

## 1. INSTRUCTIONS

## 2. SOME OF OUR FAVORITE EXPRESSIONS:

Please use the speed of light  $c = 3.0 \times 10^8$  m/s.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

$v/c$  in terms of  $\gamma$  is

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

“**Moving objects shrink**” or length contraction. Moving object of length  $L'$  contracts from its proper length  $L$  by

$$L = \gamma L'$$

“**Moving clocks run slow**” or time dilation. Moving clock  $t'$  runs slow relative to its proper time  $t$  by

$$t = \gamma t'$$

“**Simultaneity slips**” The time in a moving frame between simultaneous events in another frame is

$$T = \frac{vD}{c^2}$$

The details: If events  $E_1$  and  $E_2$  are simultaneous in one frame then in a frame moving with speed  $v$  in the direction from  $E_1$  to  $E_2$ , the event  $E_2$  occurs earlier than  $E_1$  by the time interval  $Dv/c^2$ , where  $D$  is the distance between the events in the second frame.

“**Velocity addition is modified**”. An object moves at  $u$  in a frame. In another frame moving at  $v$  with respect to this frame, the object moves at  $w$  given by

$$w = \frac{v + u}{1 + uv/c^2}$$

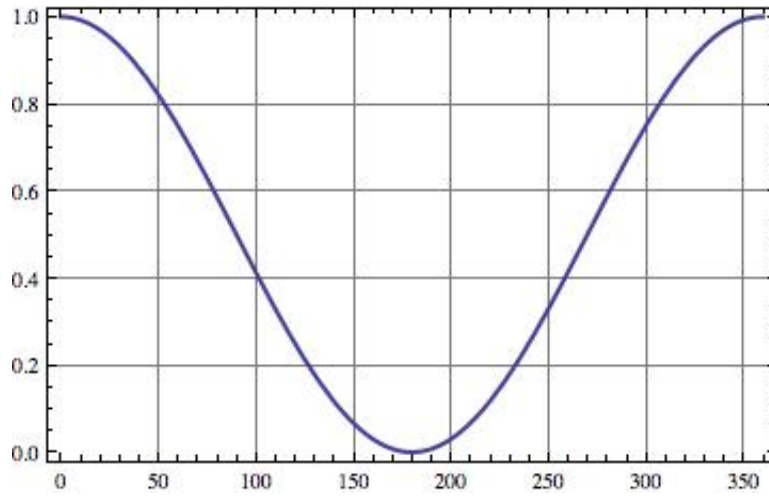
“**Light changes color**” For observers with relative velocity  $v$

$$K = \frac{T'}{T} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} = \gamma + \sqrt{\gamma^2 - 1} = z + 1$$

where  $T'$  is in the frame that receives the light,  $T$  is in the frame that emits light. For receding observers  $v > 0$ , for approaching observers  $v < 0$ . The speed in terms of  $K$  is given by

$$\frac{v}{c} = \frac{K^2 - 1}{K^2 + 1}$$

The probability chart  $P(m_\theta = +m_B | m_z = +m_B)$  vs.  $\theta$



**Quantum Mechanics** The probability for a system to transition from state  $A$  to state  $B$  is found by

- (1) **Find the paths** List the exclusive, exhaustive ways or paths to transition from  $A$  to  $B$ .
- (2) **Assign amplitudes** Determine the amplitudes for each path.
- (3) **Add amplitudes** Add the amplitudes for all possible paths “tip to tail”.
- (4) **Square to probability** Then the square of the length of the total amplitude is the probability for the transition.

“**Gravity slows clocks**” Alice and Bob are near a massive object that accelerates objects at  $g$ . If Alice is a height  $h$  above Bob then when Alice’s clock clicks off  $T_A$  then Bob’s clock ticks off  $T_B$

$$T_B = T_A \left( 1 - \frac{gh}{c^2} \right)$$

(to leading order).

The Schwarzschild radius of a black hole of mass  $M$  is

$$r_s = \frac{2GM}{c^2}$$

where  $G$  is Newton’s gravitational constant.

The Hawking temperature of a black hole of mass  $M$  is, in degrees Kelvin,

$$T_H = 6 \times 10^{-8} \frac{M_\odot}{M}$$

where  $M_\odot$  is the mass of the sun.

## 3. QUESTIONS: PLEASE ANSWER AND EXPLAIN YOUR ANSWER

- (1) Briefly explain the characteristic feature(s) of a black hole.
- (2) The mass of a black holes has a large impact on their behavior. Compare the environment near the horizon of a very massive black hole, say with a mass of a million times the mass of the sun so that  $\frac{M}{M_{\odot}} = 10^6$ , to the environment near the horizon of a black hole with the mass of Tethys, a satellite of Saturn, so that  $\frac{M}{M_{\odot}} = 3 \times 10^{-10}$ .