frame and show that they are not simultaneous in another frame. Make sure to label and explain your diagram as necessary.
3) Briefly describe in words how a lack of simultaneity leads two different observers (frames) to measure the same object and get different lengths.
4) Draw a spacetime diagram showing two different frames measuring the length of a meter stick. Show the world lines of the two ends of the stick in the reference frame moving along with the stick. Then show how a second frame measures the length of the stick. Again, label and explain your diagram as necessary.
5) Two firecrackers A and B are placed at the engine and caboose of a train that is moving in the +x direction relative to the ground frame. According to synchronized clocks on the train, both firecrackers explode simultaneously. Which firecracker explodes first according to synchronized clocks on the ground? Explain using a spacetime diagram.
6) An event occurs at t = 6.0s and x=4.0s (4 light seconds) in the O Frame. When and where does this event occur in an O' Frame moving with β=0.50 in the +x direction with respect to the O Frame? Now take your answer and transform it back into the O' frame.
7) An event occurs at t' = 6.0s and x'=4.0s (4 light seconds) in the O' Frame. When and where does this event occur in an O Frame? The O' frame is moving with β=0.60 = 3/5 in the +x direction with respect to the Home Frame? Now take your answer and transform it back into the O frame.
8) A muon is created by a cosmic-ray interaction at an altitude of 60km. Measured on earth, the muon's speed down is 0.998. The lifetime for a muon at rest is about 2μs. Note that you are being bombarded with muons as you work this problem.
   a) How far would the muon travel (in the earth's frame) in a time 2μs measured on earth?
   b) How far does the muon travel (in the earth's frame) in 2μs measured by the muon?
   c) In the earth frame it looks like time dilation allows the muon to go farther than one would expect. From the muon point of view it appears that length contraction is responsible for the muon going farther than one might expect. Explain the extra distance from both the muon and the earth's point of view.
9) I sit in my office all day long without moving. You, however, come to office hours on Thursday morning from 9:30 – 12:00 in order to ask wonderful, deep, philosophical questions about the nature of time and to discuss our mutual interest in juggling. You then go about your typical day of dancing and frolicking (or at least walking between classes). At 5pm you realize that your physics problem set is due in 17 hrs and you scurry back to my office to catch the very end of my 4 – 5:30 pm office hours in order to ask remarkably pointed questions like "Is this what you want for #2." What is the <u>difference</u> in the time measured by your watch and my watch between the two visits. Who ages faster and can relativity explain my aged visage? Assume a constant velocity for the time that you are moving. Use a Taylor expansion to avoid traumatizing your calculator.
10) Explain why the twin paradox is not really a problem with relativity. Extra Credit: Use a spacetime diagram to show where the "extra time" comes from.

11) A spaceship leaves earth, travels to Pluto (which is 5.0 hr of distance away, that is, 5 light hours), and then returns exactly 11.0 hr later. If the spaceship's acceleration time is very short, so that it spends virtually all its time traveling at a constant speed, estimate the time measured between departure and arrival as measured by the ship's clock.
12) Given the following measured values with independent uncertainties given, find the value and uncertainty in the expressions below.

    $a = 5 \pm 1$ $\quad$ $b = 5 \pm 2$ $\quad$ $c = 15 \pm 3$

    i) $R=a + b$
    ii) $S = a - b$
    iii) $T = c/a$
    iv) $W = ab$
    v) $X = a(b+c)$
    vi) $Y= 5c$
13) Estimate the diameter of your head (I don't care what direction/method you choose). Assume that the uncertainty in your diameter estimate is 1cm. Find the circumference, surface area, and volume of your (spherical) head with uncertainties.

Paradoxes and Energy

HW #3, Due 9/17/04

Read T&R Ch 2.11-2.14, Mr. Tompkins Ch. 3, Taylor 4.2-4.4

1) Explain why the twin paradox is not really a problem with relativity. Extra Credit: Use a spacetime diagram to show where the "extra time" comes from.
2) A spaceship leaves earth, travels to Pluto (which is 4.0 hr of distance away, that is, 4 light hours = 4hr*c), and then returns exactly 10.0 hr (measured on earth) later. If the spaceship's acceleration time is very short, so that it spends virtually all its time traveling at a constant speed, estimate the time measured between departure and arrival as measured by the ship's clock.
3) How fast does an object have to be moving in a given frame if its measured length is to be "significantly" different from its rest length? Assume a significant difference is 1 part in 10,000.
4) Use a two observer diagram (and text) to qualitatively explain the pole/barn paradox. Show that the farmer does see the pole in the barn, the athlete does not see the pole fit into the barn, and (most importantly) that both of these views are consistent with each other.
5) Alex has just caught a lake trout 20 inches long. Zipping by a motor boat, a game warden sees the fish as 12 inches long. Uh-oh! The minimum legal length is 16 inches.
   a) How fast is the game warden moving relative to Alex?
   b) Do you expect that Alex will have to pay a fine? Why or why not?
6) The photo below was taken in a universe where the speed of light is slower than it is in our universe. Use this picture to estimate c in the other universe. This is obviously a very rough estimate…I am looking for your train of thought more than your answer.

