Waves and Fields (PHYS 195): Week 12

Spring 2021 v1.0

Intro:

We finish our study of magnetic fields and start our brief study of light as a wave and optics.There are some wonderful (and distracting) topics related to this including rainbows, sun dogs, Galileo's original observations of Jupiter, and the "burning mirrors" of the Siege of Syracuse. But we'll see what we have time for. The key material will be (a very little) geometric optics and interference and diffraction (with phasors!).

Reading:

- **For Lab this week:** Taylor Chapter 7.
- Friday: 28-7 - 8 and HRW Chapter 33.1 but we skip pages 978 - 980.
- Monday: HRW 33.4 - 6, 34.1 - 2, 34.4, and 35.1 (If you'd like, read 34.5 on optical instruments for fun. Unfortunately we will not be able to focus on this material this spring.)
- Wednesday: HRW 35.2 - 3 Some of this will be a quite familiar!

Physics Topics:

- Light as a wave
- Optics - overview and geometric
- Ray tracing
- Huygens' principle
- Snell's law

Problems: *Due Tuesday night, April 27, at 11:59 PM on gradescope code ZR34XK*

(1) HRW 24.24

(2) The electric field in the (x, y) plane is constant and points in the $+x$ direction so that $\mathbf{E} = E_o\hat{\imath}$. Find the potential

$$V = -\int \mathbf{E} \cdot d\boldsymbol{\ell}$$

at (d, d) in two ways:

(a) First, integrate along a path from (0,0) to $(0, d)$ and then from $(0, d)$ to (d, d).
(b) Second, integrate directly from (0,0) to (d, d).
(c) Must your two answers be equal? If so, why?

In both cases assume the potential is zero at (0,0).

(3) Electric Dipoles:

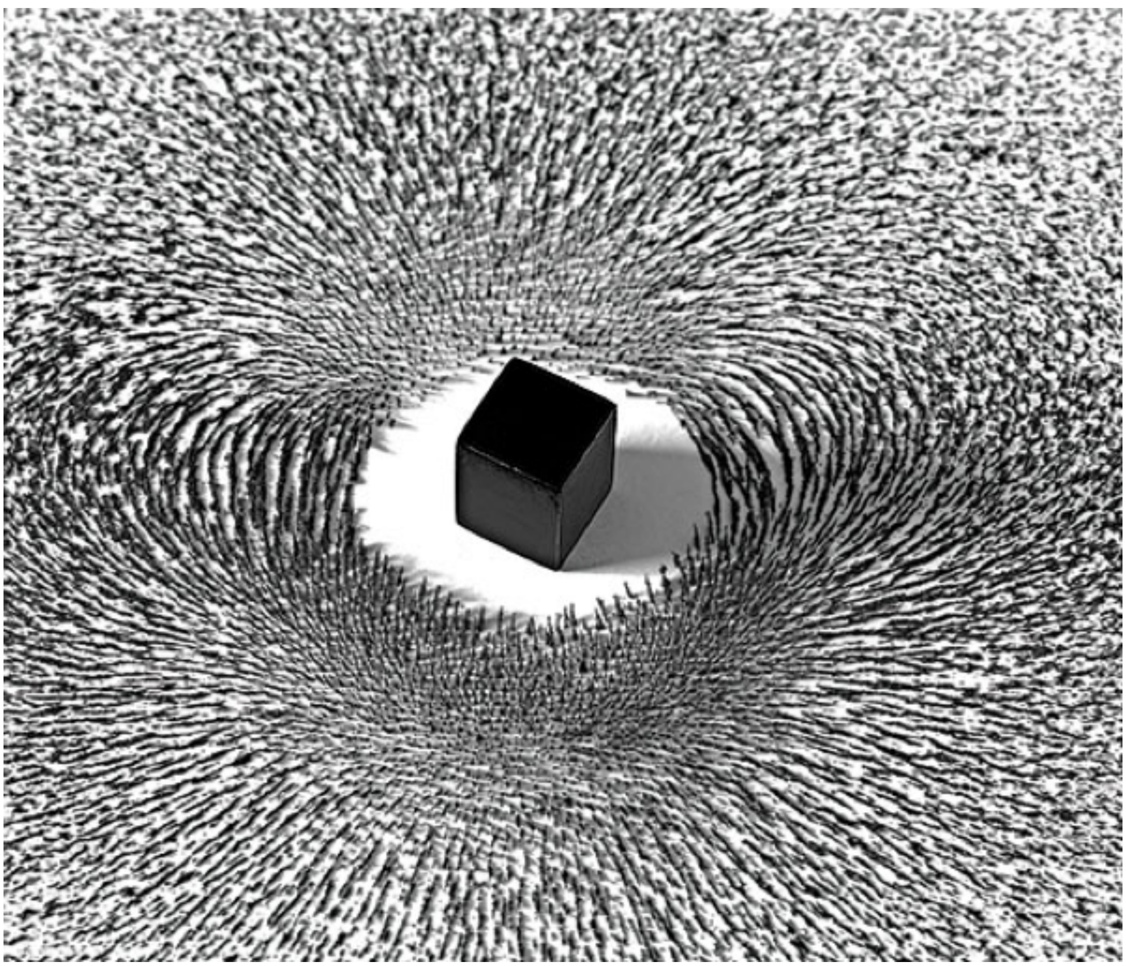
(a) Suppose you have an electric dipole with $Q = 4.20$ nC, $|\mathbf{d}| = 1.00$ cm. This dipole is placed in a uniform electric field of strength 201 N/C and is oriented 130.0 degrees from the direction of the field. What is the torque on the dipole?
(b) Find the expression for the angular frequency of oscillation of a dipole in a uniform electric field when the angle between the dipole moment and the electric field is small. Assume the dipole has a moment of inertia I and express your result in terms of E, p, and I.

