Simple Oscillation

Introduction

Last semester you learned about Hooke's Law, which says that the force exerted by a stretched spring is directly proportional to the amount of stretch. The proportionality constant, usually called *k*, characterizes the stiffness of the spring.

Consider a spring held securely at one end and hanging vertically. Suppose a mass is hung from the free end of the spring and allowed to come to rest. It is in equilibrium. Then the force exerted by the spring is equal in magnitude to the force of gravity, $F_{spring} = kx_{eq} = mg$, where x_{eq} is the extension of the spring in equilibrium.

If a mass hanging on a spring is displaced from its equilibrium position and released, it will oscillate up and down in a nearly periodic fashion (The motion is not perfectly periodic because of energy lost from the mass spring system.). Assuming the motion is periodic, we define the period, *T*, as the time to make one complete oscillation. In other words, given some initial position and velocity, it is the time to return to that same position and velocity.

In this lab we will measure the spring constant of a spring and we will determine the relationship between the period of oscillation and the amount of mass on the spring. In the process we will have a chance to review some uncertainty analysis skills learned last semester as well as apply some new ones.

Apparatus

Mass set, spring, stopwatch, support stand, sonic ranger.

Procedure

Part I Measuring k with uncertainty.

1. Hang 100 g from the spring and determine how much the spring stretches. Include a diagram and record the initial and final positions of the end of the spring.

2. Estimate the uncertainty in the stretch and the uncertainty in the mass. Note that measuring the stretch involves two measurements. Be sure to take this into account when estimating the uncertainty.

3. Determine *k* (in N/m) and determine the uncertainty in *k*.

4. Use a propagation of error rule to determine the uncertainty in *k*. Write *k* with uncertainty in standard form to the correct precision and celebrate by drawing a box around your result.

5. If the spring obeys Hooke's Law, how much should it stretch if you increase the mass to 200 g? Try it. Record your results. Is the spring linear to within the uncertainty of measuring the stretch?

Part II Measuring the period of oscillation.

1. Hang 100 g total from the spring. Lift the mass up about 10 cm above its equilibrium position and release the mass. Use the stopwatch to time *one* complete oscillation. Repeat for a total of 10 trials. Calculate the average period, the standard deviation, and the standard error. Write your result in standard form.

2. Start the mass oscillating again but now time 10 complete oscillations. Estimate the uncertainty in the time. Determine the period with uncertainty.

3. Compare the two methods used to determine the period. Is one better than the other? Explain. Discuss your thoughts with your instructor.

4. Open the document called Motion Sensor. Start the mass oscillating with the same amplitude as before and use the sonic ranger to collect 10 seconds worth of position vs. time data. Use Curve Fit to fit the data to a sine function and determine the angular frequency of the motion and its uncertainty. Determine the period and uncertainty.

5. Start the mass oscillating with an amplitude of about 1 cm instead of 10 cm and use the sonic ranger to determine the period with uncertainty. Do you see evidence that period depends on amplitude? If so, would you say there a strong dependence or a weak dependence? If you are not sure, take more data.

6. Compare the methods of determining the period, using your results and discussing which experimental methods are better or worse relative to the others.

Part III Dependence of period on mass

1. Measure period for at least 5 different masses, over as broad a range of masses as is practical. Caution: Do not exceed 400 g or you may do permanent damage to the spring.

2. Make a graph of period vs. mass. (Period on the vertical axis and mass on the horizontal axis.) Draw in by hand a best-fit line and describe the shape of the graph. Think about whether or not the graph should go through the origin.

3. With only a small number of points there are many possible curves that fit the data well. A reasonable guess of the actual relation is a power law of the form

$$T = Am^c, \tag{1}$$

where T is the period, m is the mass and A and c are constants. From your graph you can surmise that if this relationship is correct, then b is less than 1. Taking the log of both sides of this equation gives

$$\log T = \log A + c(\log m). \tag{2}$$

According to equation (2), a plot of log T vs. log m will give a straight line. Plot your data in this way. Are you convinced that your data is consistent with equation (1)? If so use the Linest function to determine the uncertainty in the slope and the intercept. State your value for c in standard form to the correct precision and celebrate your result by drawing a box around your result. What do you think the theoretical value for c might be? Discuss the agreement, or lack thereof, between the expected value and the measured value of T. Can you figure out the significance of the value of A?