SNELL'S LAW

Objectives: • Investigate refraction at a boundary of media and use this to determine the index of refraction.
• To work with weighted averages.

To Do Before Lab: • Read this lab
• Read Taylor 4.1 and 7.1

Apparatus:
Laser, D shaped water lens, piece of fiber optic cable, protractors, Excel, coffee creamer, mystery fluid

Grade: Quiz

Introduction:
Light is a wave. Hence, many of the principles of waves that we learned determine how light behaves. However, for most common situations using lenses and mirrors we can simplify our analysis using geometric optics. Geometric optics rests on three simple assumptions:

1. Light travels in straight lines, called rays;
2. Light rays cross each other with no interference between them;
3. Whenever the rays strike the interface between two media in which the speed of light is different (e.g., air-glass;.glass-air; air-water, etc.) the rays bend, by an amount which depends on the two speeds and the angle of incidence.

These assumptions have their limits: We know from diffraction that the first assumption does not work for apertures the size of the wavelength of light. The second assumption is related to the superposition principle which is true for all waves. The third assumption is the subject of this lab. Known as "Snell's law," it states that the angle of refraction is related to the angle of incidence via

$$n_I \sin(\theta_I) = n_R \sin(\theta_R) \quad (1)$$

with n_I and n_R being the index of refraction for the material on the incident side and refracted side, respectively. These quantities are shown in Fig. (1).

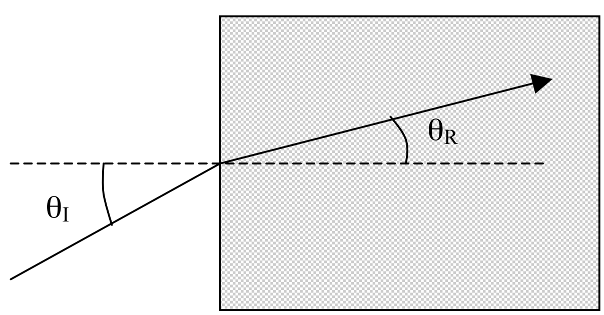


Fig. 1: Snell's Law

Part I: Data

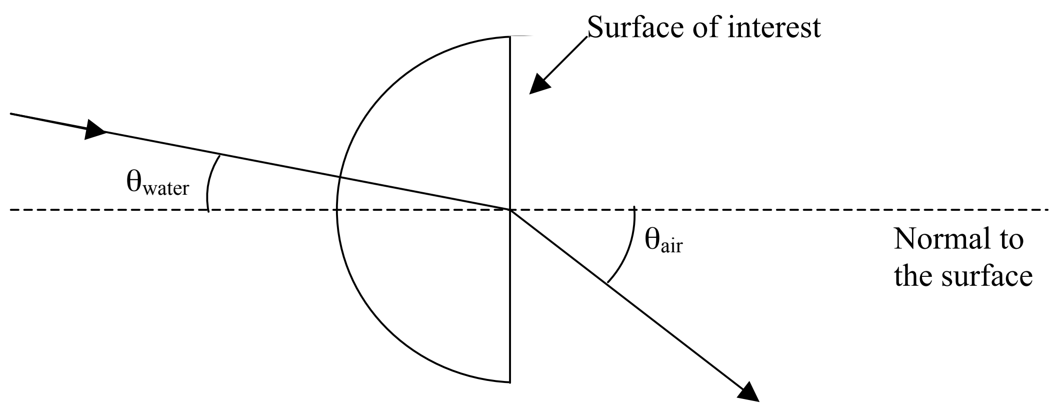
Refraction refers to the bending of light when it crosses an interface between two materials. We will discuss this from a wave point of view in class, but for now we will consider it experimentally.

(1) Arrange your laser and D shaped water dish as shown. We are interested in looking at refraction as the light passes from water-to-air. Using Eq. (1) show that the light does not bend when it crosses a surface along the normal (perpendicular) to the surface.

The beam must hit the dish at normal incidence – *perpendicular to the surface-* so there is no bending of the light at the air-to-water interface. Align the dish so that its straight edge is along the protractor axis and the dish is centered on the protractor. Align the laser so that it is centered and perpendicular to the axis of the protractor. (What happens if either of these two conditions is not met?)

Qualitatively describe what happens to θ_{air} as θ_{water} increases from 0 to 90 degrees. Now measure θ_{air} as a function of θ_{water}. You should be able to get 10 or 15 accurate points with uncertainties. Enter this data into Excel. Plot sin(θ_{water}) vs. sin(θ_{air}).

> ***ACHTUNG*! Excel Eccentricity:** Excel uses radians in "=sin(A2)". You will need to convert from degrees to radians.



Does your data look straight? Should it?

(2) As in Eq. (1) Snell's law of refraction states that $n\sin(\theta)$ is constant across an interface so

$$n_{water}\sin(\theta_{water}) = n_{air}\sin(\theta_{air}) \qquad (2)$$

where n_{water} and n_{air} are the index of refraction for water and air, and θ_{water} and θ_{air} are defined in the diagram above. For each point find n_{water} using $n_{air} = 1.00029 \sim 1$.

