RESONANCE BEHAVIOR OF A DRIVEN DAMPED OSCILLATOR

Objective: • Study resonance in a simple mechanical system.
• Find the resonance curve, Q, and the damping coefficient from data.

Prelab:
There are two ways to estimate Q-factor described in this lab.

a) If the natural frequency $\omega_0 = 10.334 \pm 0.002$ radians/second, and the damping constant $\beta = (0.331 \pm 0.005)$ /second, then what is Q in standard form?
b) If the peak frequency $f_R = 1.5848 \pm 0.0012$ Hz and the frequency width $\Delta f = 0.1009 \pm 0.0005$ Hz, then what is Q in standard form?
c) Did these two estimates of Q agree or not? How can you tell?

Apparatus: Sonic ranger, LoggerPro, spring, mass hanger, apparatus for magnetic damping, mass selection, scale, signal generator, oscilloscope, amplifier, and mechanical driver.

Introduction:
This week explore the motion of a damped oscillator and the phenomenon of resonance in a driven, damped oscillating system.

When a driving force is added to this physical system, the equation of motion can be written as

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_o^2 x = \frac{F(t)}{m} \tag{1}$$

where β is the damping constant, m is the mass, t is time, and x is position.
We use a modified loudspeaker connected to a frequency generator to produce a driving force of the form $F(t) = F_o\cos(\omega t)$. Notice that ω is the driving frequency; a parameter that we typically control. The late time, `particular', or steady state solution to this differential equation is given by

$$x_p(t) = A\cos(\omega t - \delta)\,, \tag{2}$$

with

$$A = \frac{F_o/m}{\sqrt{(\omega_o^2 - \omega^2)^2 + 4\beta^2\omega^2}}\,, \tag{3}$$

and

$$\delta = \arctan\left(\frac{2\beta\omega}{(\omega_o^2 - \omega^2)}\right). \tag{4}$$

This lab focuses on equation (3), the relationship between the amplitude of the late time solution and the driving angular frequency. In class and the reading you saw the effects of resonance and the curves of amplitude *A* and total energy as a function of frequency.

Q in a resonant system describes mechanical amplification: the ratio of the resonance amplitude to the amplitude of the end of the spring vibrated by the driver is *Q*. In this lab you will determine *Q* two ways, one from the damped motion in Part I and the second from the resonance curve in Part II.

Part I: Building your system and determining its properties

Recall that the solution to the equation of motion including drag is given by

$$x(t) = x_m e^{-\beta t}\cos(\omega_d t + \varphi) \tag{5}$$

and the effect of drag is to shift the frequency of oscillation to

$$\omega_d = \sqrt{\omega_o^2 - \beta^2} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\ . \tag{6}$$

(1) Start with the neodymium magnet and support stand *far* from the aluminum rod. Using the logger pro file "Motion Tracker.cmbl" find a way to measure the natural angular frequency, ω_o,of the oscillator. Although the mass moves through air and thus has some damping, we consider this arrangement to be our "un-damped" oscillator.

(2) Move the magnet and support stand so the magnet is 2 – 5 mm from the aluminum rod. If the space between the magnet and the rod is too large, damping will be so small as to make it difficult to obtain a resonance curve in Part II. Start the system oscillating, collect data in logger pro. Using an applicable fit, determine ω_d and β with their uncertainties.

(3) Compute ω_d in standard form from your data for ω_o and β. How different is the calculated ω_d from your logger pro measured ω_d? Is there evidence of a systematic error?

(4) Determine *Q* in standard form from your data. It is approximately

$$Q \approx \frac{\omega_0}{2\beta} \tag{7}$$

for light damping. If it is not in the range 3-20, ask your instructor to help adjust the magnet to increase or decrease damping. Once you have a system with *Q* in the right range, compute it with uncertainty. Once you have your magnet properly placed, keep it in the same place for part 2. If the magnet location changes, your ω_d will change, which means you will have to start back at part I-2.

Part II: Resonance

In this part of the lab you take frequency and amplitude data so you can create your very own resonance curve for the mechanical system in front of you!

Here's how:

(1) Turn on the frequency generator, power amplifier (switch is on the back), and the oscilloscope. Set the generators frequency to the oscillator's natural *frequency*.

(2) Measure the frequency on the oscilloscope and enter it in a table of frequencies and amplitudes. Start plotting your A^2 vs. f data right away so that you will be able to see where you need more data points.

(3) Each time the frequency is changed you must wait until the system settles down to measure the new amplitude—we say the transient has died away and we see only the late time solution of equation (2). Wait to check that you have the late time solution before you take oscillator amplitude and frequency data for your table.

(4) Set LoggerPro to take 10 seconds of data and find the amplitude and frequency of oscillation using Logger Pro to curve fit. This is your first amplitude vs. frequency data point!

(5) Explore the amplitude squared vs. frequency space by incrementing the frequency and repeating steps 2 and 4. In the end you should have about 10 points, centered on the resonance frequency, showing how amplitude depends on the driving frequency. Choose your frequencies appropriately. Make sure that you follow the curve away from resonance, above and below, to fill out both sides of your resonance curve.

(6) The quality factor, Q, is a measure of the damping in terms of the natural angular frequency. The Q of a resonant system can also be found from the resonance curve you created in step (5) above: the higher the Q the sharper the curve.

Open the Resonance Curve Fit.cmbl file on the desktop, hit "Use file as is". Now you can paste your frequency and amplitude squared data into the table. Select "curve fit" from the *analyze* drop down menu. At the bottom of the curve fit options, you will see a Resonance Curve fit. It will be fitting the following function to your data:

$$Amplitude^2 = \frac{B}{(C - f^2)^2 + Df^2}\ , \tag{8}$$

where f is the frequency, and B, C and D are fitting constants. Select it and press "try fit" to see how well it draws a curve over your data. You may have to go back and take more data to fill in the area around the peak, or not include some data point to get logger pro to successfully fit the data. Using the cursor on the resonance curve provided by the fit, estimate the width Δf of the curve at *one-half* the maximum height and the uncertainty. This will give you the "Q" of the mass-spring system with the relation

$$Q \cong \frac{f_R}{\Delta f}. \tag{9}$$

This is an important way of characterizing resonant systems. Using uncertainties compare this result for Q with what you obtained in Part I (5). Do they agree? Is there evidence of a systematic error?