Waves and Fields (PHYS 195): Example Final

Spring 2007

This is a 3 hour, closed-book exam held under the auspices of the Hamilton Honor Code. Useful information are included on the Handy Relations page. Don't forget to check your **significant figures**.

Problems:

(1) Describe the purpose of uncertainty.

(2) Derive **one** of the following:
- The equation of motion for a pendulum oscillating around equilibrium. Start with a free body diagram and $\mathbf{F} = m\mathbf{a}$.
- The wave equation for waves on a string. Assume the string is under tension F_T and start with a force diagram for a bit of string.
- The wave equation for light from the two field dynamics equations discussed in class,

$$\frac{\partial B_z}{\partial x} = -\mu_o \epsilon_o \frac{\partial E_y}{\partial t}$$

  and

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

- The simple harmonic equation for a rubber ducky (with a constant cross section) floating in water.
- The equation of motion for a mass on a spring hanging in a gravitational field. Start with a free body diagram and $\mathbf{F} = m\mathbf{a}$.

(3) Derive **one** of the following:
- The locations of the maxima for the double slit experiment, $d \sin\theta = m\lambda$. Start with a diagram and include the necessary conditions for this equation to hold.
- Sketch the derivation of Snell's relation $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Start with the boundary conditions for fields at the surface of a material.
- The locations of the dark bands for single slit diffraction $a \sin(\theta) = m\lambda$
- The conditions for constructive interference for light incident on a thin film.

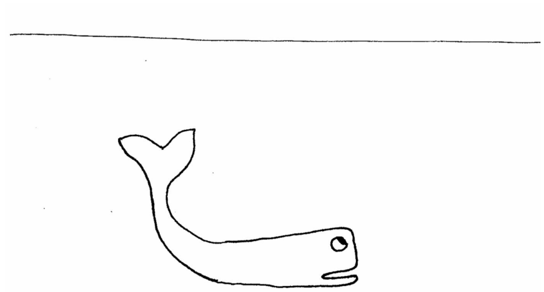
(4) Write down the wave equation, and the simple harmonic equation of motion.

(5) What length should you make an organ pipe if it is to produce a low C (262 Hz)? Assume the pipe has one open end and that $v_{sound} = 343$ m/s.

(6) Light of wavelenth 680 nm shines on two slits and produces an interference pattern on a screen 2.0 m away. If the 4th order maxima is 38 mm away from the central maxima, what is the separation of the two slits?

(7) In lecture we had a pendulum in a parallel plate capacitor. Once the plates of the capacitor were charged, the conducting ping pong ball bounced between the plates.
  (a) If the ball just bounced off the positive plate, what force does it experience?
  (b) If the ball just bounced off the negative plate, what force does it experience?
  (c) How would you describe the equation of motion?
  (d) Why is this not simple harmonic motion?

(8) On a perfectly calm day, a whale glances up and views the world above the ocean surface. In what range of angles is this view confined? Assume $n = 1.33$.



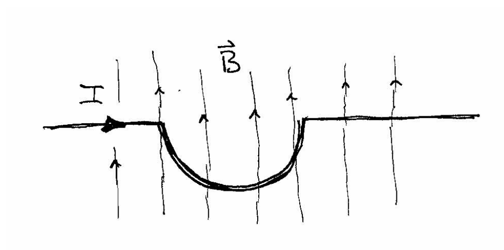
(9) A well-known manufacturer of water bottles "Tough-gene" claims that the bottles can withstand a acceleration of 70 times the magnitude of the local acceleration of gravity ($g = 9.8$ m s^{-2}) without leaking. You decide to test this by subjecting the bottle to simple harmonic motion with an amplitude of 0.10 m. What oscillation frequency will subject the bottle to a 70.0g acceleration? Where in the oscillation does this maximum acceleration occur?

(10) In the last problem if the uncertainty in the amplitude is ± 0.05m, what is the uncertainty in the frequency? Assume this is the dominant uncertainty.

(11) A police car pursues a speeding auto. Both cars are traveling 150.0 km/hr = 41.67 m/s. The siren on the police car emits a frequency of 2.000×10^3 Hz. What is the frequency heard by: (a) the driver of the speeding car? (b) by a possum watching the cars approach? Assume the speed of sound is 343 m/s.

(12) You and a friend each have mass 60.0 kg and are holding 1.0 C of charge. At what distance from each other is the magnitude of the electrical force equal to your weight on the surface of Earth?

