Intro:
This week we finish our study of damped driven oscillators - and resonance. Next week we’ll start on our study of waves, beginning with transverse waves on a string. We will have one math interlude complex numbers.

Reading: HRW Chapter 16 and Taylor sections 3.5 - 3.11
- Monday: Kleppner and Kolenkow Section 10.3 and Note 10.1 and HRW 15.6 (The treatment of resonance in HRW is superficial but still worth a read.)
- Wednesday: Kleppner and Kolenkow Section 10.3 and Note 10.2
- Friday: HRW 16.1 - 16.8

Physics Topics:
- Resonance
- The dependence of amplitude and phase of driving angular frequency $\omega_D$
- Waves - equation of motion

Math Topics:
- Complex numbers
- Wave equation (in 1 dimension)

Problems:
Due Friday, February 14 before class starts

1. Hop over to the [Phet spring simulator](#) Hang the red mass on a spring 3. Keeping the spring constant slider at its default value but moving the friction slider to the first notch above “none”, determine a procedure to find the damping coefficient “b”. Make any measurements you need and then find $b$. In your solution describe your methods and show your calculations. No need to work through the uncertainty analysis here.

2. A 13.47 m pendulum (like the one hanging in the North Atrium) takes 4.1 hours to come to rest. On one run it started with an amplitude of 0.902 m which decayed to 0.590 m in 43 minutes. From this data find the pendulum’s $Q$.

3. A carefully suspended whale hangs on a steel wire. It is given a gentle twist and swings around the steel wire. Assume that there is no damping.

(a) Using $\tau = -\kappa \theta$, where $\theta$ is the angular deflection, find the equation of motion for $\theta$ starting from $\tau = I \alpha$. Assume that the moment of inertia of the whale is $I$.

(b) What type of motion is this?
(4) In lab last week you had a physical pendulum built from a ring, string, and ball. Given your measurements in lab, find the moment of inertia $I$ of your pendulum. If you cannot find your data then after nashing your teeth in distress use these numbers: Ring diameter 2.42 cm and mass 1.0 g (for Al) or 6.8 g (for brass); String length 1.54 m and mass 2.6 g; and Ball radius 1.27 cm and mass 72.0 g.

(5) Kleppner and Kolenkow problem 10.2

(6) Kleppner and Kolenkow problem 10.10 Hint: The data given in the problem tells you the amount of energy stored in the pendulum and the rate in which energy is put into the pendulum. Since the amplitude doesn’t change this rate must be equal to the rate of energy dissipated.

(7) Equations!
   (a) Write down the equation of motion for a simple harmonic oscillator
   (b) Circle the angular frequency in the equation.
   (c) Write the general solution for SHM
   (d) Write down the equation of motion for a damped, driven harmonic oscillator
   (e) Write the general solution for a lightly damped harmonic oscillator

(8) Please correct the following statement: “If you drive a damped harmonic oscillator at a driving frequency other than $\omega_0$ then at late times it oscillates at $\omega_0$, but the amplitude of the response depends on how close the driving frequency is to the resonance frequency.”

(9) Many modern towers contain huge damped oscillator systems designed to oscillate at the same frequency as the buildings themselves. For instance the Taipei 101 tower has a 728 ton pendulum built into the 90 - 87th floors. You can view a video of the relative motion during an earthquake on this same web page.
   (a) Why are these damped oscillator systems built?
   (b) In the video the period of oscillation is about 7.1 s. Assuming a lightly damped simple pendulum, find the natural angular frequency.
   (c) Suppose that in 10 periods the amplitude of oscillation is reduced from the maximum of 1.4 m to 0.80 m. Find the effective damping coefficient $b$.

(10) Phase shift: Using two or three rubber bands and a mug (or other suitable mass) build a lightly damped oscillator. Connect the bands to make a long “spring”.
   (a) Determine the natural frequency of your oscillator.
   (b) Bounce your mug and observe the phase when you drive it below the natural frequency, at the natural frequency, and above the natural frequency. Make a sketch of phase $\phi$ versus driving frequency $\omega_D$ for your system.

Lab:
Resonance in a mechanical system - the resonance curve, $Q$, and comparing angular frequencies.

A look ahead...