Unbelievably shortened

7) T&R problem 2.48
8) How fast is a star traveling if it has a red shift of 2 (we measure its wavelength to be twice as long as the emitted wavelength.)
9) A particle has a initial speed of 0.5c. At what speed does its momentum increase by a) 1%, b) 10%, c) 100%.
10) What is the speed of a proton when its kinetic energy is equal to a) 10% of its rest mass, b) its rest mass, c) 3 times its rest mass.
11) Calculate the momentum, kinetic energy, and total energy of an electron traveling at a speed of a) 0.01c, b) 0.1c, c) 0.9c.
12) Taylor problem 4.12
13) In a desperate attempt at procrastination, you measure the length of your roomate's cat several times with the following results (in cm): 60, 65, 59, 64, 62, 57. A) Find the standard deviation in your measurements, b) estimate the uncertainty in any single measurement, c) then give the results of your measurement (length of cat ± uncertainty in length). To avoid a mind-numbing calculation, please use the standard deviation on your calculator.

Relativistic Energy, and Waves

HW #4, Not due 9/24/04

Read T&R Chapter 3 sections 3.1-3.4, 3.6

1) Which has more mass, a relaxed spring or a compressed spring?
2) T&R problem 3.73
3) A free neutron is an unstable particle and beta decays into a proton, and electron, and a (essentially) massless neutrino. How much kinetic energy is available in the decay?
4) An object has energy E= 1 MeV and momentum p=0.86 MeV/c. Find its
   a) velocity
   b) mass
   c) kinetic energy, K
5) If electrical energy costs \$0.04 per 10^6 J (\$0.15/kWhr), how fast can you make a 1g object travel for \$1million.
6) Two balls of putty, each of mass m, move toward each other with speeds of 0.95 measured in the home frame. The balls stick together forming a single motionless lump. What is the mass of the lump.
7) Compute the speeds of electrons (rest energy =511KeV) and protons (rest energy = 938MeV) with kinetic energies of 10MeV. Use relativistic and non-relativistic methods. Compare your results.
8) How much energy would be required to accelerate a particle of mass m from rest to a speed of a) 0.5 b) 0.99
9) A π- (mass 140 MeV) decays to a μ- (mass 106 MeV) and a neutrino. The neutrino is essentially massless, like a flash of light. If the pion is at rest when it decays, find the speed of the emitted muon.
10) A photon (consider this a flash of light for now) with energy E_0 hits an electron of mass m which is initially at rest. The photon scatters ("bounces") directly backward from the collision. Use conservation of momentum and energy to find the final energy of the photon E in terms of E_0 and m_e. This process is called Compton scattering.
11) Some ears can hear frequencies as low as 50 Hz and as high as 20kHz. What are the corresponding wavelengths? What is the wavelength of WHCL, the campus radio station?
12) Qualitatively describe the pattern that results when visible light goes through 1) a 10cm wide window, 2) a 20μm wide slit, 3) a 50nm wide slit. (Note: in case you have not done this yet, put your finger and thumb very close together right in front of your eye. Look through the very narrow slit at a distant street light. You should be able to see a single slit diffraction pattern!)
13) Draw, label, and very briefly explain a diagram showing 2-slit interference. Red laser light (λ=633nm) goes through two narrow slits 40μm apart. How far apart are the spots on a wall 4m away?
14) Estimate the energy of a visible photon in eV and J.
15) Estimate the number of photons that enter one of your eyes from a 60W bulb 100m away.

