LENGTH CONTRACTION V2

SPATIAL SNAPSHOTS

By choice, \( L_A = D_A \) (1)
(WITHOUT LOSS OF GENERALITY)

SPACE-TIME DIAGRAMS

By the Relativity Principle
\[
\frac{L_T}{L_A} = \frac{D_A}{D_T} \tag{2}
\]

FROM THE SPACE-TIME DIAGRAMS
(A MO THE ABSTRACT "RULE")

FROM THE TETSUYA SPATIAL SNAPSHOTS,

NOW ALGEBRA! (3) AND (4) GIVE

\[
\frac{T_A}{T_T} = \frac{L_A}{L_T} = \frac{D_A}{D_T} \tag{3}
\]

\[
\frac{T_T}{c^2} = \frac{D_T}{c^2} \tag{4}
\]

FROM (3) AND (4)

\[
D_T = L_T + V T_T \tag{5}
\]

FROM (5) AND (3)

\[
D_T = L_T + \frac{D_T V^2}{c^2} \Rightarrow L_T = D_T \left(1 - \frac{V^2}{c^2}\right) \tag{7}
\]
\[ L_T = \frac{1}{\gamma^2} L_A = \frac{1}{\gamma^2} D_A = \frac{1}{\gamma^2} D_T \quad (8) \]

**Therefore, combining (7) and (8),**

\[ L_T = \frac{1}{\gamma^2} D_T = D_T \left( 1 - \frac{v^2}{c^2} \right) \]

\[ \Rightarrow \quad \frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2} \Rightarrow \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

**So, from (6) again**

\[ L_T = \frac{1}{\gamma} L_A, \text{ in words "measured in a moving frame, an object's proper length } \]

\[ \text{L}_A \text{ is contracted by a factor of } \gamma \]

- Adapted from Mermi, *It's About Time*, pp. 58-69