

1. SOME OF OUR FAVORITE EXPRESSIONS

For the purposes of this test, let's use $c = 3.0 \times 10^8$ m/s.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

For receding observers,

$$K = \frac{T'}{T} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$

Length contraction:

$$L' = \frac{L}{\gamma}$$

Time dilation:

$$t = \gamma t'$$

Time between simultaneous events in moving frame:

$$T = \frac{vD}{c^2}$$

Velocity addition:

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + v_{AB}v_{BC}/c^2}$$

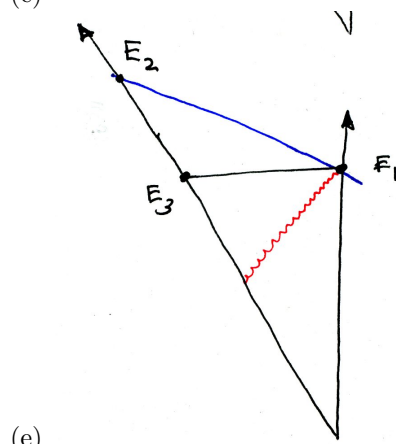
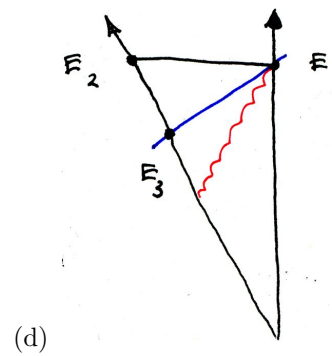
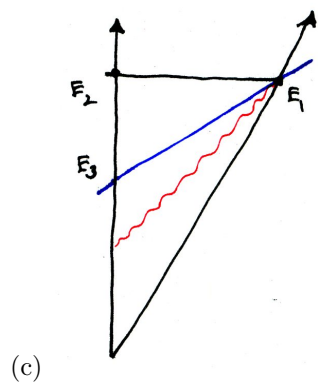
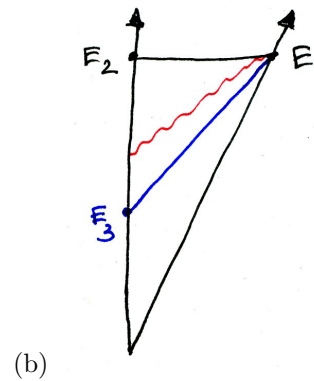
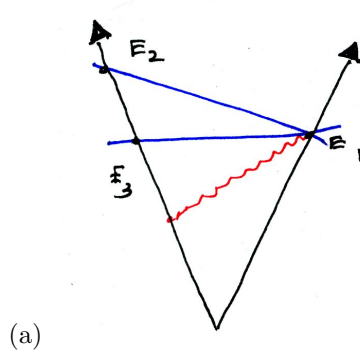
2. MULTIPLE-CHOICE QUESTIONS: ANSWER AND BRIEFLY DEFEND YOUR ANSWER

[On the actual test there will likely be fewer of these.]

- (1) Two, identical whales collide elastically (happily!). Before the collision Werther on the right moves to the left at 3.5 ms^{-1} . Charlotte on the right is stationary. In the original reference frame, what happens after the collision?
 - (a) Werther moves at $+3.5 \text{ ms}^{-1}$ while Charlotte is stationary.
 - (b) Both whales move at -3.5 ms^{-1} .
 - (c) The whales away from each other at a speed of 1.75 ms^{-1} .
 - (d) Werther is at rest and Charlotte moves at -3.5 ms^{-1} .
 - (e) The whales move away from each other at a speed of 3.5 ms^{-1} .
- (2) At 10 AM the President of Transylvania receives a message from an agent who is observing a secret base on Mars, "The missile targeting Castle Bran is set to fire at 12:15 PM." Fortunately some days before Transylvania Special Operations set explosive charges on the launch pad of the missile. Immediately the President presses the button to activate the charges, thus sending a signal to Mars. Is the missile destroyed before it launches? Assume that Mars is 20 light minutes away.
 - (a) No
 - (b) Yes
 - (c) It depends on the choice of frame in which you answer this question.

- (3) On January 1st, Alice's birthday, Alice heads away from Bob at $v = 4/5c$. (This means that $K = 3$ and $\gamma = 5/3$.) Bob calculates that he needs to send a "Happy Birthday!" message on February 11 so that it arrives on the following January 1st, according to Bob. When does the message arrive, according to Alice?
- (a) After one year, on January 1st.
 - (b) After $5/3$ years or about August 1st of the following year.
 - (c) After $3/5$ of a year or about July 6
 - (d) After $4/5$ of a year or about September 18.

(4) What does the space-time diagram of the history look like from Alice's frame?



- (5) Redshift is determined by
- length contraction
 - the change in color of the light received as compared to the emitted light
 - time dilation
 - the change in how bright the received light is as compared to the emitted light
 - simultaneity
- (6) Redshift is useful because you can use it to determine
- how fast an object emitting light is moving.

- (b) how fast any object is moving.
 - (c) the rate of clocks in a moving frame
 - (d) the apparent length contraction of moving objects
- (7) Muons, moving $0.99c$ relative to the ground, are created in the upper atmosphere, at 20 km. In the ground frame the muon half-life of $1.52 \mu\text{s}$ is dilated to about $7.1 \times 1.52 = 10.8 \mu\text{s}$ and 1 % of the particles arrive at the surface of earth. What is the description of this in the muon rest frame?
- (a) Time runs slowly on earth so the half-life is now approximately $11 \mu\text{s}$ and more particles make it to sea level.
 - (b) The atmosphere is contracted to a depth of approximately $20 \text{ km} / 7$ or 2.8 km , hence the fraction that make it is still about 1%.
 - (c) The observers disagree. The resolution is left up to experiment.
 - (d) The muons don't make it to the earth's surface because the half-life is $1.52 \mu\text{s}$ in their rest frame and only 5×10^{-12} % survive.
 - (e) The atmosphere is dilated to a depth of approximately 142 km and only 3×10^{-95} % survive.

3. QUESTIONS: ANSWER CAREFULLY AND COMPLETELY [ON THE ACTUAL TEST THERE WILL LIKELY BE MORE OF THESE.]

- (8) Sketch the space-time diagram of question 1 in the original frame.
- (9) A steel cable connects two space-suited whales at rest. Well-equipped, whales agree to fire identical rockets at the same instant, as determined by the original frame. The trains accelerate in such a way as their velocities, as measured in the original frame, are always equal. The cable will snap if it is stretched by as much as 1%. Explain what happens. Include a space-time diagram.