Relativistic Energy, and Waves
HW #5, Due 10/8/04

Read T&R Finish Ch. 3, Mr. Tomkins Chapter 8

1) Photoelectric Effect: Assume that the intensity of the monochromatic (one color=one frequency) light shining on a cathode is the same for all frequencies. Plot the following experimental results and explain your plots using the photon model.
   a) How does the kinetic energy of emitted electrons vary as a function of the frequency of the light.
   b) How does the current of emitted electrons vary as a function of the frequency of the light. Include a wide range of frequencies.
2) Photoelectric Effect: Assume that monochromatic light is shining on the cathode in a photoelectric effect experiment. Plot (or describe) the following experimental results and explain your plots using the photon model.
   a) How does the kinetic energy to vary with light intensity?
   b) How does the current of emitted electrons to vary with light intensity?
3) T&R 3.32: The human eye is sensitive to a pulse of light containing as few as 100 photons. For yellow light of λ = 580 nm how much energy is contained in the pulse. Provide your answer in eV.
4) How hot is the surface of the sun? Assume that the most intense wavelength from the sun is in the visible region of the spectrum.
5) Paint your favorite professor completely black, then estimate the power of the thermal radiation from him/her. Compare your answer to a typical 60W lightbulb. Is your favorite professor rather bright? Or pretty dim?
6) Estimate the wavelength and frequency of the maximum in the thermal radiation spectrum coming from your roomate's cat. If I am "white with fury" and this problem set makes your "blood boil," who is hotter?
7) A 10cm x 10cm black plate at a temperature of 300 K is place next to a similar plate at 200 K. What is the net amount of power transferred between them through thermal (blackbody) radiation. How much does your answer change if the cooler plate is close to 0 K?
8) Use the frequency-space treatment that we used in class to consider the number of standing wave modes for a two dimensional square crystal whose sides are 0.1m long. Assume that the speed of sound is 5000m/s in this material. Estimate the number of modes with frequencies between 1MHz and 1.1MHz.
9) When low energy X-rays scatter backward off a material, you can get two different energies (frequencies) in the backscattered beam.
   a) Explain why using the Compton scattering formula.
   b) Describe what happens to a pingpong ball if you bounce it off a) a golf ball and b) a bowling ball.
   c) Relate part a to part b.
10) In a photoelectric experiment similar to the one described in class, I measured the stopping voltage of the photoelectrons as a function of the wavelength of light shining on the cathode. From the following data, find the value for Planck's constant, h, and the work function, W. Remember that E=hν and use eV for your unit of energy. You

can use a TRENDLINE from Excel to find the slope and intercept. For extra credit use the LINEST function to evaluate the uncertainty in your results.

Wavelength (nm)	Stopping Voltage (V)
300	2.1
350	1.45
420	0.9
500	0.4
520	0.3

11) T&R 3.18: Use a computer to calculate Planck's radiation law (from the book) for a temperature of 3000K, which is the temperature of a typical tungsten filament in an incandescent light bulb. Plot the intensity versus wavelength.
   a) Estimate (roughly) the fraction of the power in the visible region.
   b) Compare the intensity at 400nm, 700nm, and at the peak of the spectrum.

Bohr Model

HW #6, Due 10/15/04

Read T&R Chapter 4, Mr. Tompkins Ch. 9

1) An electron and a proton in an atom are 10^{-10}m apart.
   a) Find the Coulomb force between the particles.
   b) Find the electrical potential energy between the particles.
2) Two protons in a nucleus are 10^{-15}m apart.
   a) Find the Coulomb force between the particles.
   b) Find the electrical potential energy between the particles.
3) Use Excel to plot the electrical potential energy between an electron and a proton which are separated by a distance r where r ranges from 10^{-11} m to $5\text{x}10^{-10}$ m.
4) The Bohr atom:
   a) List the primary assumptions of the Bohr model. (You can use my assumptions or the book's assumptions.)
   b) Calculate the classical relationship between electron speed and radius using a "planetary" (purely classical) model of the hydrogen atom.
   c) Calculate the relationship between electron speed and radius using Bohr's angular momentum assumption.
   d) Use the above to find possible radii for electron orbits in the Bohr model. Calculate the value of the smallest Bohr radius in nm.
   e) Find the allowable energies for the hydrogen atom using the Bohr model.
5) Derive the Rydberg equation from the allowed Bohr energies. Plug in numbers to find the Rydberg constant from fundamental constants.
6) Use an energy level diagram to show (label) the longest wavelength transitions in the Balmer series. Use a second diagram to show (label) the longest wavelength transitions in the Lyman, Balmer, and Paschen series of the hydrogen atom.
7) Use the allowed Bohr energies to calculate the energy of the first (lowest energy) line in the Balmer series of the hydrogen atom. From this energy, calculate the wavelength of this line.
8) Spend a moment being thankful that there are no uncertainty calculations on this problem set. List creative ways to celebrate.