We also need the uncertainty in the index of refraction *for each point*. We'll call this σ_{nwater} . Looking at Eq. (2) we see that to compute σ_{nwater} at each point we need to find the uncertainty in a ratio of sines. This can be done in two steps: first find the uncertainty in $\sin(\theta_I)$ and in $\sin(\theta_R)$, then find the uncertainty in their ratio. With Excel compute the uncertainty at each point. What do you notice about the size of the uncertainty as the angle increases?

To find the final uncertainty due to *all* the points, we could just take the average of our measurements to get our final value for n_{water}. However, you just found that the errors are not all the same. The value of n is more accurate for large values of θ. We can take this into account using a weighted average rather than a straight average. The weighted average uses a weighting factor of $1/\sigma^2$ which counts measurements with smaller uncertainties more strongly. For a set of N measurements, n_i, each with error σ_i, the weighted average and its error is given by

$$n_{ave} = \frac{\sum_i \frac{n_i}{\sigma_i^2}}{\sum_i \frac{1}{\sigma_i^2}}$$

$$\sigma^2{}_{ave} = \frac{1}{\sum_i \frac{1}{\sigma_i^2}}$$

(Note that if σ is the same for all N measurements this reduces to the usual average where the denominator is the number of measurements.

$$n_{ave} = \frac{\sum_i n_i}{N}$$

$$\sigma^2{}_{ave} = \frac{1}{N}\sigma^2$$

These are just the usual formulas for an average and its uncertainty.)

Part II: Results

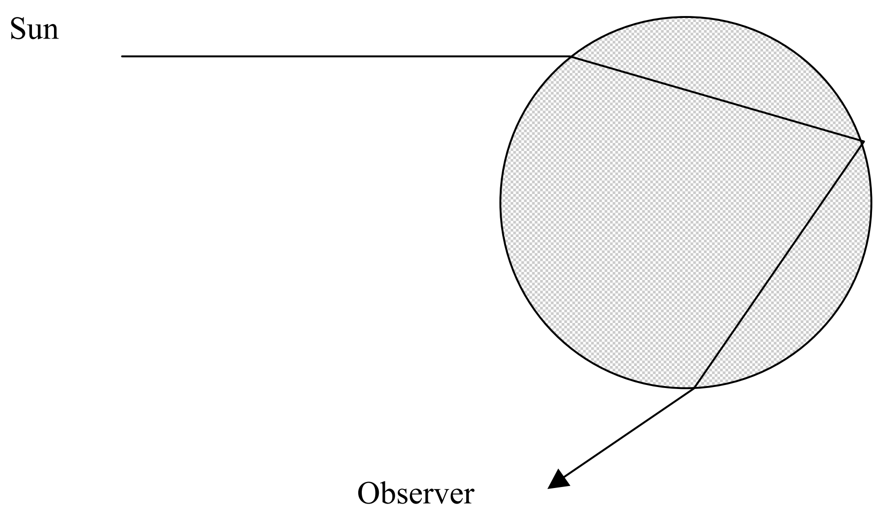
(1) Plot your values for n_{water} as a function of θ_{water}. Include error bars.

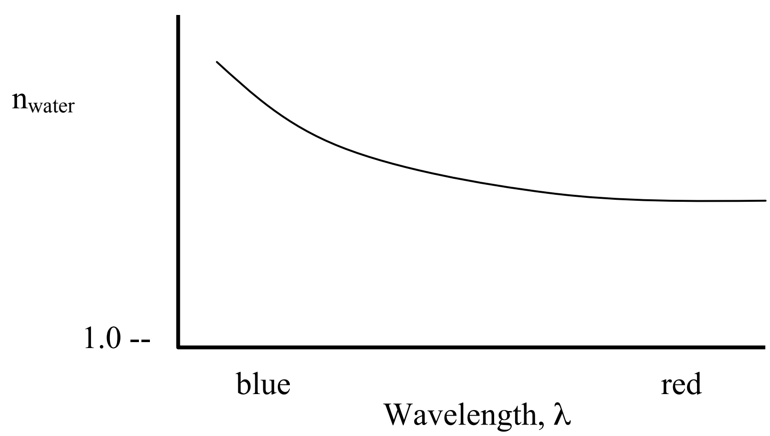
(2) Do you see any evidence for a systematic error in the graph? Use Excel to find out what would happen to your graph if your D shaped lens were tipped by 3 degrees away from the proper alignment. You should be able to see the advantages of a weighted average.

Part III: Total internal reflection

What happens at large angles of incidence? Experimentally explore the angles around the point where the amount of transmission changes drastically. This angle is called the critical angle. What is the condition for the critical angle? Find an expression for the critical angle in terms of the index of refraction of water. Compare your expression to a measured value.

Extra: *Rainbows*: A rainbow is caused by light entering into many small raindrops, reflecting off the back surfaces, and leaving the droplets to arrive at your eye. Refraction happens when the sunlight enters and exits the raindrops. Different wavelengths of light have slightly different indices of refraction. Using the diagram below and the plot of n vs. wavelength, figure out if red or blue is on the inside of the rainbow.





Lab Quiz: Identifying a fluid

While a particular index of refraction does not uniquely determine a fluid, it can help you find out what you have. With your lab group, devise a simple method using the equipment you have to determine the index of refraction of a fluid. Once you have a satisfactory method ask your instructor a trial fluid. Determine the index of refraction. Individually write up your results in a concise summary. Please include: (1) A description of your procedure (2) Your result with uncertainty stated as a complete measurement, i.e. $n \pm \sigma_n$.