RESONANCE IN A MECHANICAL SYSTEM

Objectives: • Study resonance in a simple mechanical system.
• Find the resonance curve, Q, and the damping coefficient from data.

To Do Before Lab: • Read this lab and K&K 10.3
• Review Taylor Ch 3

Apparatus: Sonic ranger, LoggerPro, k~3.5 Nm^{-1} spring, 50 g mass hanger, mass selection, cardboard (~13 cm square), speaker, function generator, oscilloscope, stopwatch, scale, cables, Excel.

Introduction:
In this lab we explore the phenomenon of resonance in a driven, damped oscillating system. The system is just what we used two weeks ago, the familiar mass on a spring, but this time there is a bit of extra drag provided by cardboard.

As you have seen in class, the equation of motion for this driven, damped oscillating system is

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \omega_o^2 x = \frac{F(t)}{m}. \qquad (1)$$

We will use a modified loudspeaker connected to a frequency generator to produce a driving force of the form $F(t) = F_o \sin(\omega t)$. Notice that ω is the driving frequency; a parameter that we can control. The late time or steady state solution to this differential equation is given by

$$x_p(t) = A\cos(\omega t - \varphi) \qquad (2)$$

with

$$A = \frac{F_o}{\sqrt{m^2(\omega_o^2 - \omega^2)^2 + \omega^2 b^2}} \qquad (3)$$

and

$$\varphi = \tan^{-1}\left(\frac{\omega b}{m(\omega_o^2 - \omega^2)}\right). \qquad (4)$$

The lab focuses on the relationship between the amplitude of oscillation and the driving frequency.

Part I: Your system's properties
In this section we determine the natural frequency, the damping constant, and the resonant frequency of the system.

(1) Determine the spring constant with uncertainty.

(2) Measure the masses of the hanger with the cardboard and the spring.

(3) Calculate the natural angular frequency of the undamped system and its uncertainty using the rules for error propagation in Taylor. Box this result.

(4) Determine *b*: Set the mass oscillating with an amplitude of about ~10 cm and record the motion with the sonic ranger for 15 s. What is the equation that describes this damped motion? Use the curve fit feature of LoggerPro to find a best fit for the sonic ranger data. From the equation for the best fit determine *b*. Consult your instructor if the fitted parameters seem inappropriate (LoggerPro can go badly wrong with this fit).

(5) Calculate the damped angular frequency ω_d. How does this angular frequency compare with the natural angular frequency? Now find the resonant angular frequency and convert it to a regular frequency.

Part II: Resonance Data

In class and the reading you saw the effects of resonance and the curves of amplitude *A* and total energy as a function of frequency. In this part of the lab you take frequency and amplitude data so you can create your own resonance curve for the mechanical system in front of you! Here's how:

(1) Turn on the frequency generator and the oscilloscope. Set the frequency generator so that it generates a sine curve at the approximate resonant frequency you calculated above in Part I. Change the frequency a bit and see the effect on the mass hanger. Ooo-ahh!

(2) Check to be sure that you have a nice signal on the scope. Measure the frequency on the oscilloscope and its uncertainty. Record this in a table of frequencies and amplitudes.

(3) Set LoggerPro to record about 100 seconds of data. Start the mass from approximately its equilibrium position and start taking data. Sit back and enjoy the show! Towards the end of the 100 seconds the system should settle down to oscillations at a constant amplitude.

(4) Measure the amplitude using Logger Pro to curve fit. This is your first amplitude data point.

(5) Explore the amplitude vs. frequency space by repeating steps (2) through (4). In the end you should have about 10 points showing how amplitude depends on the driving frequency. Choose your frequencies appropriately. Typical range is around 0.4 Hz. Each data point will take 100 seconds to find. You may assume that the uncertainties for all your amplitudes and all your frequencies are the same. Why?

(6) Enter your data into Excel and plot the squared amplitude – proportional to energy - vs the frequency.

(7) Add *x* and *y* error bars to your graph. Print this plot and include it in your notebook.

(8) This is it! Does your prediction for the resonant frequency agree with your results? Do not worry if they differ but do write a celebratory epistle or discuss what might have gone awry.

(9) Estimate the width Δf of the curve at one-half the maximum height. This will give you the "Q" of the mass-spring system with the relation $Q \cong f_R / \Delta f$. Box this important result. Compare this result for Q with what you obtain from the relation, "$Q = m\omega/b$".