de Broglie, Uncertainty Principle
HW #7, Due 10/22/04

Read T&R Chapter 5, Mr. Tompkins Ch. 10

1) Of the following quantities, which increase and which decrease for the electron in the Bohr model as n increases? a) frequency of revolution, b) speed, c) de Broglie wavelength, d) potential energy, e) kinetic energy, f) total energy? Derive your results using our results from class, or briefly explain your reasoning.
2) Consider a helium nucleus with one electron (singly ionized helium).
   a) Calculate the energy of the photon from the n=2 to n=1 transition.
   b) How much does the reduced mass correction change your result?
3) Find an approximate value for the wavelength of
   a) Yourself walking to class
   b) A 100 eV electron
   c) A thermal (room temperature) neutron
   d) An ultra-cold neutron traveling at 3m/s. Compare the size of these neutrons (approximately its wavelength) to the size of an atom.
4) Find an approximate (within a factor of 10 or so) value for *h* in the quantum jungle using the gazelles. (see Tomkins p. 120)

Gazelle Pic Here

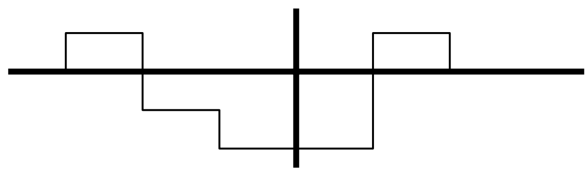
5) In my work, I deal with neutrons with wavelengths between 0.05 nm and 0.5nm. What energies (in eV) and velocities do these correspond to.
6) 0.5 nm neutrons Bragg scatter off a graphite crystal and come out perpendicular to the incident beam. What is the spacing of the Bragg planes in the graphite? Draw a diagram showing the Bragg planes and the angle that you used in your calculation. Note: this is how we made a monoenergetic beam of neutrons in my old research group at NIST. It is a very common technique in neutron physics.
7) Electrons are accelerated in an electric field caused by a 22 volt potential difference. They are traveling in a mercury vapor, which has its first excited state at an energy of 5 eV. Describe what happens to the electrons along their path. What is their final energy if they start at the negative electrode?
8) Fourier Transforms:
   a) A flute plays a very short note at a nominal frequency of 880 Hz. The note lasts only 1/32 of a second. What is the approximate range of frequencies contained in the sound pulse?
   b) If a phone line is only capable of transmitting a range of frequencies accessible to the human ear (50 Hz – 20,000 Hz) what is the shortest pulse that can be transmitted over the line. (Think about slow modems.)

9) If an exited state of an atom has a lifetime of 10^{-7} s, what is the uncertainty in the energy of photons emitted by such atoms as they decay to the ground state? This is called the natural line width.
10) Consider an electron confined in an atom. (see General Knowledge for size)
    a) What is the uncertainty in the momentum of the electron?
    b) If the electron's momentum is equal to Δp from part a, what is the electron's energy?
11) Using the same reasoning as the above question, what is the lowest energy allowed for a proton (mass = 938 MeV/c2) confined in a nucleus? Note the difference in energy scale between atoms and nuclei.
12) The Bohr magneton: A current going around a loop of wire creates a magnetic field. Use variables throughout this problem.
    a) The current is essentially the amount of charge that passes a given point in one second. The current I due to a charge e moving around in a circle with a frequency f is ef because the charge passes a given point on the circle many times in one second. (frequency of orbit=1/period of orbit. See problem 1) Find the current due to the electron in the first Bohr orbit.
    b) The magnetic moment of a current loop is IA, where A is the area of the loop. Find the magnetic moment of the electron in the first Bohr orbit. This magnetic moment is called a Bohr magneton and is the typical unit for measuring atomic magnetic moments. You should get $\mu_B = e\hbar/2m$. The front cover of your book lists the actual value in several sets of units.

Schrodinger Equation
HW #8, Due 10/29/04

Read T&R Chapter 6.1-6.4

1) T&R Question 6.9 (not problem 6.9) Can a wave packet be formed from a superposition of wave functions of the type $\psi(x,t)=Ae^{i(kx-\omega t)}$? Can it be normalized? Answer yes/no, yes/no, then relate this to the fact that a short musical note requires a range of frequencies, and to the fact that a confined particle requires a range of momenta (wavelengths).
2) T&R Problem 6.3: Show that the free particle wavefunction, $\psi(x,t)=Ae^{i(kx-\omega t)}$, satisfies the Schrodinger equation.
3) Given the following (highly unlikely) wavefunction of an electron, what is the probability that we will find (measure) the electron at a position greater than 0? Remember to normalize.

