Waves and Fields (PHYS 195): Week 1

Spring 2021 v1.0

Intro:

Our course starts with an in-depth study of oscillations: "motion that returns to the same place at the same momentum every period". The tools are familiar from last semester - Newtonian mechanics - although the emphasis will be different. In addition to the physics of oscillations (with added features damping and driving forces), we'll also study the mathematics of $\mathbf{F} = m\mathbf{a}$ *- the idea that this equation is a "differential equation".*

I highly recommend that you read the "Reading" before lecture. Although you need not read the relevant sections in detail (that can wait until later), to "view" the new material before it is discussed will be a great help to you. Speaking for myself, I wouldn't expect to fully understand the lectures without doing this reading in advance. I will try to keep the reading for lecture up to date, but I do find that the schedule slips a bit, especially when predicting further into the future. Please don't hesitate to ask for the current reading.

Reading:

- Taylor Chapters 1, 2 (sections 1-6), and 4. Much of this is a review of uncertainties discussed in lab last fall. Where the material is new read with care. Taylor is a great book and you can get a lot from it. **We will not be using the "Provisional Rules" discussed in Chapter 2. Instead we will follow what was done in Phys 190 and use quadrature.**
- During the week:
  - Monday (Feb 1) : HRW 15.1 - 15.2 and the first part of 10.1 in Kleppner and Kolenkow
  - Wednesday : HRW 15.2 and 10.1 in K&K
  - Friday : HRW 15.3 - 15.5
  - Monday (Feb 8): the Simmons e-Reserve reading and K&K "Small Oscillations of Bound Systems" on e-Reserves

Physics Topics:

- Simple harmonic oscillation: springs, pendula, torsion, pendula...
- Energy in SHM
- k_{eff}

Math Topics:

- Calculus! Derivatives, Chain rule, Derivatives of Trig functions
- $\mathbf{F} = m\mathbf{a}$ as a differential equation

Problems: (Due on Tuesday February 9 at 11:59 PM on gradescope - code ZR34XK)

(1) Taylor Problem 2.3. It's on page 36. I assign it just to be sure we're all on the same page about standard form of results with uncertainties.

(2) Consider the position function or displacement

$$x(t) = (13\ \text{cm}) \cos(4\pi t - \pi/2)$$

(a) Find the velocity dx/dt.
(b) Find the acceleration.
(c) On the same plot, please sketch $x(t), v(t)$, and $a(t)$.

Assume that the time is in units of seconds. Feel free to check your results with Wolfram Alpha.

(3) For simple harmonic oscillators when (if ever) are the displacement ("$x(t)$" with the origin at equilibrium) and velocity vectors in the same direction? When are the displacement and acceleration vectors in the same direction?

(4) HRW Problem 15.12

(5) A 81 g nuthatch lands on a 890 g bird feeder that is hung from a spring. When the nuthatch lands the feeder lowers by 0.75 cm. What is the natural angular frequency of oscillation once the nuthatch takes off?

(6) In class I mentioned that the vertical mass-on-a-spring has the same equation of motion as the horizontal one we discussed in class. In this problem you demonstrate how this comes about.
  (a) Sketch a vertically hanging spring and set your vertical or "y" coordinate system's origin.
  (b) Now imagine adding a mass to the spring. Sketch the mass in equilibrium and call that position y_{equi}. Using the variables in the diagram, make a free body diagram and write down the equilibrium condition, $F_{total} = 0$.
  (c) Obtain an expression for y_{equi}, the vertical position of the center of the mass in equilibrium.
  (d) Now let's move the mass! Make another sketch of the mass when it is pulled *down a distance y from equilibrium* and (just) released.
  (e) Make a free body diagram for this position.
  (f) From the diagram, and the coordinates in the diagram, obtain an equation of motion.
  (g) Show that, indeed, the equation of motion is for **simple harmonic motion**,

$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0.$$

(7) Use Wolfram Alpha or Mathematica to find

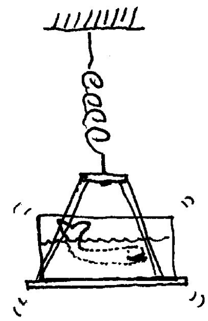
$$\frac{d}{dt}\sin(\omega t) \text{ and } \frac{d^2}{dt^2}\cos(\omega t).$$

If in doubt, check the results using another method. State which method(s) you used.

(8) HRW 15.24

(9) A distant star orbits a compact object with a period of 10.1 years.
  (a) Assuming the star is in a circular orbit with radius $r = 3.65 \times 10^{12}$ m, find its angular angular frequency and speed.
  (b) Using Newton's law of gravitational force find the mass of the compact object. Express your answer in terms of numbers of solar masses.

(10) A baby whale (comfortable in a tank) oscillates from a vertically hanging light spring once every 0.55 s. Assume that the tank, water, whale and all have a mass of 450 kg.



(a) Write down the equation giving the vertical position as a function of time (with + upward), assuming the whale started from rest after being lifted 10 cm from equilibrium ($y = 0$).
(b) How long will it take to reach equilibrium for the first time?
(c) What will be the whale's maximum speed?
(d) What will be the maximum acceleration enjoyed by the whale? Where will this occur?

Lab:

Labs start next week on Tuesday February 9. We will be working with a not-so-simple pemndulum.

A look ahead. . .

Next week we add some spice and complication to SHM by adding damping and driving forces. For a preview see HRW 15.8 and 15.9. The material is given a full treatment in KK 10.3.