Error Analysis in a Nutshell

Uncertainty in a single parameter (Taylor 2.1-2.3, 2.8)

For a measured value:

Estimate the uncertainty as best you can. Note that the uncertainty is usually larger than your ability to read the scale.

Sig Figs! One digit uncertainty. Quote value to same precision as σ.

For a slope:

use the LINEST function in Excel

Select a two-by-two grid

=LINEST(y values, x-values, true, true)

Control-shift-enter (windows) or command-enter (mac)

Slope	y-intercept
Slope uncertainty	y-intercept uncertainty

Uncertainty in repeated measurements

Standard Deviation: (Taylor 4.2-4.3)

The standard deviation gives the expected uncertainty for a single measurement. It is derived from the scatter in the values from many repeated measurements. The Excel function for a standard deviation is STDEV().

Standard Error: (Taylor 4.4)

The standard error is the uncertainty in the average of repeated measurements. You would expect the average of N measurements to be more accurate than a single measurement, but for a random process, 2 measurements is not twice as good as 1 measurement. Repeating a measurement N times reduces your uncertainty by a factor of $\frac{1}{\sqrt{N}}$, so

$$Standard\ Error = \frac{1}{\sqrt{N}} Standard\ Deviation$$

Systematic vs. Statistical (Random) Errors: (Taylor 4.1, 4.6)

Statistical errors, caused by random fluctuations, can be reduced by repeating a measurement so the random fluctuations average to zero. Statistical techniques (Std. Deviation, Std. Error, Error Propagation, etc.) apply to statistical errors. Systematic errors are reproducible so they do not average to zero. Systematic errors are difficult to deal with because they can't be improved by repeated measurements and each case must be handled differently.

Error Propagation: (Taylor 3.5-3.8)

Errors can be propagated through any equation using only three rules.

- Sum/Difference Rule: For c = a+b or a-b, $\sigma_c = \sqrt{\sigma_a^2 + \sigma_b^2}$
- Product/Quotient Rule: For c = ab or a/b, $\frac{\sigma_c}{c} = \sqrt{\left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2}$
- Function of One Variable: For c = f(a), $\sigma_c = |f'(a)|\, \sigma_a$

For complex equations, these rules must be applied in steps.

For c = (a+b)*c, find the uncertainty in the quantity (a+b) first, then use the product rule to combine (a+b) $\pm\sigma_{(a+b)}$ times c $\pm\sigma_c$.

LINEST hints

Enter your numbers for x and y values in 2 columns
Select a block of blank cells 2 columns wide by 5 rows high
Type =linest(your y-values,your x-values,,true), Note: no spaces!
To enter "your y-values", click on the box with the first value and drag to the last value.
Repeat this for "your x-values".

Then type control-shift-enter on windows OR command-enter on mac.

You should end up with a 2 by 2 table of numbers as shown below.

slope	**y-intercept**
uncertainty in slope	**uncertainty in y-intercept**

Uncertainty Review Guide

Taylor, "An Introduction to Error Analysis."

These topics will be covered at some point during the semester.

Significant Figures:		
Basic Sig Figs:	Sect 2.1-2.3	
Uncertainty more accurate than sig figs:	Sect 2.8	
Propagation of errors:		
Add/Subtract:	Sect. 3.5	Eqn. 3.16
Multiply/Divide:	Sect. 3.6	Eqn. 3.18
Function of 1 Variable:	Sect. 3.7	Eqn. 3.23
"Chain Rule", Dominant Errors:	Sect. 3.8	
Standard Deviations:		
Counting Statistics:	Sect. 3.2	Eqn. 3.2
Standard Deviation:	Sect. 4.2, 4.3	use calculator
Standard Error (Stand. Dev. of the Mean)	Sect. 4.4	Eqn. 4.14
Systematic Errors:	Sect. 4.1, 4.6	

Advanced Topics (Look up when necessary)	
Rejecting Data:	Ch. 6
Weighted Average:	Ch. 7