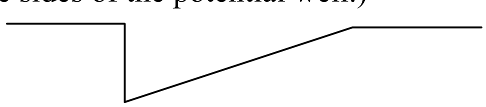
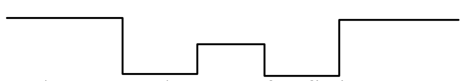


4) Normalize the following wavefunctions. *A* and *a* are constants. You may use a math table (remember Schaum's Outline that you bought at the beginning of the year?).
   a) $\psi(x) = Are^{-r/a}$ (consider only the region from r=0 to infinity)(T&R prob 6.5)
   b) $\psi(x) = Ae^{-ax^2}$ (consider all x)
5) T&R 6.15: For the infinite square-well potential, find the probability that a particle in its ground state is in each third of the one-dimensional box of length *L*.
6) Explain in words what is meant by normalization of a wave function.
7) Find values of θ such that $e^{i\theta} = -1$, $e^{i\theta} = +i$, $e^{i\theta} = -i$.
8) What are the three lowest energies allowed for an electron confined to a box that is roughly the diameter of an atom? What is the lowest energy allowed for a proton confined in a box the size of a nucleus? Note the difference in energy scale between atoms and nuclei.
9) Imagine a particle in a box whose wavefunction at time t=0 is a superposition of the box's 1st and 2nd energy eigenfunctions, $\psi_{E1}(x)$ and $\psi_{E2}(x)$:
$$\psi(x) = A\psi_{E1}(x) + B\psi_{E2}(x)$$
where A and B are real constants.
   a) Find $\psi(x,t)$. (add the time component of the wavefunction)
   b) Show that the absolute square of the wave function is not time-independent, but instead oscillates with $\omega=(E2-E1)/\hbar$. In other words, show that at least one term of $\psi^*\psi$ oscillates. You can reduce your result using Schaum, but you do not need to.
   c) What is the physical interpretation of this result?

Square Wells, Expectation Values, Qualitative Solutions

HW #9, Not Due 11/5/04

Read T&R Chapter 6.4-6.7

1) Consider an electron wavefunction given by $\psi(x) = A\sin(\pi x)$ for $0<x<3$ and given by $\psi(x) = 0$ everywhere else. Determine the probability of finding (measuring) the electron between x=0 and x=0.5.
2) What are the three lowest allowed energies for an electron confined to a box that is roughly the diameter of an atom? What is the lowest energy allowed for a proton confined in a box the size of a nucleus? Note the difference in energy scale between atoms and nuclei.
3) Infinite square well potential:
   a) Find the normalized wavefunctions for a one-dimensional infinite square well (box) of length *L*.
   b) Find the expectation value of the position, $\langle x\rangle$, for the n=2 wavefunction.
   c) Find the expectation value of the momentum, $\langle p_x\rangle$, for the n=2 wavefunction.
   d) Find the expectation value for the momentum squared, $\langle p_x^2\rangle$, for the n=2 wavefunction. Why this is different than $\langle p_x\rangle^2$? (think about it physically).
4) Consider the wavefunctions ψ and $\psi' = e^{i\theta}\psi$, where θ is real. Show that these wavefunctions will yield the same probability of finding any given expectation value, $\langle o\rangle$, (using any operator, $\hat{o}$). This means that there is no physical way to distinguish between them and they are completely equivalent: The overall complex phase of the wavefunction is arbitrary.
5) (T or F) For a given potential, there is a valid $\psi(x)$ for every value of energy.
6) Sketch a qualitatively valid wavefunctions (correct shape) for bound states in the following potentials. Also sketch qualitatively valid wavefunctions (correct shape) for unbound states in the following potentials. (an unbound state has more energy than the sides of the potential well.)
   a) 
   b) 
7) Consider the ground state of a finite square well potential. What happens to the energy as the height of the walls gets higher? Hint: First, draw a wavefunction for a finite well of length L. Then draw a similar wavefunction for an infinite square well. What happens to the wavelength?
8) The potential for a simple harmonic oscillator is given by $V(x) = \frac{1}{2} m\omega^2 x^2$. (Recall from class that classically $\omega^2 = k_{spring}/m$.)
   a) Try $\psi(x) = Ae^{-ax}$ as a solution. Is there any value of *a* that makes this work for all values of *x*?
   b) Try $\psi(x) = Ae^{-ax^2}$ as a solution.
   c) Find the value of *a* for the improved guess in part b).
   d) Find the value of *A* that normalizes the wavefunction.
   e) Write the COMPLETE (all parts!) wavefunction for the ground state of a simple harmonic oscillator.

