PENDULUM MOTION: MOVING AWAY FROM SIMPLE MOTION

Objective: • Determine how the period of a pendulum depends on amplitude.

Prelab: • Read this handout.

- 1. Consider the two pairs of data points shown at right with uncertainty bars. Is pair A or pair B more likely to come from a measurement of the same value? Explain.
- 2. Suppose an ideal massless spring with spring constant $k = 6.48 \pm 0.02$ N/m is compressed from its equilibrium position by $0.303 \pm .002$ m. What is the magnitude of the force holding the spring compressed? Use quadrature for error propagation and give your answer in standard form.

Apparatus: Stopwatch, string, mass, clamp, meter sticks, protractor

Introduction: A *simple pendulum* is an idealization consisting of a point mass, swinging at the end of a massless string. In class you have learned that for small angles of swing, the period of a simple pendulum is *approximately* given by the formula

$$T = 2\pi \sqrt{\frac{\ell}{g}} , \qquad (1)$$

where ℓ is the length of the string and g is the acceleration of gravity. As the amplitude of oscillation approaches 0, the period of a simple pendulum approaches the value given by eq. 1. In this lab we will investigate what happens to the period when the small angle approximation is not valid.

Procedure:

Given the tools available, design an experiment that will allow you to determine the period T as a function of angular amplitude θ_m , with minimal uncertainty in each quantity. To determine θ_m , you may use the protractor or trigonometry. Measure the period of the pendulum for as broad a range of angular amplitudes as practical. Plot T vs. θ_m including error bars for both T and θ_m . Be sure to explain your methods, including estimates of error bars, in your notebook.

Later in your physics education you may be lucky enough to derive the relation $T = T_0 \cdot Q(\theta_m)$ where T_0 is the value given by eq. 1 and $Q(\theta_m)$ is the amplitude correction. The function $Q(\theta_m)$ can be written as a power series in θ_m ,

$$Q(\theta_m) = 1 + a_1\theta_m + a_2\theta_m^2 + a_3\theta_m^3 + \dots$$

Note that some of the a_i may be zero. If θ_m is not too large ($\theta_{radians} \le 1$), the higher order terms in the power series become progressively smaller very quickly and only the first non-zero term after the 1 is important. Determine the power of the first non-zero term after the 1 by plotting your data in such a way that it will give a straight line if the first term is of order θ_m , θ_m^2 , or θ_m^3 . See if you can convince your instructor that the correction term is linear, quadratic, or cubic using your data. Once you settle on the power of the term, determine the value of the leading non-zero coefficient, ($a_1, a_2, \text{ or } a_3$) and its uncertainty.

