Pendulum Motion:
Moving away from Simple Motion

Objective: • Determine how the period of a pendulum depends on amplitude.

To Do Before Lab: • Read this handout.

Apparatus: Stopwatch, string, mass, clamp, meter sticks, protractor

Introduction: A simple pendulum is an idealization consisting of a point mass, swinging at the end of a massless string. In class you have learned that for small angles of swing, the period of a simple pendulum is approximately given by the formula

\[ T = 2\pi \sqrt{\frac{\ell}{g}} , \]

where \( \ell \) is the length of the string and \( g \) is the acceleration of gravity. As the amplitude of oscillation approaches 0, the period of a simple pendulum approaches the value given by eq. 1. In this lab we will investigate what happens to the period when the small angle approximation is not valid.

Procedure:
Given the tools available, design an experiment that will allow you to determine the period \( T \) as a function of angular amplitude \( \theta_m \), with minimal uncertainty in each quantity. To determine \( \theta_m \), you may use the protractor or trigonometry. Measure the period of the pendulum for as broad a range of angular amplitudes as practical. Plot \( T \) vs. \( \theta_m \) including error bars for both \( T \) and \( \theta_m \). Be sure to explain your methods, including estimates of error bars, in your notebook.

Later in your physics education you may be lucky enough to derive the relation

\[ T = T_0 \cdot Q(\theta_m) \]

where \( T_0 \) is the value given by eq. 1 and \( Q(\theta_m) \) is the amplitude correction. The function \( Q(\theta_m) \) can be written as a power series in \( \theta_m \),

\[ Q(\theta_m) = 1 + a_1 \theta_m + a_2 \theta_m^2 + a_3 \theta_m^3 + \ldots \]

Note that some of the \( a_i \) may be zero. If \( \theta_m \) is not too large (\( \theta_{\text{radians}} \leq 1 \)), the higher order terms in the power series become progressively smaller very quickly and only the first non-zero term after the 1 is important. Determine the power of the first non-zero term after the 1 by plotting your data in such a way that it will give a straight line if the first term is of order \( \theta_m, \theta_m^2 \), or \( \theta_m^3 \). See if you can convince your instructor that the correction term is linear, quadratic, or cubic using your data. Determine the value of the leading non-zero coefficient, (\( a_1, a_2, \) or \( a_3 \)) and its uncertainty. You get a bonus point if you can measure a value for ‘\( a \)’ (within 20% of the right value and with a relative uncertainty of less than 20%). Two bonus points are available for ‘\( a \)’ within 2% and with an uncertainty of less than 2%. 