SOLUTIONS FOR DRIVEN, DAMPED OSCILLATORS

THE EQ'N OF MOTION IS

$$\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m}\cos(\omega t) \qquad (1)$$

LET'S LOOK FOR A LATE TIME (OR 'PARTICULAR') SOL'N: FIRST, TRY $x = A\cos(\omega t - \delta)$ AND SEE WHAT HAPPENS.

$$\frac{dx}{dt} = -\omega A\sin(\omega t - \delta)$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

EOM, EQ'N (1), GIVES THEN

$$-\omega^2 x + 2\alpha(-\omega A\sin(\omega t - \delta)) + \omega_0^2 x = \frac{F_0}{m}\cos(\omega t)$$

$$\Rightarrow (\omega_0^2 - \omega^2)A\cos(\omega t - \delta) - 2\alpha\omega A\sin(\omega t - \delta) = \frac{F_0}{m}\cos(\omega t)$$

TRIG IDENTITIES! TO MATCH UP LIKE WIGGLES

$$(\omega_0^2 - \omega^2)[A(\cos(\omega t)\cos\delta + \sin(\omega t)\sin(\delta))] - 2\alpha\omega A(\sin\omega t\cos\delta - \cos\omega t\sin\delta)$$
$$= \frac{F_0}{m}\cos(\omega t)$$

GATHERING TERMS IN $\cos\omega t$ AND $\sin\omega t$,

$$(\omega_0^2 - \omega^2)\cos\omega t\cos\delta + 2\alpha\omega\cos\omega t\sin\delta + (\omega_0^2 - \omega^2)\sin\omega t\sin\delta$$
$$- 2\alpha\omega\sin\omega t\cos\delta = \frac{F_0}{Am}\cos\omega t$$

NOW THE $\cos\omega t$ COEFFICIENTS AND $\sin\omega t$ COEFFICIENTS