Waves and Fields (PHYS 195): Week 3

Spring 2021 v1.0

Intro:

This week we continue to study how the dynamics of oscillation changes due to damping, particularly when we add driving forces and encounter the phenomenon of **resonance***: In driven, lightly damped systems the amplitude depends on the driving frequency and can grow quite large. We will see that oscillating systems driven at resonance can yield a breakdown of the system (for example the simple mass-on-a-spring and the Tacoma Narrows Bridge (for which the situation is more subtile - see videos and discussion on Wednesday or Friday next week). At resonance such a system cannot dissipate all the energy that it gains from the driving force. In nature, driven oscillating systems frequently interact with their environment, dissipating energy through the production of waves - removing the destructive extra energy - which form our next main topic.*

Reading:

- Friday: K&K 10.3 and HRW 15.6
- Monday: I suspect we'll continue with material in K&K 10.3
- Wednesday: A start on HRW 16 and some material in Ch 11

Physics Topics:

- Oscillators with light damping
- Q
- Resonance

Math Topics:

- Solutions to a driven, damped oscillator

Problems: Due Tuesday February 23 at 11:59 PM on gradescope code ZR34XK

(1) HRW 15.62

(2) More practice with the Taylor series and one Super-Handy Identity:

(a) Using Taylor series show that

$$(1+x)^n \simeq 1 + nx$$

for $|x| \ll 1$. Hooray the first term of the binomial expansion! This is also a very useful approximation.

(b) As an application of this Super-Handy Identity let's find what happens to the relativistic energy of a mass at low momentum. For a particle with rest mass m the energy is

$$E = \sqrt{p^2c^2 + m^2c^4} \text{ or, equivalently, } E = mc^2\sqrt{1 + \frac{p^2}{m^2c^2}}$$

If the momentum p is small compared to the rest m then what should we use for n and x in the above handy identity?

(c) Use the identity to find the approximate energy. Physically interpret both terms. Hint: It you don't see it, use $p = mv$.

(3) The potential energy of a pendulum of length ℓ and mass m may be written as

$$U(\theta) = mg\ell(1 - \cos\theta).$$

(a) Find the first five terms of the Taylor expansion around $\theta = 0$. (You'll find that many of these terms vanish.) Identify the term that gives the restoring force and the term we often neglect when we make the small angle approximation.

(b) If we neglect this last term, what is the percent error in the potential energy at 30 degrees?

(c) You found in the first lab that the amplitude correction to the period was proportional to θ_m^2. The correction is

$$\frac{1}{16}\theta_m^2.$$

How large is this fractional correction to the period for the angle in part (b)? If you had used a pendulum with an amplitude of 30° in the second lab, would this have affected your result?

(4) A spring of negligible mass and constant $k = 312$ N/m is suspended vertically and a 0.511 kg pan is hung from the lower end. A baker drops a 2.57 kg lump of bread dough from rest onto the pan from a height of 0.480 m - plop! The bread dough makes a totally inelastic collision with the pan, which is at rest. Please neglect damping.

(a) What is the speed of the pan immediately following the collision?

(b) Find the equation of motion.

(c) Determine the solution $y(t)$ describing the motion.

(5) A 15.47 m pendulum takes 4 hours to come to rest. On one run it started with an amplitude of 0.902 m which decayed to 0.590 m in 43 minutes. From this data find the pendulum's Q.

(6) Go to the Phet spring simulator. Choose the "Lab" window. Set the orange mass to $m = 206$ g and the "Spring Constant 1" to 4 notches to the right of "Small". Check that the "Damping" slider is set to the first notch above "None". Hang the mass on the spring and observe the motion. Note that you have a stopwatch and ruler.

(a) Devise a procedure to find Q. You may want to write your answer up after you have successfully found Q.

(b) Determine the quality factor Q for this system.

(7) View the video showing masses on springs. The rod is clamped on one end. It is not being driven by anything outside of what you can see in the video.

(a) Describe what happens during the video.

(b) What happens to the energy in the two oscillators?

(c) What is the purpose of the paper clips on the right mass?

(8) K & K problem 10.10 Hint: The data given in the problem tells you the amount of energy stored in the pendulum and the rate in which energy is put into the pendulum. Since the amplitude doesn't change this rate must be equal to the rate of energy dissipated.

(9) **Optional** Extra for fun (and participation points): We discovered that a pendulum has a period that depends on the amplitude of the swing, which is easy to see at larger amplitudes. This is not ideal for clocks; we want them to tick evenly, and not depend on the amplitude of the swing. So a circular path is *not* the correct one for an amplitude-independent period (or an *isochronous* pendulum). What is the correct trajectory for the so that there are no amplitude corrections? How would a 17th century you make a pendulum that follows this path? Please write up your solutions to this one separately and hand it in directly to me.

(10) **Bonus** Extra for fun (and participation points) A question that came up in lab: How far above the surface of earth would you have to go to detect a part in 10^3 change in our value of g? Here the difference is between your g measured with care in the lab and the value of "$g(h)$" at some height h above the surface of Earth, as determined by Newton's law of gravitation.

Lab:

Resonance in a mechanical system - the resonance curve, Q, and comparing angular frequencies.

A look ahead. . .

Next week we turn to waves and sound. To look ahead see HRW Chapters 16