(4) HRW 28.3

(5) HRW 28.17

(6) **Magnet testing** You have two identical magnets in your motor kit. This problem allows you to map out the magnetic field of these magnets. You can measure the (relative) strength of the magnetic field with a paperclip: Place a paperclip on a level surface. Hold a ruler perpendicular to the surface, and hold the magnet so that it is parallel with the surface. Slowly lower the magnet down towards the paperclip. At some point the paper clip "leaps" up to the magnet. Record the heights at which this occurs for all three orientations of the magnet. In class I will have a mapping tool available, too. Based on your results, carefully sketch the magnetic field of your magnet. Use a two dimensional sketch of the field lines showing north and south poles.

(7) Based on this photograph by Ahmed Mater titled *Magnetism IV* determine the locations of the poles. (Since we are working with iron filings we can identify the locations of the poles tell but not which is "North" and which is "South".)



(8) Find the magnetic field along the axis of a circular wire with current I and radius R (we started this one in class), showing that

$$B(z) = \frac{\mu_o I R^2}{2\left(z^2 + R^2\right)^{3/2}}$$

(9) HRW 28.56

(10) (2 pts.) **J.J. Thomson discovers the electron!**

> *"The electrified particle theory has, for purposes of research, a great advantage over the aetherial theory, since it is definite and its consequences can be predicted..."* - J. J. Thomson (1897)

J.J. Thomson found the charge to mass ratio of the particle we now call electrons.

(a) Examine the schematic of the Thomson apparatus on the next page. The distance L is from the origin of the x axis (on the left of the capacitor) to the screen on the right. Notice the accelerating potential ΔV_a on the left hand side and the cross section of the capacitor with potential ΔV_p. Sketch the electric field lines, including a couple of fringing field lines, and equipotentials of the capacitor.

(b) Show that while the particle is between the plates its y position is

$$y(x) = \frac{1}{2}\frac{qE}{m}\frac{x^2}{v_x^2}.$$

(c) Show that the position on the screen where the particle lands is given by

$$y(L) = \frac{qE}{m}\frac{b}{v_x^2}\left(L - \frac{b}{2}\right).$$

(d) Sketch the particle's trajectory.

(e) The problem with the above equation for position is that it involves *two* unknowns q/m (what Thomson wished to find) and v_x. Ugg. Thomson reduced the number of unknowns by an ingenious way of measuring v_x - the "velocity selector". He introduced a magnetic field until the spot on the screen returned to the $y = 0$ position on the screen. Find the direction of the magnetic field which restores the beam to its original undeflected position on the screen. Explain your reasoning.