9) Imagine a particle in a box whose wavefunction at time t=0 is a superposition of the box's 1st and 2nd energy eigenfunctions, $\psi_{E1}(x)$ and $\psi_{E2}(x)$:

$$\psi(x) = A\psi_{E1}(x) + B\psi_{E2}(x)$$

where A and B are real constants.

a) Find $\Psi(x,t)$. (in other words, add the time component of the wavefunction)

b) Show that the absolute square of the wave function is not time-independent, but instead it oscillates with $\omega=(E_2-E_1)/\hbar$. In other words, show that at least one term of $\Psi^*\Psi$ oscillates. You can reduce your result using Schaum, but you do not need to.

c) What is the physical interpretation of this result?

10) The wavefunction corresponding to the ground state in a simple harmonic oscillator is given by $\psi(x) = Ae^{-ax^2}$ where $a = m\omega/2\hbar$.

a) NOTE: an 'even' function is symmetric about x=0 so that f(x) = f(-x). For an odd function, f(x) = -f(-x). You should be able to convince yourself that $\int_{-\infty}^{\infty} odd\_function \cdot dx = 0$ and this may make your life easier.

b) Find the expectation value $\langle x \rangle$.

c) Find the expectation value $\langle p_x \rangle$.

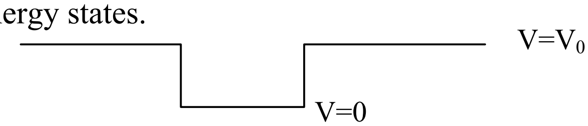
d) Explain why these are the results that you would expect classically.

e) Find the expectation value of $\langle x^2 \rangle$.

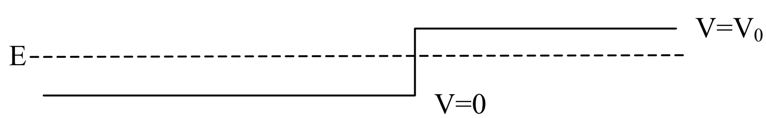
Tunneling and Transitions
HW #10, Due 11/19/04

Finish T&R Chapter 6

1) Imagine a particle in a box whose wavefunction at time t=0 is a superposition of the box's 1st and 2nd energy eigenfunctions, $\psi_{E1}(x)$ and $\psi_{E2}(x)$:
$$\psi(x) = A\psi_{E1}(x) + B\psi_{E2}(x)$$
where A and B are real constants.
   a) Find $\Psi(x,t)$. (in other words, add the time component of the wavefunction)
   b) Show that the absolute square of the wave function is not time-independent, but instead it oscillates with $\omega=(E_2-E_1)/\hbar$. In other words, show that at least one term of $\Psi^*\Psi$ oscillates. You can reduce your result using Schaum, but you do not need to.
   c) What is the physical interpretation of this result?
2) Allowed Region: Case I: Show that the general solution, $\psi_I(x) = Ae^{ik_Ix} + Be^{-ik_Ix}$, is a solution to the Schrodinger equation for $V(x) = V_0$ assuming that $E>V_0$. Find the value of k in terms of E and V_0. For $E>V_0$, finding k_I requires taking the square root of a positive number.
3) Forbidden Region: Case II: Show that the general solution, $\psi_{II}(x) = Ce^{-\alpha x} + De^{\alpha x}$, is a solution to the Schrodinger equation for $V(x) = V_0$ assuming that $E<V_0$. Find the value of α. How is this solution qualitatively different from the solution for $E>V_0$ that you worked with in question 2?
4) If you use the allowed solution ($k=\text{sqrt}[2m(E-V_0)/\hbar^2]$) in the forbidden region, then the argument inside the square root is negative and k becomes complex. Show that this method still gives the forbidden region solution above.
5) For a finite square well, write the possible solutions in the three regions. Write the boundary conditions that can be used to solve for any coefficients. You need *n* equations with *n* unknowns to make the system solvable.
6) For a barrier with height $V_0 > E$, write the possible solutions in the three regions. Write the boundary conditions that can be used to solve for any coefficients. You need *n* equations with *n* unknowns to make the system solvable. It may be helpful to consider making the incoming wave into a wave packet so that it can be normalized.
7) Consider a finite square well potential with its bottom at potential V=0 and its walls at potential V_0. Describe the difference in energy level spacing for $E< V_0$ (bound states) as compared to $E>V_0$. Consider the curvature of the wavefunction outside the well to describe why this is true. For $E>V_0$ you may want to consider wave packets instead of pure energy states.

