

RESONANCE BEHAVIOR OF A DRIVEN DAMPED OSCILLATOR

- Objective:**
- Study resonance in a simple mechanical system.
 - Find the resonance curve, Q , and the damping coefficient from data.

Apparatus: Sonic ranger, LoggerPro, spring, mass hanger, apparatus for magnetic damping, mass selection, scale, signal generator, oscilloscope, amplifier, and mechanical driver.

Introduction:

This week explore the motion of a damped oscillator and the phenomenon of resonance in a driven, damped oscillating system.

When a driving force is added to this physical system, the equation of motion can be written as

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_o^2x = \frac{F(t)}{m}. \quad (1)$$

We use a modified loudspeaker connected to a frequency generator to produce a driving force of the form $F(t) = F_o \cos(\omega t)$. Notice that ω is the driving frequency; a parameter that we typically control. The late time, 'particular', or steady state solution to this differential equation is given by

$$x_p(t) = A \cos(\omega t - \delta), \quad (2)$$

with

$$A = \frac{F_o/m}{\sqrt{(\omega_o^2 - \omega^2)^2 + 4\beta^2\omega^2}}, \quad (3)$$

and

$$\delta = \arctan\left(\frac{2\beta\omega}{(\omega_o^2 - \omega^2)}\right). \quad (4)$$

This lab focuses on equation (3), the relationship between the amplitude of the late time solution and the driving angular frequency. In class and the reading you saw the effects of resonance and the curves of amplitude A and total energy as a function of frequency.

Q in a resonant system describes mechanical amplification: the ratio of the resonance amplitude to the amplitude of the end of the spring vibrated by the driver is Q . In this lab you will determine Q two ways, one from the damped motion in Part I and the second from the resonance curve in Part II.

Part I: Building your system and determining its properties

With help of you instructor if needed, build a damped oscillator with the spring, metal damping rod, mass hanger and mass. You should have a total mass between 40 g and 100 g.

Recall that the solution to the equation of motion including drag is given by

$$x(t) = x_m e^{-\beta t} \cos(\omega_d t + \varphi)$$

and the effect of drag is to shift the frequency of oscillation to

$$\omega_d = \sqrt{\omega_o^2 - \beta^2} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$

(1) Start with the neodymium magnet and support stand *far* from the aluminum rod. In your lab group come up with a procedure to measure the natural angular frequency, ω_0 , of the oscillator consisting of the spring and aluminum rod with a mass hanger. Although the mass moves through air and thus has some damping, we consider this arrangement to be our “un-damped” oscillator.

(2) Move the magnet and support stand so the magnet is 5 - 12 mm from the aluminum rod.¹ When the oscillator is at rest, the magnet should be next to the middle of the aluminum rod. Now determine ω_d and β with their uncertainties.

(3) Compute ω_d from your data for ω_0 and β .

(4) What is the relative shift in angular frequency compared to the natural angular frequency ω_0 ? i.e. what is $(\omega_0 - \omega_d) / \omega_0$?

(5) Determine Q from your data. It is approximately

$$Q \approx \frac{\omega_0}{2\beta}$$

for light damping. If it is not in the range 10-20, ask your instructor to help adjust the magnet to increase or decrease damping. Once you have a system with Q in the right range, compute it with uncertainty.

Part II: Resonance

¹ If the spacing to the magnetic damper is too large, damping will be so small as to make it difficult to obtain a resonance curve in Part II. Thus, the resonance will be very narrow and hard to scan given the resolution of the frequency generator.

In this part of the lab you take frequency and amplitude data so you can create your very own resonance curve for the mechanical system in front of you!

Here's how:

(1) Turn on the frequency generator, power amplifier (switch is on the back), and the oscilloscope used to measure the frequency of the generator. The generator has two controls of interest for us: the amplitude and the frequency. The generator's amplitude controls the amplitude of the mechanical driver. With the generator's amplitude turned down, set its natural *frequency*. Adjust the amplitude so that the function generator does not exceed 10 V peak to peak. (You can read this amplitude from the oscilloscope.) We want to insure that the speaker is not overdriven. The amplifier will show a red light if this happens. If you see the red light on then turn down the amplitude.

(2) Measure the frequency on the oscilloscope and enter it in a table of frequencies and amplitudes. Start plotting your A^2 vs. f data right away so that you will be able to see where you need more data points.

(3) Set LoggerPro to record about 100 seconds of data. Start the oscillator from its equilibrium position and start recording data. Sit back and enjoy the (early time solution) show! Towards the end of the 100 seconds the system should settle down to oscillations with constant amplitude. Each time the frequency is changed you must wait until the system settles down—we say the transient has died away and we see only the late time solution of equation (2). Wait to check that you have the late time solution before you take oscillator amplitude and frequency data for your table.

(4) Set LoggerPro to take 10 seconds of data and find the amplitude and frequency of oscillation using Logger Pro to curve fit. This is your first amplitude vs. frequency data point!

(5) Explore the amplitude squared vs. frequency space by incrementing the frequency and repeating steps 2 and 4. In the end you should have about 10 points, centered on the resonance frequency, showing how amplitude depends on the driving frequency. Choose your frequencies appropriately. Make sure that you follow the curve away from resonance, above and below, so that you encompass the amplitude dropping to $\frac{1}{2}$ the maximum amplitude on each side.

(9) The quality factor, Q , is a measure of the damping in terms of the natural angular frequency. The Q of a resonant system can also be found from the resonance curve you created in step (5) above: the higher the Q the sharper the curve.

Your instructor can show you how to use LoggerPro to fit a resonance curve to your data,

$$\text{Amplitude}^2 = \frac{B}{(C - f^2)^2 + Df^2},$$

where f is the frequency, and B , C and D are fitting constants. Using the cursor on the resonance curve provided by the fit, estimate the width Δf of the curve at *one-half* the maximum height. This will give you the “ Q ” of the mass-spring system with the relation

$$Q \cong \frac{f_R}{\Delta f}.$$

This is an important way of characterizing resonant systems. Using uncertainties compare this result² for Q with what you obtained in Part I (5). Do they agree?

² Make sure the magnet-aluminum rod spacing doesn't change between part I and part II. If it does change the comparison of these two methods of finding Q loses its significance.