(f) Draw a free body diagram and show that the velocity is given by

$$v_x = \frac{E}{B}.$$

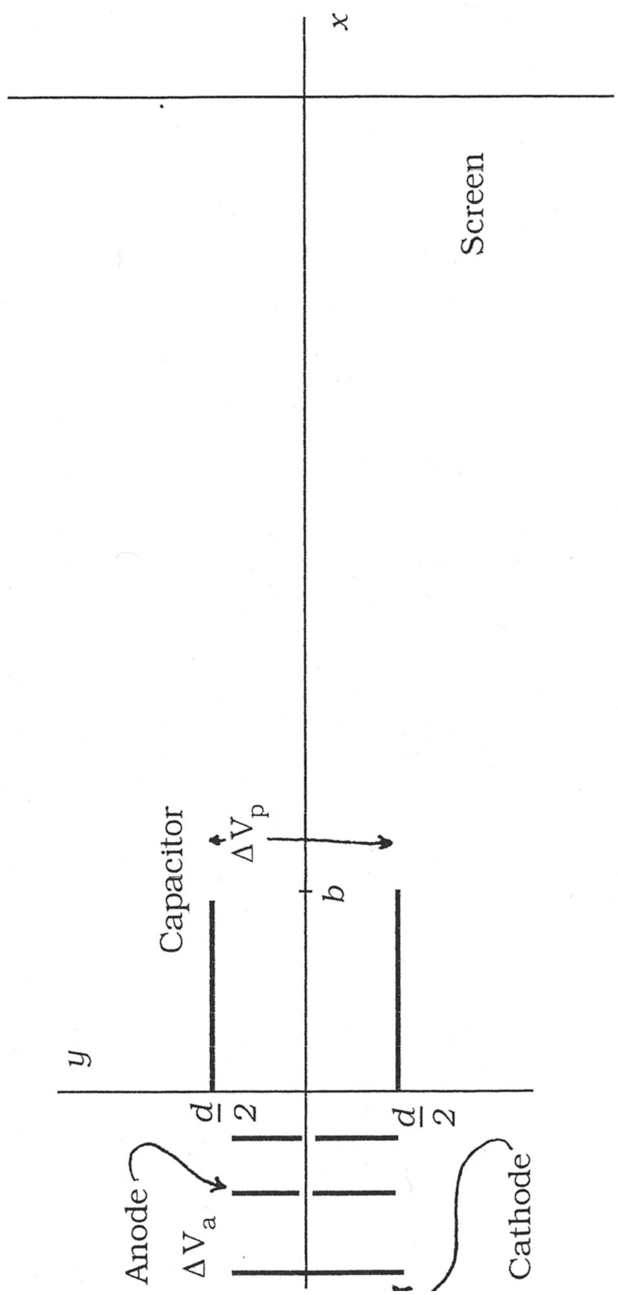
Since the spot remains well defined in the magnetic field, what can you infer about the velocities of the particles in the beam?

(g) If you combine all the relevant expressions the ratio of charge to mass is

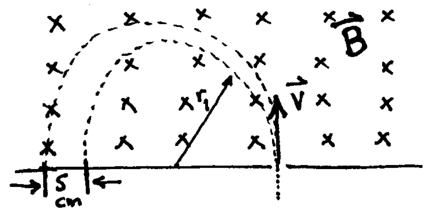
$$\frac{q}{m} = \frac{y(L)\,\Delta V_p}{\left(L - \frac{b}{2}\right) b\, B^2 d}$$

Suppose that b and L are 4.00 cm and 20.00 cm, respectively, and that the spacing between the deflecting plates is 1.50 cm. Under a potential difference of $\Delta V_p = 150$ V, the deflection of the spot on the screen is observed to be $y(L) = 2.6$ cm. The magnetic field which restores the spot to the center has a strength of 4.5 $\times 10^{-4}$ Tesla. Calculate the velocity of the beam particles and the charge-to-mass ratio.

The Thomson Experimental Setup



(11) As we saw in class, a mass spectrometer is an apparatus that can separate ionized atoms with different charge to mass ratios (q/m) using a magnetic field. A beam of ionized atoms of carbon, each with charge $+e$, enters a magnetic field perpendicular to the beam. All the ions in the beam have the same speed (arranged with the velocity selector discussed in problem 10). The ions accumulate 5.00 cm apart. The more abundant $^{12}_{6}\mathrm{C}$ isotope (which has atomic mass number 12) traces a path of smaller radius, 15.0 cm. What is the atomic mass number of the other isotope? Hint: Use $qv \times B$ to make a ratio containing the mass number.



Lab:
Snell's Law

A look ahead...
Interference and Diffraction - Chapters 35 and 36