(13) A wire with a semicircular section is in a magnetic field. If the wire is free to rotate around the two straight segments show which way the loop will move when the current is switched on as shown.



(14) Use the Taylor expansion to show that, around $\theta = 0$, $\sin\theta \approx \theta$.

(15) MC Escher drew a reflecting ball, perhaps you are familiar with it,

Suppose the ball has a diameter of 14 cm and that Escher holds the ball 40 cm away from is face. (Recall that the focal length is half the radius.) Using a rays and algebra show that the image is upright. What is the magnification? Is the image virtual or real?

(16) *Did Escher make a mistake??* In the last problem Escher's hand is holding the ball. Should the image of his hand be inverted? Defend your answer using rays and/or algebra.

(17) Charge separation usually exists across the boundary membrane of nerve cells. Outside there is a low concentration of potassium ions (K^+) while the inside has a high concentration of sodium ions (Na^+). The typical potential difference is 90 mV across an insulating boundary of 5 nm. What is the magnitude of the electric field, assuming it is constant across the boundary (like a parallel plate capacitor)?

(18) As we saw in class, when two flat pieces of glass are separated at so they form a thin wedge, interference occurs. Suppose the plates are separated by a piece of paper on one end. When illuminated with 500 nm light you see 42 dark bands. How thick is the paper? Assume that you look directly along the normal.

Handy Relations

General:

$$\sum \tau = I\alpha$$
$$F_x = -\frac{\partial U}{\partial x}$$
$$\Delta P = \frac{F}{A} = -B\frac{\Delta V}{V}$$
$$F_B = \rho g V$$

The Taylor series of a function $f(x)$ around $x = 0$ is

$$f(x) = f(0) + \left.\frac{df}{dx}\right|_{x=0} x + \frac{1}{2}\left.\frac{d^2 f}{dx^2}\right|_{x=0} x^2 + \frac{1}{6}\left.\frac{d^3 f}{dx^3}\right|_{x=0} x^3 + \ldots$$

Oscillations:

$$F = -kx$$

For spring-like SHM $\omega_o = \sqrt{\frac{k}{m}}$.

$$T = 2\pi/\omega$$
$$x(t) = x_o \sin(\omega t + \phi)$$
$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$
$$x(t) = x_o e^{-bt/2m}\sin(\omega t + \phi)$$
$$x(t) = A\sin(\omega t + \phi)$$
$$A = \frac{F_o}{m\sqrt{(\omega^2 - \omega_o^2)^2 + b^2\omega^2/m^2}}$$
$$\phi = \arctan\frac{\omega_o^2 - \omega^2}{\omega(b/m)}$$
$$Q = m\omega/b$$
$$E = \frac{1}{2}m\omega^2 A^2$$

Waves:

$$v = \lambda f$$

For waves on a string

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$P = 2\pi^2 \mu v f^2 A_o^2$$

Standing wave condition for waves fixed at both ends

$$L = \frac{n\lambda}{2}, n = 1, 2, 3, \ldots$$

$$D(x,t) = D_o \sin(kx - \omega t)$$
$$k = \frac{2\pi}{\lambda}, v = \frac{\omega}{k}$$

Standing wave condition for waves fixed at both ends

$$L = \frac{n\lambda}{2}, n = 1, 2, 3, \ldots$$

$$f' = f\left(\frac{v \pm v_O}{v \mp v_S}\right)$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$M = \frac{h_i}{h_o}$$

angular magnification $m = \theta_i/\theta_o$

$$\beta = 10 \log \frac{I}{I_o}$$

Fields:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mu_o = 4\pi \times 10^{-7}\,\mathrm{Tm/A},\ \epsilon_o = 8.85 \times 10^{-12}\,\mathrm{C^2/Nm^2}$$

$$Q = CV,\ V = IR,\ P = IV,\ C = \frac{\epsilon_o A}{d}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}\hat{r}$$

$$V = \frac{1}{4\pi\epsilon_o}\frac{Q}{r}$$

$$V_{tot} = V_1 + V_2 + ...$$

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\ell, \text{ and } E_x = -\frac{\partial V}{\partial x}$$

$$B = \frac{\mu_o I}{2\pi r}$$

$$\tau = \mu \times B\,,\ \mu = IA$$

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

$$\mathbf{F} = I\ell \times \mathbf{B}$$

$$c = \omega/k = 1/\sqrt{\epsilon_o \mu_o}$$