Waves and Fields (PHYS 195): Week 4

Spring 2009 v1.0

Intro:

We'll start our study of waves with transverse waves living in one dimension (i.e. on a string). Beginning with basic properties we'll then find the equation of motion for waves on a string - the **wave equation***. We will flesh out the study of waves by studying the solutions to the wave equation, including harmonics, standing waves, the principle of superposition, boundary conditions, and Fourier series.*

Due Monday, February 16

Reading: HRW Chapter 16

- Wednesday: HRW 16.1 - 16.6, and pages 425 - 6
- Friday: HRW 16.7 - 16.11 (we may not discuss phasors...)
- Monday: HRW 16.12 - 16.13
- For next Wednesday: HRW 17.1 -17.3

Physics Topics:

- Resonance
- Transverse waves
- Standing waves
- Waves carry energy and momentum!
- Harmonics

Math Topics:

- Partial derivatives
- Wave equation (in 1 dimension)
- Boundary consitions
- Superposition
- Fourier series

Problems:

From material in classes through Friday, February 13.

(1) Using Maple, plot the resonance curve (amplitude^2 vs. driving ω) from $\omega = 0$ to $\omega = 2\omega_0$ for a lightly damped system with $Q = 35$. To do this first re-express the amplitude in terms of Q and ω, the driving frequency. Since your horizontal axis will be units of $\omega)o$ you can set $\omega_o = 1$. Submit a printout of the Maple output with your solutions.

(2) Let $z = x^3y + e^{xy}$.
   (a) Compute $\frac{\partial z}{\partial x}$.
   (b) Compute $\frac{\partial z}{\partial y}$.
   (c) Compute $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$. Comment on this.
   (d) Compute $\frac{\partial^2 z}{\partial x^2}$.

(3) A lightly damped system with a natural angular frequency of 7.7 1/s and a Q of 50 is driven at an angular frequency of 9.0 rad/s. What is the phase difference between the driving force and the system?

(4) Open the wave on a string Phet simulation.
  (a) Give the end of the string a good wiggle. Describe what happens on the right end of the string for each of the three possible boundary conditions: Fixed End, Loose End, and No End. Feel free to adjust the damping so you can more clearly see the motion.
  (b) Fix the right end, decrease the damping to zero, and add the driving force with the "Oscillate" button. Wait a little while. What is happening?

(5) What kind of waves did the wind excite on the Tacoma Narrows Bridge?

(6) Suppose you have a string with linear mass density 0.100 kg/m and a wave,

$$y(x,t) = 0.050\ \sin(6.00x + 12.0t),$$

propagating on this string. y and x are measured in meters and t is measured in seconds.
  (a) Is this a right moving or left moving wave?
  (b) Find (or record) k, ω, λ, f, and v.
  (c) What are the minimum and maximum speeds of the string?
  (d) Find the average rate of energy transport by this wave.

(7) A sinusoidal wave traveling on a string in the negative x direction has amplitude 1.00 cm, wavelength 3.00 cm, and frequency 200 Hz. At $t = 0$, the particle of string at $x = 0$ is displaced a distance 0.80 cm above the origin and is moving upward. (a.) Sketch the shape of the wave at $t = 0$. (b.) Determine the function $y(x,t)$ describing the wave.

(8) Show that, for any "suitably nice" (one you differentiate twice) function f , $y(x,t) = f(kx+\omega t)$ satisfies the wave equation if $v = \omega/k$.

(9) **Bonus** We derived the wave equation for no $g = 0$ - no gravity. Re-derive the wave equation, or its generalization, for the case that there is gravity. *I will not post a solution to this problem. You have until the end of the semester to complete it. Please submit this bonus solution directly to me. Spectacular solutions will receive more points.*

Lab:

Waves on a String: Investigating harmonics, speed of propagation, and other properties of 1D waves. It will be useful to have read 16.12-16.13 before lab.

A look ahead. . .

Next week we turn to more on waves and sound. To look ahead see Chapter 17.