Waves and Fields (PHYS 195): EXAMPLE Quiz I

Spring 2009 v1.0

This is a closed book exam held under the auspices of the Hamilton Honor Code. Useful bits of information are included in the problems and on the Handy Relations page. Do not hesitate to ask questions!

Problems:

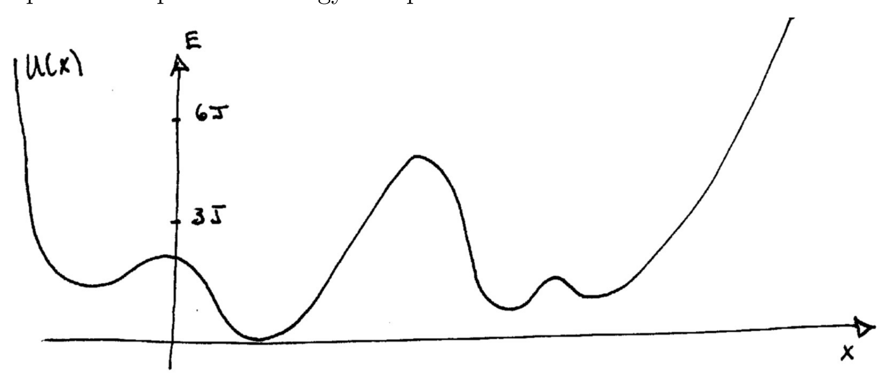
(1) Before you in the room is an inertial balance, used to determine mass without weighing anything.
   (a) Without any additional mass in the plate, characterise the oscillator by finding its natural angular frequency.
   (b) Finish the characterization by finding Q.
   (c) Suppose that the oscillator is driven, sketch the resulting resonance curve, i.e. the amplutude versus driving frequency. Use part (b.) and (a.).

(2) Your classmate Heather writes from the Hamilton Antarctica trip, "Hi Seth, Well, I am literally surrounded by physics!!! Crossing the Drake Passage, we had about 55 foot waves ..."
   (a) Suppose these waves were sinusoidal and that they passed every 25.0 ± 0.2 seconds. How much energy was stored in the vertical oscillations of the ship? Assume the ship has a mass of $3.841 \pm 0.001 \times 10^6$ kg.
   (b) Where does this energy come from?

(3) A mass $m = 40.0$ g hangs from a $k = 3.5$ N/m spring. Neglect damping and assume you have 2 significant figures throughout this problem.
   (a) What is the angular frequency of oscillation?
   (b) If the amplitude of oscillation is $A = 12$ cm then how much energy is stored in the system?
   (c) When $t = 0$ the mass is at equilibrium and is moving upward. Find the solution which describes the displacement.

(4) Using a simple pendulum you find the local acceleration of gravity. After careful data taking you find that the period is $T = 2.027 \pm 0.002$ s and the length of the pendulum is $\ell = 102.2 \pm 0.1$ cm. What is your result for g? Does it agree with $g = 9.810$ m/s^2?

(5) Here is a plot of the potential energy of a particle.



   (a) Circle the stable equilibrium points.
   (b) If a particle has 3 J of energy sketch how the motion would appear on this plot. Identify the turning points.
   (c) In this case, is the motion simple harmonic? Explain your answer.

(6) In lab you had a damped, mass on a spring system. Suppose it had an initial amplitude of 15 cm. After 8.0 s the amplitude reduces to 5.5 cm. The displacement from equilibrium can be expressed as

$$x(t) = x_o e^{-\alpha t} \sin(\omega t).$$

Find α and the oscillator's Q. Assume $k = 3.3$ N/m and m is exactly 50 g.

(7) **Reducing resonance!** The steady state solution for driven, damped oscillation can be written as

$$x_S(t) = A_S \cos(\omega t + \phi_S)$$

where ω is the driving frequency. The amplitude A_S is a function of the driving frequency

$$A_S = \frac{F_o}{m\sqrt{\left(\omega^2 - \omega_o^2\right)^2 + b^2\omega^2/m^2}}$$

At LIGO, the huge gravitational wave detectors in Washington and Louisiana, folks are trying to measure tiny deflections in hanging mirrors. These mirror systems are essentially pendula. Noise is a constant problem. To avoid spurious signals they require vibrational isolation (so that $x_S(t) \approx 0$). They have little control over ω and F_o. When designing the suspention systems what quantities should be made large to reduce the unwanted oscillations?

(8) A traveling wave has the form

$$D(x,t) = \sin(2\pi x - \pi t).$$

Assume everything is in SI units.

(a) Find the wavelength, frequency and speed of the wave.
(b) Sketch $D(x,t)$ when $t = 0$.
(c) Sketch $D(x,t)$ at $x = 1$.
(d) Label your sketches with wavelength or period as appropriate.

(9) A string fixed at both ends has a tension $F_T = 3$ N, a length $L = 2$ m, and a linear mass density of $\mu = 1.2 \times 10^{-3}$ kg/m. It is being driven with a speaker as you saw in lab.

(a) Find the phase velocity.
(b) Sketch the first two harmonics.
(c) What are the frequencies of the harmonic modes?

Handy Relations General:

$$\sum \tau = I\alpha$$
$$F = -\frac{dU}{dx}$$
$$\Delta P = \frac{F}{A} = -B\frac{\Delta V}{V}$$
$$F_B = \rho g V$$

The Taylor series of a function $f(x)$ around $x = 0$ is

$$f(x) = f(0) + \left.\frac{df}{dx}\right|_{x=0} x + \frac{1}{2}\left.\frac{d^2 f}{dx^2}\right|_{x=0} x^2 + \frac{1}{6}\left.\frac{d^3 f}{dx^3}\right|_{x=0} x^3 + \dots$$

Oscillations:

$$F = -kx$$

For spring-like SHM $\omega_o = \sqrt{\frac{k}{m}}$. For a simple pendulum $\omega_o = \sqrt{\frac{g}{\ell}}$. $T = 2\pi/\omega$ $\omega = 2\pi f$

$$Q = m\omega_o/b$$
$$E = \frac{1}{2}m\omega_o^2 A^2$$

For a driven system at late times $x(t) = A(\omega)\cos[\omega t - \delta(\omega)]$ with

$$A(\omega) = \frac{F_o/m}{[(\omega_o^2 - \omega^2)^2 + 4\alpha^2\omega^2]^{1/2}} \text{ and } \delta(\omega) = \arctan\left(\frac{2\alpha\omega}{\omega_o^2 - \omega^2}\right)$$

where $\alpha = b/2m$.

Waves:

For waves on a string

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$y(x,t) = y_m \sin(kx \pm \omega t)$$
$$k = \frac{2\pi}{\lambda}, v = \frac{\omega}{k}$$

Standing wave condition for waves fixed at both ends

$$L = \frac{n\lambda}{2}, n = 1, 2, 3, \dots$$
$$v = \lambda f$$
$$P = 2\pi^2 \mu v f^2 A_o^2$$

Uncertainty:

For a calculated quantity $q = q(x, ..., z)$ then

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + ... + \left(\frac{\partial q}{\partial x}\delta z\right)^2}$$