Intro:
After discussing what happens to charges and magnetic moments in a magnetic field, we move into geometric optics, perhaps familiar from previous courses, including ray tracing, mirrors, lenses, rainbows and other optical phenomenon.

Reading:
• Friday: HRW 28.1 - 5 and on magnetic materials 32.4-5, 32.8
• Monday: HRW 28.7 - 8 on loops of current
• Wednesday: HRW 33.1 (except the derivation using Maxwell's equations) and 33.2.

Physics Topics:
• Motion of magnetic moment in a magnetic field
• Light as a wave
• Optics - overview and geometric

Math Topics:
• More cross products using unit vectors

Problems: Due at the beginning of class on Wednesday April 18
(1) Red paint used in painting murals often contains iron oxide, which is magnetic. When the paint is wet the oxide can move in the fluid, like the iron filings or grains of rice I have used in the class demos to show fields. When the paint dries the iron oxide is no longer able to move. Explain how this observation can be used in archaeology to date cave paintings or ancient murals. This technique doesn’t work on paintings that can be moved.

(2) Using the Biot-Savart law find the magnetic field in the center of a circular loop or radius $R$.

(3) HRW 29.8

(4) A mass spectrometer is a apparatus that separates ionized atoms with different charge to mass ratios ($q/m$) using a magnetic field. A beam of ionized atoms of carbon, each with charge $+e$, enters a magnetic field perpendicular to the beam. All the ions in the beam have the same speed (arranged with the velocity selector discussed in class and in problem 8). The ions accumulate 5.00 cm apart. The more abundant $^{12}_6$C isotope (which has atomic mass number 12) traces a path of smaller radius, 15.0 cm. What is the atomic mass number of the other isotope? Hint: Use $qv \times B$ to make a ratio containing the mass number.
(5) **POSTPONED TO NEXT WEEK:** A rectangular current loop is in a *uniform* magnetic field pointed along the positive $y$-axis as shown.

![Diagram of a rectangular current loop in a uniform magnetic field]

(a) What is the total force on the loop? Please do this by finding the $I\ell \times \mathbf{B}$ force for each side.
(b) Find the total torque on the current loop.
(c) If the magnetic field is 2.0 gauss (1 gauss is $10^{-4}$ T), $a = 2.1$ cm, $b = 3.5$ cm, and $I = 1.2$ A, then what is the torque on the current loop?

(6) **POSTPONED TO NEXT WEEK:** Let’s find the force and torque on a square current loop with side length $a = 1.2$ cm in an *inhomogeneous* (or, equivalently, non-uniform) $\mathbf{B}$-field (a field that has different magnitudes and directions in different locations). We will orient the coordinates so that the magnetic field points along the positive $z$-axis in the middle of the loop and so that the $x$ axis points to the right. The current is $I = 15$ A.

Assume that

\[
\mathbf{B} = (0.0045 \text{ T}) \frac{\hat{i}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}} \text{ on the left side of the loop}
\]

and

\[
\mathbf{B} = (0.0055 \text{ T}) \frac{\hat{i}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}} \text{ on the right side.}
\]

The unit vector notation says “The magnetic field is pointing $45^\circ$ above the horizontal”.

(a) Find the force on the left side of the current loop.
(b) Find the force on the right side of the current loop.
(c) Discuss, but do not calculate, the forces on the other two sides.
(d) What is the net force on the loop?
(e) What is the torque on the loop if

\[ \vec{\mu} = I \vec{A} = I a^2 \left( \cos \frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{k} \right) \]

Use the average of the magnetic fields on the left and right for “\( \vec{B} \)” in your \( \vec{\mu} \times \vec{B} \) calculation.

(f) Briefly describe in words the resulting motion.

(7) HRW 29.56 A pair of loops (“Helmholtz coils”) with rather nice \( \vec{B} \)-field.

(8) **J.J. Thomson discovers the electron!**

“The electrified particle theory has, for purposes of research, a great advantage over the aetherial theory, since it is definite and its consequences can be predicted...” - J. J. Thomson (1897)

J.J. Thomson found the charge to mass ratio of the particle we now call electrons.

(a) Examine the schematic of the Thomson apparatus. The distance \( L \) is from the origin of the \( x \) axis (on the left of the capacitor) to the screen on the right. Notice the accelerating potential on the left hand side and the cross section of the capacitor. (We saw this in action last week.) Sketch the electric field lines and equipotentials in the capacitor.

(b) Show that while the particle is between the plates its \( y \) position is

\[ y(x) = \frac{1}{2} \frac{qE}{m} \frac{x^2}{v_x^2} \]

(c) Show that the position on the screen where the particle lands is given by

\[ y(L) = \frac{qE}{m} \frac{b}{v_x^2} \left( L - \frac{b}{2} \right) \]

(d) Sketch the particle’s trajectory.

(e) The cathode beam does not smear out but remains a sharply defined spot. If this beam consists of many particles, what must the host of particles have in common?

(f) The problem with the above equation for position is that it involves two unknowns \( q/m \) (what Thomson wished to find) and \( v_x \). Ugg. Thomson reduced the number of unknowns by an ingenious way of measuring \( v_x \). He introduced a magnetic field until the spot returned to its initial position on the screen. Find the direction of the magnetic field which restores the beam to its original undeflected position on the screen. Draw a free body diagram and show that the velocity is given by

\[ v_x = \frac{E}{B} \]

Since the spot remains coherent in the magnetic field, what can you infer about the velocities of the particles in the beam?

(g) If you combine all the relevant expressions the ratio of charge to mass is

\[ \frac{q}{m} = \frac{y(L) \Delta V_p}{(L - \frac{b}{2}) b B^2 d} \]

Suppose that in a given tube \( b \) and \( L \) are 4.00 cm and 20.00 cm, respectively, and that the spacing between the deflecting plates is 1.50 cm. Under a potential difference of \( \Delta V_p = 150 \) V, the deflection of the spot on the screen is observed to be 2.6 cm. The magnetic field which restores the spot to the center has a strength of \( 4.5 \times 10^{-4} \) Tesla. Calculate the velocity of the beam particles and the charge-to-mass ratio. (Remember your significant figures.)
(9) **Bonus**  Show that with $s = R$ in HRW 29.56 that the $B$-field is locally uniform and has vanishing first and second derivative, i.e. is ‘flat’ in the center of the coils. For the end result see the lab this week.

(10) **Bonus**  It would be nice to have some sense of how large a Coulomb of charge is, like saying 1 J is the energy stored in lifting a Nalgene 10 cm. What is your favorite sense of 1 C? Thanks to Josh for suggesting such a question!
(11) **Bonus** Thinker from class: If magnetic forces can do no work, what does the work?
Lab:
“$qv \times B$” or measuring $e/m$ for an electron.

A look ahead...
On to lenses - Chapter 34