Intro:

This week we continue to study how the dynamics of oscillation changes due to damping, particularly when we add driving forces and encounter the phenomenon of resonance: In driven, lightly damped systems the amplitude depends on the driving frequency and can grow quite large. We will see that oscillating systems driven at resonance can yield a breakdown of the system (for example the simple mass-on-a-spring and the Tacoma Narrows Bridge (for which the situation is more subtle - see videos and discussion on Wednesday). At resonance such a system cannot dissipate all the energy that it gains from the driving force.

In nature, driven oscillating systems frequently interact with their environment, dissipating energy through the production of waves - removing the destructive extra energy - which form our next main topic.

Due Wednesday, February 7 before class starts

Reading:

- For the week if moment of inertia is rusty HRW 10.4 and 10.5
- Monday: HRW 15.6 and K&K 10.3
- Wednesday: K&K 10.3
- For next week: HRW 16 and some material in 11

Physics Topics:

- Oscillators with light damping
- Q
- Resonance

Math Topics:

- Solutions to a driven, damped oscillator

Problems:

From material in classes through Monday, February 5.

1. HRW 15.62

2. HRW 15.95 Please assume 2 sig figs.

3. Taylor 4.22

4. More practice with the Taylor series and one Super-Handy Identity:
   (a) Using Taylor series show that
   \[(1 + x)^n \simeq 1 + nx\]
   for \(|x| \ll 1\). Hooray the first term of the binomial expansion!
(b) As an application of this Super-Handy Identity let’s find what happens to the relativistic energy of a mass at low momentum. For a particle with rest mass \( m \) the energy is

\[
E = \sqrt{p^2c^2 + m^2c^4} \quad \text{or, equivalently,} \quad E = mc^2\sqrt{1 + \frac{p^2}{m^2c^2}}
\]

If the momentum \( p \) is small compared to the rest \( m \) then what should we use for \( n \) and \( x \) in the above handy identity?
(c) Use the identity to find what the approximate energy. Physically interpret both terms. Hint: If you don’t see it, use \( p = mv \).

(5) A gymnastically inclined whale, with moment of inertia \( I \), swings gently from a pivot as shown. Use rotational dynamics (so \( \tau = I\alpha \)) to find the equation of motion. For small angles \( \theta \) find the general solution, \( \theta(t) \).

(6) Universality! The potential energy of a pendulum of length \( \ell \) and mass \( m \) may be written as

\[
U(\theta) = mg\ell(1 - \cos \theta).
\]
(a) Compute the Taylor expansion around \( \theta = 0 \). Identify the term in the Taylor expansion that gives the harmonic restoring force of a simple pendulum.
(b) If we take the small angle approximation and neglect all other terms, what is the percent error in the potential energy at 40 degrees?

(7) A glider on an air track is connected by springs to either end of the track (as in the lecture demo on Monday, Jan. 29). Assume that both springs have the same spring constant, \( k = 3.4 \) N/m, and the mass the glider is \( m = 317 \) g.
(a) Extra alert in class, you count 25 oscillations before the amplitude has decreased by one half. What do you estimate for the value of the damping constant \( \beta \)?
(b) How long should it be before the oscillations have an amplitude one quarter of the initial value?

(8) A 15.47 m pendulum takes 4 hours to come to rest. On one run it started with an amplitude of 0.902 m which decayed to 0.590 m in 43 minutes. From this data find the pendulum’s \( Q \).
There is a Foucault pendulum in the North Atrium by the stairs. It is quite long. Using a stopwatch, determine the length of the pendulum. To do this use \( g = 9.80 \pm 0.01 \text{ m/s}^2 \) (our “Official 195 Spring 2018 Measurement of \( g \) on the Hill” is not ready yet) and a suitable number of periods so that the uncertainty in the length is less than 0.5%.

A grandfather clock uses a pendulum 89.0 cm long to keep time. The clock is driven by a 2.10 kg weight that falls 72.0 cm every seven days. (Someone must wind the clock and lift this mass once a week.) The amplitude of the pendulum swing is 0.035 radians or about 2 degrees. The pendulum bob has a mass of 0.400 kg. What is the \( Q \) of the clock?

Lab:
Resonance in a mechanical system - the resonance curve, \( Q \), and comparing angular frequencies.

**A look ahead...**
Next week we turn to waves and sound. To look ahead see HRW Chapters 16