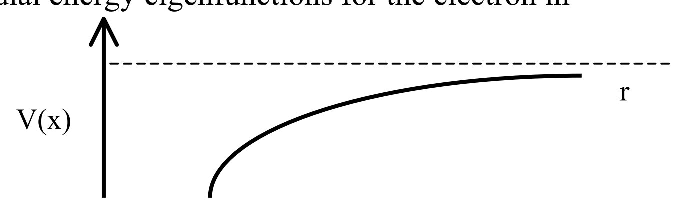


8) Sketch the wavefunction for the following potential assuming $E<V_0$. Use (and modify) your sketch to briefly describe tunneling.

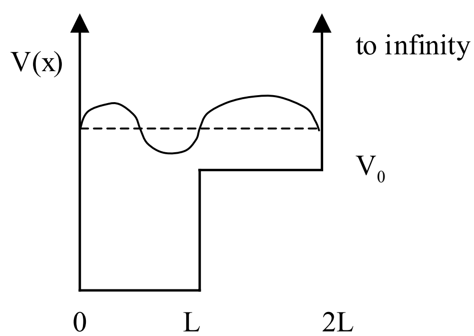


Tunneling and Transitions
HW #11, Due 13/3/04
T&R Chapter 7, Mr. Tompkins Chapter 11

1) Consider an electron moving radially toward or away from a proton. Having a radial separation of less than zero is physically impossible, so the electron's wavefunction must go strictly to zero at *r=0* as if there were an infinite barrier there. The potential energy function of the electron at other values of *r* due to its electrostatic attraction to the proton is given by $V(r) = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$
   a) List the qualitative "rules" for the shape of the wavefunction that we found from using curvature arguments. Include the number of bumps, bump amplitude vs. V(x), wavelength vs. V(x).
   b) For each "rule" give a brief justification using the curvature of ψ, $\frac{\partial^2}{\partial x^2}\psi$, and the Schrodinger equation.
   c) Qualitatively sketch the first three radial energy eigenfunctions for the electron in a hydrogen atom.



2) A particle of mass m moves in a one-dimensional potential V(x) as shown. The wavefunction for one of the energy states is sketched as well.



$V(x) = 0 \quad 0 < x < L$
$V(x) = V_0 \quad L < x < 2L$
$V(x) = \infty$ elsewhere

The particle has energy E and wavefunction as sketched:

$\psi_1(x) = A \sin(k_1 x) \quad 0 \le x \le L$
$\psi_2(x) = B \sin(k_2 x) L \le x \le 2L$
$\psi(x) = 0$ elsewhere
Note that $\psi(0) = \psi(L) = \psi(2L) = 0$

   a) Determine E in eV if $V_0 = 1$ eV. Hint: find the ratio k_1/k_2 in terms of L and also in terms of E and V_0.
   b) From the continuity conditions (for ψ(x) and ψ′(x)) at x = L, derive the ratio of the amplitudes in the 2 regions.

c) Find the probability that the particle would be found between 0 and L.
d) Write down the complete wave function Ψ(x,t) for this particle between 0 and L.
e) Do you think the function ψ(x) described above corresponds to the lowest energy (ground) state of the system? If not, then sketch the shape of ψ(x) for the ground state.

Extra Credit: Give upper and lower limits for the energy of the ground state (think about wavelengths). Briefly explain you reasoning.

Hint: Note that $\int_{0+a}^{\pi+a} \sin^2(\theta)d\theta = \int_{half\_period} \sin^2(\theta)d\theta = \frac{1}{2}$

3) Spherical coordinates:
   a) Show that a volume element in spherical coordinates is given by $dv = r^2\sin(\theta)d\rho\, d\theta\, d\phi$.
   b) Find the volume of a sphere of radius R by integrating $\int V(r)dv$ over all space where $V(r) = \begin{matrix} 1 & for & r \le R \\ 0 & for & r > R \end{matrix}$
   c) Find the expectation value for <Lz> for the $n=2,\ l=1,\ m_l=1$ wavefunction of the hydrogen atom. The Lz operator is given by $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$. Find the angular wavefunction from the table of Spherical Harmonics on p. 227. The radial wavefunction can be found on p. 243, but you do not need to use it explicitly.

   Note that this is much simpler than it might appear. Remember that for normalized wavefunctions $1 = \int \psi^*\psi dv$

1) Show that the ground state of hydrogen given below is normalized over all space.

$$\psi_{100}(r,\theta,\varphi) = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}$$

2) Use *separation of variables* to find the time-independent Schrodinger equation from the time-dependent Schrodinger equation. (Yes, this is in your notes and in the book but I want to make sure that you understand the separation of variables technique before you try 6.)
   a) Guess a solution Ψ(x,t) = ψ(x)T(t). Plug this into the time dependent Schrodinger Eqn.
   b) Divide by ψ(x)T(t). Isolate all of the *x* terms on one side and the *t* terms on the other side.
   c) Each side must equal a constant. Let the constant equal E. Try $T(t)=e^{iEt/\hbar}$ in the time equation.
   d) What is left is the time independent Schrodinger Equation.

