Waves and Fields (PHYS 195): Week 2

Spring 2021 v1.0

Intro:

This week we will see why all (energetically) conservative systems exhibit SHM around stable equilibrium. Later in the week our study of oscillation continues with a detailed look at models of oscillation that include a drag force or air resistance.

In lab this week we explore the relation between period and amplitude in not-so-simple pendula.

Reading:

- For the week in Taylor (John Taylor's "An introduction to error analysis") Sections 3.1, 3.3 - 3.4, Chapter 4.
- Wednesday: HRW 15.3 - 15.5 and the Taylor and Simmons reading on Blackboard. These cover Taylor series and the universality of simple harmonic motion.
- Friday: K & K 10.2 on Blackboard and HRW 15 section 5
- Monday: We may be still discussing damped motion or we may be on to resonance see HRW 15.6 and K&K 10.3 on Blackboard

Physics Topics:

- Rotational motion and how it relates to SHM
- Oscillations as a universal phenomenon
- Oscillation with damping
- Q

Math Topics:

- Small angle approximation to trig functions
- Taylor's expansion
- Late time solutions and transient solutions for oscillations with damping

Problems: Due Tuesday February 16 at 11:59 on gradescope code ZR34XK

From material in classes through Friday, February 12.

(1) In some recent experiments, researchers Celestini and Raufaste of the University of Côte d'Azur, France were able to build simple devices consisting of a cylinder with a squishy lower section that jumped rather high when placed on an oscillating stage. See this APS news story. The pictures and video are cool. They found that the "optimal performance occurs when the duration of the platform's upward acceleration is approximately equal to the natural oscillation period of the vibrating soft layer [the squishy bit], so that the most energy can be transferred from platform to projectile. The best improvements were for soft layers of up to about 30% of the total cylinder height where the efficiency could be improved threefold."
   (a) If the stage oscillates at 76 Hz, what is the "duration of the upwards acceleration"?
   (b) From the above description what must be the natural angular frequency of the squishy gel? It may help to think of the gel as a spring.

(2) In our colloquium the first week of classes Dr. Prescod-Weinstein explained one reason we expect that there is "dark matter" (non-luminous matter) - the speed at which stars orbit galaxies. In this problem we'll explore this a bit further.

(a) Suppose that a galaxy consists of a central mass M and smaller mass m stars that orbit this large central mass. Assuming the stars are on a circular orbit, use Newton's law of gravitation to find the speed of the stars. This is will be an algebraic relation.
(b) Notice that this speed is a function of radius r. Sketch or plot the expected v vs. r plot for such a model galaxy.
(c) Jump over to the course webpage to view the galaxy rotation curve for M33, where the curve is superimposed on a photograph of the galaxy M33. Compare your prediction to the data. In particular, discuss the discrepancies at large radius (where the story concerns dark matter) and at small radius (which is about how central masses are clustered).
(d) How could you modify this simple galaxy model so that the velocity curves would agree at large radius?

(3) Taylor 2.19 Please show all your work.

(4) Taylor 3.40

(5) As you know a solution for SHM is

$$x(t) = x_m \cos(\omega_o t + \varphi)$$

Suppose that a whale undergoes SHM with a period of 7.0 s. At $t = 0$ the whale's position is $x(0) = 1.2$ m and velocity is $v(0) = 0.34$ m/s. Find x_m and φ and write out the specific solution for these initial conditions.

(6) Click on the Phet spring link (click here or use the class website) and select the energy panel. Set up the sim by moving the friction slider to "none" and set the orange mass to 143 g. Please keep all other settings at the default value.
(a) Using the handy stopwatch and anything else that you find useful, find the period of oscillation for the orange mass? Use the stopwatch (made available by dragging it out of the box on the right). Assume that the middle entry is in seconds.
(b) Find the spring constant.
(c) What position did the programers choose for the zero of the gravitational potential energy? (You can check you answer by moving the mass.)
(d) In our work we showed that the energy sloshed between kinetic and spring potential energy. On this sim, why is the kinetic energy (shown in green) so small?
(e) Move the friction slider up to more than "none" and restart the motion. Describe what happens. What is "E_{Therm}"?

(7) Energy in oscillations:
(a) Given the relation $F = -kx$, integrate this to find the form of the potential $U(x)$. Recall that $F = -\frac{dU}{dx}$
(b) Sketch the potential energy well (by which I mean "the potential energy"), assuming $k = 2$ N/m.
(c) Use your graph to show the turning points of the motion of a mass with 4 J of *total* energy oscillating in this well.

(8) We approximated, $\sin\theta \approx \theta$. Here we'll be more careful about what we lost in this approximation.
(a) Find the first three non-vanishing terms in the Taylor expansion of $\sin(\theta)$ around $\theta = 0$.

(b) Make a table giving the percent error in the small angle approximation $\sin\theta \approx \theta$ at 60, 50, 40, 30, 20, 10 and 5 degrees. The error is the ratio of the neglected term(s) and actual value. Working in radians is best for this last column of your table.

(c) Using your results from part (a) or (b), or another method, find the angle, in degrees, at which the error in the approximation is 5%. From this what would you conclude about what a "small angle" is?

(9) A particle of mass m moves in a potential

$$U(x) = 6x^2 - 8x^4$$

(a) What is k_{eff} equal to?

(b) For a mass $m = 3$ kg, what is the angular frequency of small oscillations around equilibrium?

(10) (Optional) Have you filled out the 195 getting to know you survey?

Lab:

We'll study the oscillations in a pendulum and deviations away from simple harmonic motion

A look ahead. . .

Next week we learn more about damping, driving and **resonance**!