Waves and Fields (PHYS 195): Week 2

Spring 2009 v1.0

Intro:

Our study of oscillation continues with a detailed look at "more realistic" models of oscillation. We saw on Monday how all conservative systems exhibit SHM around equilibrium. This week we discuss how to include some "damping" (a kind of energy loss that depends on velocity, like air drag) and "driving" forces which pump energy into a system.

The most significant outcome of these two changes is the phenomenon of **resonance**. *This will be the theme for the end of this week, next week, and the lab next week. The best source for this material is Kleppner and Kolenkow on e-Reserves; HRW treat this much too simply.*

This week in lab we measure g to 1 part on 10^3!

Due Monday, February 2

Reading:

- The story so far: HRW 15 sections 1-7 and Simmons about the Taylor series on e-Reserves
- Wednesday: We'll be studying different oscillators [HRW 12.7 (reading on *Jello*)], differential equations, and a little on damping. To get started on this last topic read HRW section 15.8 and Kleppner and Kolenkow 10.2 on e-Reserves.
- Friday: Kleppner and Kolenkow 10.2 on e-Reserves
- For next Monday: Kleppner and Kolenkow 10.3 on e-Reserves

Physics Topics:

- Oscillations as a universal phenomenon
- Oscillation with damping
- Driving forces and Resonance

Math Topics:

- Equations of motion (or "$\mathbf{F} = m\mathbf{a}$") as differential equations
- Solutions to differential equations

Problems:

From material in classes through Friday, January 30.

(1) Taylor 3.39
(2) For the oscillators shown in figures HRW 15-23(b) and 15-24(b) at $t = 0$, sketch position vs. time and velocity vs. time.
(3) Compute the first 4 terms of the Taylor expansion of e^{-bx} around $x = 0$. a is a constant.
(4) In class we saw that, for small angles, $\sin\theta \approx \theta$.
    (a) Use the Taylor expansion to show this.
    (b) Use the Taylor expansion again to find the angle, in degrees, at which the error in the approximation is 10 %. From this what would you conclude about what a "small angle" is?

(5) The potential energy of a pendulum of length ℓ and mass m may be written as

$$U(\theta) = mg\ell(1-\cos\theta).$$

Find the first five terms of the Taylor expansion around $\theta = 0$. (You'll find that many of the terms vanish.) Identity the term from which we derive the restoring force and the term we neglect when we make the small angle approximation. If we neglect the last term, what is the percent error in the potential energy at 50 degrees?

(6) HRW 15.20

(7) Go to the Phet spring simulator.Hang the red mass on on spring 3. Keeping the spring constant slider at its default value but moving the friction slider to the first notch, find the damping coefficient "b".

(8) What your favorite flavor of ice cream? (If you don't like ice cream substitute "bagel" for "ice cream".)

(9) Consider the differential equation

$$\frac{d^2u}{dt^2} + \alpha\left(\frac{du}{dt}\right)^2 + \beta u = 0$$

(a) What order is this equation?
(b) Is it linear?
(c) This equation could arise in Newtonian mechanics as an equation of motion. What is an interpretation of each term?

(10) Using Maple plot the potential energy function

$$U = (x^2-2)^2.$$

Locate the stable equilibria. On your plot sketch the energy for systems which deviate only slightly from these equilibria points.

(11) Universal?!? In class you heard that "SHM is universal so $U = \frac{1}{2}kx^2$" But in the Week 1 solutions (and the Phet problem above) a correct potential energy had two terms

$$U = \frac{1}{2}k(y-y_o)^2 + mgh$$

What's going on? There would be no mystery if the two expressions are the same. How can we interpret the two potential energy expressions so that they are physically equivalent?

(12) **Bonus** Extra for fun: *Jello!!* Jello sits on your lunch tray. As you wind your way to a table in Commons you notice that it jiggles (as advertised). Sitting at your table you do an experiment. You displace the rectangular shape perpendicular to the short (4 cm) side and watch the resulting oscillatory motion. Estimate the frequency of oscillation for the $4.0 \times 8.0 \times 8.0$ cm block. Assume Jello has a denisty of $\rho = 1300$ kg/m^3 and a shear modulus of 520 N/m^2.

Lab:

We'll measure our own local g to a part in 10^3 using a pendulum. The big deal of the lan is to isolate the largest sources of uncertainty and make them as small as possible...

A look ahead...

Next week we learn some systems get rid of potentially destructive energy at resonance - through waves! To look ahead see HRW Chapter 16.