3) A particle of mass m is perfectly confined to a square box whose edges are L long (see below). To solve this (2-D version of the *infinite well*) you have to start with the 2-D time-independent Shrodinger equation written in Cartesian coordinates:

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+V(x,y)\psi = E\psi$$

Use the *separation of variables technique* to derive the wavefunctions and allowed energies for a particle in this 2-D box. Note that V(x,y,z) = $V_x(x) + V_y(y)$ where $V_x(x)$ is an infinite square well potential in the x direction, with $V_x(x)=0$ inside the well and $V_x(x)$=infinity outside the well.

1st step: Assume $\Psi(x,y)=X(x)Y(y)$ and $V(x,y) = V_x(x) + V_y(y)$.

2nd step: Plug $\Psi(x,y,z)$ into the time-independent Schrodinger Eqn. Divide both sides by X(x)Y(y). Isolate all the x terms, but only the x terms, on one side.

3rd step: Note that your equation must equal a constant (say…E_x). Set the x terms equal to the constant. Try $X(x) = \sin(k_x x)$ in this x differential equation. (Remember that $V_x(x) = 0$ inside the well.)

4th step: Use the walls as constraints to find the possible values of k_x. You may want to use n_x. (consider the 1D square well for help.) Find E_x in terms of n_x.

5th step: Try $Y(y) = \sin(k_y y)$ in the y differential equation. Again, constrain k_y using the walls and by introducing n_y.

6th step: What is E_{total} in terms of n_x and n_y. How does this compare to the 1D square well?

Hydrogen Quantum Numbers, Multielectron Atoms

HW #12, Not Due 12/10/04

T&R Chapter 8

1) Make a table showing the quantum numbers for all of the possible states of the one electron atom for n=1, 2, and 3. Also indicate the number of electrons that each energy level can hold.
2) For the following hydrogen states given by (n, l, m_l, m_s) that are possible, make a table showing the values of E, L, Lz, S, Sz. For the states that are not possible, tell why they are not possible.
   a) (1, 0, 0, ½)
   b) (1, 1, 1, ½)
   c) (3, 0, 0, -½)
   d) (2, 1, -1, -½)
   e) (3, -1, 0, ½)
   f) (3, 2, 2, ½)
   g) (3, 2, 1, 0)
   h) (2, 0, 1, -½)
3) For the 3s and 3d levels, make a table showing all of the possible values of E, L, Lz, S, Sz.
4) For a 3d state, calculate the smallest angle that the angular momentum (or magnetic moment) can make with the z-axis.
5) (T&R7:30) For the hydrogen atom, determine whether the following transitions are allowed, and if they are, find the energy involved (in terms of E0) and whether a photon is absorbed or emitted.
   a) (5, 2, 1, ½) → (5, 2, 1, -½)
   b) (4, 3, 0, ½) → (4, 2, 1, -½)
   c) (5, 2, -2, -½) → (1, 0, 0, -½)
   d) (2, 1, 1, ½) → (4, 2, 1, ½)
6) List the allowed transitions (4p → 3s, etc) in a one electron atom between the n=4 and n=3 energy levels.
7) What is the difference between an atomic energy level, an atomic state, and an atomic spectral line?
8) Draw an energy level diagram for the first 4 states of hydrogen with separate columns for s, p, d… levels. Indicate all allowed transitions.
9) Briefly describe the mechanism for spin-orbit coupling and list the n=3 states in order of their energies.
10) Draw an energy level diagram showing how the energy of s and p states change when you include a) spin-orbit interactions with no external field and b) an external field.
11) Briefly describe the mechanism for hyperfine splitting.
12) In multi-electron atoms, why do the 4s states fill up before the 3d states?
13) Consider three angular momenta, $L_1 = 1$, $L_2 = 2$, and $L_3 = 3/2$. Give the possible total angular momenta that you could get by adding the following together. For part a), also give all the possible Lz values for each possible value of $L_1 + L_2$.
    a) $L_1 + L_2$
    b) $L_2 + L_3$
    c) $L_1 + L_3$
14) Define L, S, and J for a multielectron atom in an LS coupling scheme.
15) Describe the difference between LS coupling and JJ coupling.
16) (T&R 8:13) For the hydrogen atom in the 3d excited state find the possible values of l, m_l, j, s, m_s, m_j. Give the spectroscopic notation for each possible configuration.